# $\mathbb{O}$*SMU23*

**Octonions, Standard Model, and Unification**
February 24 – December 15, 2023

*an online lecture series presented by*

**Archive Trust for Research in Mathematical Sciences and Philosophy**

www.archmathsci.org (Oxford)

and

**Inter-University Centre for Astronomy and Astrophysics**

www.iucaa.in (Pune)

Motivation:

Several physicists believe that quantum theory, as presently understood, is likely to be approximate, not exact. In particular, our understanding of space-time structure in quantum field theory might be in for an overhaul. Twistor theory, trace dynamics, and the number system known as the octonions, will possibly play an important role in this improved understanding of space-time. Octonions have also emerged significantly in recent years in the investigation of elementary particles, the standard model, and its unification with gravitation. The present lecture series presents ongoing research from eminent physicists and mathematicians working in these fields. Each session will be two hours long, with one hour for the talk, followed by an hour long question-answer session.

Hosts: Tejinder P. Singh (IUCAA, Pune) and Michael Wright (ATRMSP, Oxford)

Advisors: Latham Boyle (Perimeter), Felix Finster (Regensburg), Cohl Furey (Berlin), Niels Gresnigt (Xi'an Jiaotong-Liverpool University), Basil Hiley (University College London), Roger Penrose (Oxford)

**SPEAKERS**

Roger Penrose (*Oxford*)
Basil Hiley (*University College London*)
Albert Schwarz (*UC Davis)*
Niels Gresnigt (*Xi'an Jiaotong-Liverpool University*)
Felix Finster (*University of Regensburg*)
Cohl Furey *(Humboldt University, Berlin)*
Stephen Adler (*IAS, Princeton)*
John Barrett (*University of Nottingham*)
Tejinder P. Singh (*IUCAA, Pune*)
Ashutosh Kotwal (*Duke University*)
Angelo Bassi (*University of Trieste*)
Hendrik Ulbricht (*University of Southampton*)
Tevian Dray (*Oregon State University*)
Latham Boyle (*Perimeter Institute*)
David Chester (*Quantum Gravity Research*)
Corinne Manogue (*Oregon State University*)
Anthony Lasenby (*Cavendish Laboratory, Cambridge*)
Matej Pavsic (*J. Stefan Institute, Ljubljana*)
Robert Wilson (*Queen Mary University of London*)
Kirill Krasnov (*University of Nottingham)*
Ivan Todorov (*Bulgarian Academy of Sciences*)

**SCHEDULE**

All talks will be on Fridays, once every two weeks, at 4 pm London time

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|---|---|---|
| February 24, 2023 | Roger Penrose<br>*Basic twistor theory, bi-twistors, and Split-octonions*<br>(Abstract on page 6) | Video Recording |
| March 10, 2023 | Basil Hiley<br>*Dyson's three-fold way, quantum processes and the split-quaternions*<br>(Abstract on page 7) | Video Recording |
| March 24, 2023 | Albert Schwarz<br>*Jordan algebras, quantum theory, and particle physics*<br>(Abstract on page 8) | Confirmed |
| April 7, 2023 | Niels Gresnigt<br>*Cayley-Dickson algebras, braids, and the Standard Model*<br>(Abstract on page 9) | Confirmed |
| April 21, 2023 | Felix Finster<br>*The theory of causal fermions* | Confirmed |
| May 5, 2023 | Cohl Furey<br>*Octonions and particle physics* | Confirmed |
| May 19, 2023 | Stephen Adler<br>*Trace dynamics and implications for my work of the last two decades*<br>(Synopsis on page 10) | Confirmed |
| June 2, 2023 | John Barrett<br>*The spectral action principle in non-commutative geometry* | Confirmed |
| June 16, 2023 | Tejinder P. Singh<br>*Trace dynamics, octonions, and unification* | Confirmed |

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| June 30, 2023 | Ashutosh Kotwal<br>*Particle physics: what we know and what we do not know* | Confirmed |
| July 14, 2023 | Angelo Bassi<br>*Is quantum theory exact or approximate?* | Confirmed |
| July 28, 2023 | Hendrik Ulbricht<br>*Experimental tests of quantum mechanics for large systems* | Confirmed |
| September 1, 2023 | Tevian Dray<br>TBA | Confirmed |
| September 15, 2023 | Latham Boyle<br>TBA | Confirmed |
| September 29, 2023 | David Chester<br>*The exceptional Lie group E_8* | Confirmed |
| October 13, 2023 | Corinne Manogue<br>TBA | Confirmed |
| October 27, 2023 | Anthony Lasenby<br>*Geometric algebra, octonions, and the standard model* | Confirmed |
| November 3, 2023 | Matej Pavsic<br>*TBA* | Confirmed |
| November 10, 2023 | Robert Wilson<br>*Einstein's nightmare: is physics fundamentally discrete?* | Confirmed |
| November 24, 2023 | Kirill Krasnov<br>TBA | Confirmed |
| December 8, 2023 | Ivan Todorov<br>TBA | Confirmed |

## ZOOM link

Topic: OSMU 23 ( Octonions Standard Model + Unification Meeting )

Join Zoom Meeting
https://us06web.zoom.us/j/89956931861?pwd=YkN4bUg0NE1STFlRd0MwY0NUVjcwdz09

Meeting ID: 899 5693 1861
Passcode: 451794

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*The hosts request that questions from the audience may kindly be postponed to the discussion session. Questions can also be left in the chat box. Thank you. We look forward to having you with us in this lecture series.*

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Talks and discussion sessions will be recorded. We hope to make the recording available within two weeks after the lecture. URL www.archmathsci.org

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Communications Partner

Curt Jaimungal

*Podcast Host*
Theories of Everything https://www.youtube.com/TheoriesOfEverything

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Abstracts of talks are on the following pages.

TITLE: Basic Twistor Theory, Bi-twistors, and Split-octonions.

Roger Penrose, 2023

Abstract:

Twistor theory was introduced in the mid 1960s as an approach to combining quantum theory with space-time structure. A driving force behind the introduction of Twistor Theory was to combine the quantum-field theoretic requirement of positive frequency with the structure of space-time. In order to achieve this, the notion of twistor space was introduced to codify the structure of space-time in a way which related it to the splitting of the twistor space into two halves, one representing positive frequency, and the other representing negative frequency. Standard twistor theory involves a complex projective 3-space PT which naturally divides into two halves $PT^{+}$ and $PT^{-}$, joined by their common 5-real-dimensional boundary PN. The points of the space PN represent light rays in Minkowski space-time. However, this splitting has two quite different basic physical interpretations, namely positive/negative helicity and positive/negative frequency, which ought not to be confused in the formalism, and the notion of "bi-twistors" is introduced to resolve this issue. It is found that quantized bi-twistors have a previously unnoticed $G_2$* structure, which enables the split-octonion algebra to be directly formulated in terms of quantized bi-twistors, once the appropriate complex structure is incorporated.

*Dyson's three-fold way, quantum processes and the split-quaternions.*

*B. J. Hiley.*

My talk is based on a long neglected paper by Brice DeWitt in which he shows that in order to write the Kline-Gordon equation in a pseudo-Riemannian space in a fully relativistic covariant manner, the quantum Hamiltonian must contain an additional energy term, $\hbar^2 Q$ which he calls the "quantum potential". This term is manifested as a scalar curvature term in the geodesic. As is well known, a similar potential appears in the real part of the Schrödinger equation under polar decomposition of the wave function in Bohm's approach. Indeed a relation between the two potentials has been established.

If we include the arguments that Dyson outlines in his 1962 paper "The three-fold way" we find that the imaginary units demanded by quantum mechanics arise, not through the complex field, but through the basis elements ( *i*, *j* and *k )* of the real quaternions. This provide a rich structure allowing a purely algebraic approach to quantum phenomena rather than the usual Hilbert space formalism and no collapse of the wave function is necessary. The geometrical aspects of the dynamics emerge from the split quaternions, which combine the orthogonal and symplectic structure in a new way. This provides a link between the dynamical mass and the rest mass opening up the possibility of new ways of exploring the rest mass problem. If time allows I will show the relationship between the Dirac spinor and the semi-spinor of the conformal Clifford, the twistor.

1. DeWitt, B. S., Point Transformations in Quantum Mechanics, Phys. Rev. 85 (1952) 653-661.
2. Freeman J. Dyson, The Threefold Way. Algebraic Structure of Symmetry Groups and Ensembles in Quantum Mechanics, J. Math. Phys. **3**, (1962) 1199-1215
3. Bohm, D. and Hiley, B. J., The Undivided Universe: An Ontological Interpretation of Quantum Mechanics, Routledge, London, (1993).
4. Bohm D. J. and Hiley, B. J., Generalisation of the Twistor to Clifford Algebras as a Basis for Geometry, Revista Brasilera de Fisica, Vol. Especial Os 70 anos de Mario Schönberg, 1-26, (1984).

Albert Schwarz March 24, 2023

Title: Jordan algebras, quantum theory, and particle physics

Abstract

In the geometric approach to quantum theory that was suggested recently in my papers we are taking as a starting point the set of states (a bounded convex set of normalized states or a convex cone of not necessarily normalized states). In this framework one can prove a generalization of decoherence and use this proof to derive probabilities from the first principles ( from random interaction with the environment). Fixing an action of time and spatial translations on the cone of states one can define a notion of elementary excitation of translation invariant state and of scattering of elementary excitations. (Elementary excitations of ground state should be identified with particles, in general case elementary excitations should be regarded as quasiparticles.)

The geometric approach allows us to construct physical theories corresponding to Jordan algebras. This construction is based on the remark that one can define a cone of positive elements of a Jordan algebra. (In Jordan Banach algebras this is a homogeneous cone consisting of all squares.) In the talk I will analyze elementary excitations and scattering in physical theories based on Jordan algebras.

**Cayley-Dickson algebras, braids, and the Standard Model**

Niels Gresnigt

The initial part of this talk will review the applications of the quaternions and octonions to describing the particle content and symmetries of the Standard Model. Most of this review will be based on the works of Dixon and Furey. In the second half of the talk I will discuss some of my own ideas relating to algebraic unification.

It is proposed that going beyond the division algebras, and considering the Cayley-Dickson algebra of sedenions provides a natural means of generalizing existing results from a single generation to exactly three. The additional $S_3$ automorphisms of the sedenions might then be interpreted as a flavor symmetry, incorporating neutrino oscillations and quark mixing.

I will finish by discussing a curious association between algebraic unification based on the division algebras, and a topological representation of fermions in terms of simple braids. The structure of the Clifford algebras generated from the quaternions and octonions in this case provides a justification for many of the ad hoc assumptions in the topological model.

Throughout the presentation, the focus will primarily be on the underlying ideas, recognizing that many of the ideas presented are still a work in progress. The intent is to generate stimulating discussion.

Stephen L. Adler, May 19, 2023

Title: Trace dynamics and implications for my work of the last two decades

Synopsis: Trace dynamics is a Lagrangian theory of completely non-commuting variables q (generalized coordinate) and p (generalized momentum), which can be bosonic (Grassmann even) or fermionic (Grassmann odd). The Lagrangian is formed as a trace over polynomials in the q's and p's, and validity of cyclic permutation under the trace is assumed. From a trace action principle one directly finds the equations of motion, in Lagrangian form, and forming a trace Hamiltonian by a Legendre transform one finds the equations of motion in Hamiltonian form. When the trace Lagrangian is assumed invariant under global unitary transformation of the q's and p's, which requires that it contain no noncommuting coefficients, then there is a conserved Noether charge, given by the sum over bosonic variables of the commutators [q,p], minus the sum over fermionic variables of the anticommutators {q,p}. Forming a canonical ensemble, and taking averages over this ensemble, one finds that equipartition arguments give an emergent structure with the form of quantum field theory, provided the underlying theory develops a scale hierarchy in which variations of the trace Hamiltonian decouple, and provided that the numbers of bosonic and fermionic variables are essentially equal. In this theory the ensemble average of the conserved Noether charge plays the role of the complex unit i. The general framework is described in my 2004 Cambridge University Press book ``Quantum Theory as an Emergent Phenomenon''.

Much of my work since 2004 has been motivated by ideas coming from this book: (i) Thermodynamics has Brownian motion corrections, which suggests that these play a role in quantum measurements. This in turn suggests a connection at the phenomenological level with the Ghirardi-Rimini-Weber-Pearle model of objective reduction, with the aim of confronting experiment. (ii) The effective complex unit coming from trace dynamics is a 2X2 real matrix representation, which acts as i on one sector and -i on the other. Perhaps ``dark matter'' is a -i sector coupled to the standard model i sector only by gravitation. (iii) The equality of bosonic and fermionic variable numbers in trace dynamics suggests that extending the standard model with exact supersymmetry may be too strong a requirement. Perhaps one should only impose the weaker condition of boson-fermion balance, suggesting alternative unification models. (iv) The underlying trace dynamics theory naturally admits a Weyl scaling invariance of the metric, and this suggests that the non-derivative part of the induced gravitational action is also a Weyl scaling invariant, giving a novel model for "dark energy''. In gravity including this action, black holes no longer have horizons; near and inside the nominal horizon, the time-time component of the metric becomes very small but remains positive, never changing sign. (v) This motivates exploring horizonless ``dynamical gravastar'' models in which radii where structural changes occur come just from the relativistic Tolman-Oppenheimer-Volkoff equations with an assumed jump in the equation of state at high pressure to one with pressure plus energy density approximately zero.  Continuity properties of the TOV equations require the jump to be in the energy density, not in the pressure as assumed in prior work. This model may describe the structure of astrophysical ``black holes''.  Since material getting in can also emerge,  such objects could play a role in galaxy formation, leading to a simple formula for the disk galaxy scale radius, depending only on properties of atomic hydrogen.