

OSMU23

Octonions, Standard Model, and Unification

February 24 – December 15, 2023

an online lecture series presented by

Archive Trust for Research in Mathematical Sciences and Philosophy

www.archmathsci.org (Oxford)

and

Inter-University Centre for Astronomy and Astrophysics

www.iucaa.in (Pune)

Motivation:

Several physicists believe that quantum theory, as presently understood, is likely to be approximate, not exact. In particular, our understanding of space-time structure in quantum field theory might be in for an overhaul. Twistor theory, trace dynamics, and the number system known as the octonions, will possibly play an important role in this improved understanding of space-time. Octonions have also emerged significantly in recent years in the investigation of elementary particles, the standard model, and its unification with gravitation. The present lecture series presents ongoing research from eminent physicists and mathematicians working in these fields. Each session will be two hours long, with one hour for the talk, followed by an hour long question-answer session.

Hosts: Tejinder P. Singh (IUCAA, Pune) and Michael Wright (ATRMSP, Oxford)

Advisors: Latham Boyle (Perimeter), Felix Finster (Regensburg), Cohl Furey (Berlin), Niels Gresnigt (Xi'an Jiaotong-Liverpool University), Basil Hiley (University College London), Roger Penrose (Oxford)

SPEAKERS

Roger Penrose (*Oxford*)
Basil Hiley (*University College London*)
Albert Schwarz (*UC Davis*)
Niels Gresnigt (*Xi'an Jiaotong-Liverpool University*)
Felix Finster (*University of Regensburg*)
Cohl Furey (*Humboldt University, Berlin*)
Stephen Adler (*IAS, Princeton*)
John Barrett (*University of Nottingham*)
Tejinder P. Singh (*IUCAA, Pune*)
Ashutosh Kotwal (*Duke University*)
Angelo Bassi (*University of Trieste*)
Hendrik Ulbricht (*University of Southampton*)
Tevian Dray (*Oregon State University*)
Latham Boyle (*Perimeter Institute*)
David Chester (*Quantum Gravity Research*)
Corinne Manogue (*Oregon State University*)
Anthony Lasenby (*Cavendish Laboratory, Cambridge*)
Matej Pavsic (*J. Stefan Institute, Ljubljana*)
Robert Wilson (*Queen Mary University of London*)
Kirill Krasnov (*University of Nottingham*)
Ivan Todorov (*Bulgarian Academy of Sciences*)

SCHEDULE

All talks will be on Fridays, once every two weeks, at 4 pm London time

February 24, 2023	Roger Penrose <i>Basic twistor theory, bitwistors, and Split- octonions</i> (Abstract on page 6)	Video Recording
March 10, 2023	Basil Hiley <i>Dyson's three-fold way, quantum processes and the split-quaternions</i> (Abstract on page 7)	Video Recording

March 24, 2023	Albert Schwarz <i>Jordan algebras, quantum theory, and particle physics</i> (Abstract on page 8)	Video Recording
April 7, 2023	Niels Gresnigt <i>Cayley-Dickson algebras, braids, and the Standard Model</i> (Abstract on page 9)	Video Recording
April 21, 2023	Felix Finster <i>The theory of causal fermions</i> (Abstract on page 10)	Video Recording
Postponed from May 5 to: May 12, 2023	Cohl Furey <i>Bott periodic particle physics</i>	Confirmed
May 19, 2023	Stephen Adler <i>Trace dynamics and implications for my work of the last two decades</i> (Synopsis on page 12)	Confirmed
June 2, 2023	John Barrett <i>Non-commutative geometry and the standard model</i> (Abstract on p. 13)	Confirmed
June 16, 2023	Tejinder P. Singh <i>Trace dynamics, octonions, and unification</i> (Summary on page 14)	Confirmed
June 30, 2023	Ashutosh Kotwal <i>Particle physics: what we know and what we do not know</i>	Confirmed

July 14, 2023	Angelo Bassi <i>Is quantum theory exact or approximate?</i>	Confirmed
July 28, 2023	Hendrik Ulbricht <i>Experimental tests of quantum mechanics for large systems</i>	Confirmed
September 1, 2023	Tevian Dray TBA	Confirmed
September 15, 2023	Latham Boyle TBA	Confirmed
September 29, 2023	David Chester <i>The exceptional Lie group E_8</i>	Confirmed
October 13, 2023	Corinne Manogue TBA	Confirmed
October 27, 2023	Anthony Lasenby <i>Geometric algebra, octonions, and the standard model</i>	Confirmed
November 3, 2023	Matej Pavsic TBA	Confirmed
November 10, 2023	Robert Wilson <i>Einstein's nightmare: is physics fundamentally discrete?</i>	Confirmed
November 24, 2023	Kirill Krasnov TBA	Confirmed
December 8, 2023	Ivan Todorov TBA	Confirmed

ZOOM link

Topic: OSMU 23 (Octonions Standard Model + Unification Meeting)

Join Zoom Meeting

<https://us06web.zoom.us/j/89956931861?pwd=YkN4bUg0NE1STFIRd0MwY0NUVjcWdz09>

Meeting ID: 899 5693 1861

Passcode: 451794

The hosts request that questions from the audience may kindly be postponed to the discussion session. Questions can also be left in the chat box. Thank you. We look forward to having you with us in this lecture series.

Talks and discussion sessions will be recorded. We hope to make the recording available within two weeks after the lecture. URL www.archmathsci.org

Video recordings are available at

<https://www.youtube.com/@archivetrustforresearch/featured>

Communications Partner

Curt Jaimungal

Podcast Host

Theories of Everything <https://www.youtube.com/TheoriesOfEverything>

Abstracts of talks are on the following pages.

TITLE: Basic Twistor Theory, Bi-twistors, and Split-octonions.

Roger Penrose, 2023

Abstract:

Twistor theory was introduced in the mid 1960s as an approach to combining quantum theory with space-time structure. A driving force behind the introduction of Twistor Theory was to combine the quantum-field theoretic requirement of positive frequency with the structure of space-time. In order to achieve this, the notion of twistor space was introduced to codify the structure of space-time in a way which related it to the splitting of the twistor space into two halves, one representing positive frequency, and the other representing negative frequency. Standard twistor theory involves a complex projective 3-space PT which naturally divides into two halves PT^+ and PT^- , joined by their common 5-real-dimensional boundary PN . The points of the space PN represent light rays in Minkowski space-time. However, this splitting has two quite different basic physical interpretations, namely positive/negative helicity and positive/negative frequency, which ought not to be confused in the formalism, and the notion of “bi-twistors” is introduced to resolve this issue. It is found that quantized bi-twistors have a previously unnoticed G_2^* structure, which enables the split-octonion algebra to be directly formulated in terms of quantized bi-twistors, once the appropriate complex structure is incorporated.

Dyson's three-fold way, quantum processes and the split-quaternions. B. J.

Hiley.

My talk is based on a long neglected paper by Bryce DeWitt in which he shows that in order to write the Kline-Gordon equation in a pseudo-Riemannian space in a fully relativistic covariant manner, the quantum Hamiltonian must contain an additional energy term, $\hbar^2 Q$ which he calls the "quantum potential". This term is manifested as a scalar curvature term in the geodesic. As is well known, a similar potential appears in the real part of the Schrödinger equation under polar decomposition of the wave function in Bohm's approach. Indeed a relation between the two potentials has been established.

If we include the arguments that Dyson outlines in his 1962 paper "The three-fold way" we find that the imaginary units demanded by quantum mechanics arise, not through the complex field, but through the basis elements (i, j and k) of the real quaternions. This provides a rich structure allowing a purely algebraic approach to quantum phenomena rather than the usual Hilbert space formalism and no collapse of the wave function is necessary. The geometrical aspects of the dynamics emerge from the split quaternions, which combine the orthogonal and symplectic structure in a new way. This provides a link between the dynamical mass and the rest mass opening up the possibility of new ways of exploring the rest mass problem. If time allows I will show the relationship between the Dirac spinor and the semi-spinor of the conformal Clifford, the twistor.

1. DeWitt, B. S., Point Transformations in Quantum Mechanics, Phys. Rev. 85 (1952) 653-661.
2. Freeman J. Dyson, The Threefold Way. Algebraic Structure of Symmetry Groups and Ensembles in Quantum Mechanics, J. Math. Phys. **3**, (1962) 1199-1215
3. Bohm, D. and Hiley, B. J., The Undivided Universe: An Ontological Interpretation of Quantum Mechanics, Routledge, London, (1993).
4. Bohm D. J. and Hiley, B. J., Generalisation of the Twistor to Clifford Algebras as a Basis for Geometry, Revista Brasileira de Fisica, Vol. Especial Os 70 anos de Mario Schönberg, 1-26, (1984).

Albert Schwarz March 24, 2023

Title: Jordan algebras, quantum theory, and particle physics

Abstract

In the geometric approach to quantum theory that was suggested recently in my papers we are taking as a starting point the set of states (a bounded convex set of normalized states or a convex cone of not necessarily normalized states). In this framework one can prove a generalization of decoherence and use this proof to derive probabilities from the first principles (from random interaction with the environment). Fixing an action of time and spatial translations on the cone of states one can define a notion of elementary excitation of translation invariant state and of scattering of elementary excitations. (Elementary excitations of ground state should be identified with particles, in general case elementary excitations should be regarded as quasiparticles.)

The geometric approach allows us to construct physical theories corresponding to Jordan algebras. This construction is based on the remark that one can define a cone of positive elements of a Jordan algebra. (In Jordan Banach algebras this is a homogeneous cone consisting of all squares.) In the talk I will analyze elementary excitations and scattering in physical theories based on Jordan algebras.

Cayley-Dickson algebras, braids, and the Standard Model

Niels Gresnigt April 7, 2023

The initial part of this talk will review the applications of the quaternions and octonions to describing the particle content and symmetries of the Standard Model. Most of this review will be based on the works of Dixon and Furey. In the second half of the talk I will discuss some of my own ideas relating to algebraic unification.

It is proposed that going beyond the division algebras, and considering the Cayley-Dickson algebra of sedenions provides a natural means of generalizing existing results from a single generation to exactly three. The additional S_3 automorphisms of the sedenions might then be interpreted as a flavor symmetry, incorporating neutrino oscillations and quark mixing.

I will finish by discussing a curious association between algebraic unification based on the division algebras, and a topological representation of fermions in terms of simple braids. The structure of the Clifford algebras generated from the quaternions and octonions in this case provides a justification for many of the ad hoc assumptions in the topological model.

Throughout the presentation, the focus will primarily be on the underlying ideas, recognizing that many of the ideas presented are still a work in progress. The intent is to generate stimulating discussion.

Title: Causal fermion systems and octonions

Felix Finster April 21, 2023

Abstract:

The theory of causal fermion systems is an approach to describe fundamental physics. It gives quantum mechanics, general relativity and quantum field theory as limiting cases and is therefore a candidate for a unified physical theory. Moreover, causal fermion systems provide a general framework for modelling and analyzing non-smooth spacetime structures. The dynamics of a causal fermion system is described by a nonlinear variational principle, the causal action principle.

The aim of the talk is to give a simple introduction, with an emphasis on the underlying concepts. At the end of the talk, I will explain how octonions arise naturally by acting on the vacuum measure.

Stephen L. Adler, May 19, 2023

Title: Trace dynamics and implications for my work of the last two decades

Synopsis: Trace dynamics is a Lagrangian theory of completely non-commuting variables q (generalized coordinate) and p (generalized momentum), which can be bosonic (Grassmann even) or fermionic (Grassmann odd). The Lagrangian is formed as a trace over polynomials in the q 's and p 's, and validity of cyclic permutation under the trace is assumed. From a trace action principle one directly finds the equations of motion, in Lagrangian form, and forming a trace Hamiltonian by a Legendre transform one finds the equations of motion in Hamiltonian form. When the trace Lagrangian is assumed invariant under global unitary transformation of the q 's and p 's, which requires that it contain no noncommuting coefficients, then there is a conserved Noether charge, given by the sum over bosonic variables of the commutators $[q,p]$, minus the sum over fermionic variables of the anticommutators $\{q,p\}$. Forming a canonical ensemble, and taking averages over this ensemble, one finds that equipartition arguments give an emergent structure with the form of quantum field theory, provided the underlying theory develops a scale hierarchy in which variations of the trace Hamiltonian decouple, and provided that the numbers of bosonic and fermionic variables are essentially equal. In this theory the ensemble average of the conserved Noether charge plays the role of the complex unit i . The general framework is described in my 2004 Cambridge University Press book "Quantum Theory as an Emergent Phenomenon".

Much of my work since 2004 has been motivated by ideas coming from this book: (i) Thermodynamics has Brownian motion corrections, which suggests that these play a role in quantum measurements. This in turn suggests a connection at the phenomenological level with the Ghirardi-Rimini-Weber-Pearle model of objective reduction, with the aim of confronting experiment. (ii) The effective complex unit coming from trace dynamics is a 2×2 real matrix representation, which acts as i on one sector and $-i$ on the other. Perhaps "dark matter" is a $-i$ sector coupled to the standard model i sector only by gravitation. (iii) The equality of bosonic and fermionic variable numbers in trace dynamics suggests that extending the standard model with exact supersymmetry may be too strong a requirement. Perhaps one should only impose the weaker condition of boson-fermion balance, suggesting alternative unification models. (iv) The underlying trace dynamics theory naturally admits a Weyl scaling invariance of the metric, and this suggests that the non-derivative part of the induced gravitational action is also a Weyl scaling invariant, giving a novel model for "dark energy". In gravity including this action, black holes no longer have horizons; near and inside the nominal horizon, the time-time component of the metric becomes very small but remains positive, never changing sign. (v) This motivates exploring horizonless "dynamical gravastar" models in which radii where structural changes occur come just from the relativistic Tolman-Oppenheimer-

Volkoff equations with an assumed jump in the equation of state at high pressure to one with pressure plus energy density approximately zero. Continuity properties of the TOV equations require the jump to be in the energy density, not in the pressure as assumed in prior work. This model may describe the structure of astrophysical "black holes". Since material getting in can also emerge, such objects could play a role in galaxy formation, leading to a simple formula for the disk galaxy scale radius, depending only on properties of atomic hydrogen.

John Barrett

Friday, June 2, 2023

Title: Non-commutative geometry and the standard model

Abstract: The talk will discuss the spectral triple approach to non-commutative geometry as applied to the standard model of particle physics. The aim is to explain two things: 1. the non-commutative algebraic structure of the standard model, and 2. the spectral action principle. Most of the talk is an introduction to well-established ideas from the 2000s and before, but there may be a few remarks on more recent developments.

Trace dynamics, octonions, and unification

There ought to exist a reformulation of quantum field theory in which evolution does not depend on classical time. This is because the principle of quantum linear superposition does not permit a point structure for spacetime, except in the approximation that the universe is dominated by macroscopic classical bodies. These remarks are true irrespective of the energy scale under consideration. The fact that the Dirac operator is also the gradient operator on quaternionic space is already an indicator that space-time noncommutativity plays a key role in fermion dynamics, at all energies.

We describe our group's ongoing research program to develop the aforesaid reformulation of quantum theory, which also opens the route to quantum gravity and its unification with the standard model of particle physics. Instead of real numbers, split biquaternions and split bioctonions are used to construct the space inhabited by fermions and bosons. These obey a matrix-valued Lagrangian dynamics (generalized trace dynamics). The role of time parameter is played by the Connes time, a feature of non-commutative geometries resulting from the Tomita-Takesaki theory. The Lagrangian of the theory is based on the Chamseddine-Connes spectral action principle and is essentially trace of the squared Dirac operator which also includes Yang-Mills fields. The use of split bioctonions permits chiral fermions to arise, including three sterile neutrinos. The associated quadratic form describes two copies of 4D spacetime overlaid by two vector bundles. The dynamics is deterministic and non-unitary, though norm preserving; quantum theory emerges from coarse-graining this underlying dynamics. The Lagrangian is assumed to obey an $E_8 \times E_8$ symmetry and describes a left-right symmetric extension of the standard model. The assumed branching $E_8 \rightarrow SU(3) \times E_6$ for each of the E_8 leads to the emergence of the standard model, along with an $SU(2)_R$ pre-gravitation symmetry, and two new forces, $SU(3)_{\text{grav}}$ and $U(1)_{\text{grav}}$.

The particle content of the theory can also be deduced from Clifford algebras possessing the associated dynamical symmetries. It is known from earlier research that $Cl(6)$, associated with complex octonions, describes one generation of standard model fermions under the unbroken symmetry $SU(3)_c \times U(1)_{\text{em}}$. We generalize this to show that $Cl(7)$, associated with complex split bioctonions, and being a direct sum of two copies of $Cl(6)$, describes one generation of chiral fermions under $SU(3)_c \times U(1)_{\text{em}} \times SU(3)_{\text{grav}} \times U(1)_{\text{grav}}$. Whereas the quantum number associated with $U(1)_{\text{em}}$ is electric charge, the quantum number associated with $U(1)_{\text{grav}}$ is square root of mass (in Planck mass units). Each of the $Cl(6)$ is extended by a $Cl(2)$ to include the $SU(2)_L$ and $SU(2)_R$ symmetries, giving rise to two copies of $Cl(8)$. Between them they account for three fermion generations, and the resulting $Cl(9)$ algebra appears to be the algebra of unification. We predict twelve new gauge bosons associated with the dark

(gravitational) sector $SU(3)_{\text{grav}} \times SU(2)_R \times U(1)_{\text{grav}}$, as also a BSM charged Higgs. This Higgs assigns electric charge to the right-handed fermions, whereas the standard model Higgs assigns mass to the left-handed fermions, upon electroweak symmetry breaking, which in this theory is also left-right symmetry breaking.

Our investigations suggest that the electroweak scale is also the scale for the unification of the standard model with gravitation. Prior to this scale the universe undergoes an inflationary phase which resets the Planck scale from 10^{19} GeV to the electroweak scale. Symmetry breaking gives rise to the emergence of $U(1)_{\text{em}}$ as well as $U(1)_{\text{grav}}$, along with their associated quantum numbers – electric charge and square-root mass, respectively. The exceptional Jordan algebra (EJA) of 3×3 Hermitian matrices with octonionic entries describes a family of three generations of left-handed fermions (electric charge eigenstates) / right-handed fermions (square root mass eigenstates), once the value of the electric charge / square-root mass is specified. The characteristic equation of the EJA is a cubic, and its eigenvalues carry information about the values of fundamental constants of the standard model. In particular, by expressing square-root mass eigenstates as superpositions of electric charge eigenstates, we deduce the observed mass ratios of charged fermions, and the CKM matrix parameters, in addition to the low energy fine structure constant and the weak mixing angle.

Quantum systems do not live on spacetime, but on split bioctonionic space. We suggest that the Ghirardi-Rimini-Weber-Pearle (GRWP) process of spontaneous localization leads to the emergence of our 4D classical universe, and quantum theory on a classical spacetime background overlaid with a vector bundle describing the unbroken symmetries of strong force and electromagnetism. The weak force appears to be a spacetime symmetry associated with a second 4D spacetime which has microscopic dimensions but is also a part of our universe, and which might be acting as a (hitherto undetected) channel for EPR communication. Along this channel, communication obeys relativistic causality, but any two points of our universe are only a microscopic distance apart (of the order of the range of the weak force).

We end by outlining open questions, and the considerable work which remains to be done, to carry this program of unification to completion.

The research articles associated with this program are listed at <https://www.tifr.res.in/~tpsingh>