Gamma-ray Bursts – A Review & Some Results

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Abstract

We see how the BATSE results have influenced our understanding of the Gamma-ray Burst phenomena. The observed values of $\left\langle \frac{V}{V_{max}} \right\rangle$, $\langle sin^2 b \rangle$, and $\langle cos\theta \rangle$ seems to rule out any Galactic disk model. Galactic halo models will require a halo core radius greater than about 35 kpc, to expain the observed isotropy. No hitherto known population has such a distribution. A test has been suggested to verify the plausibility of Galactic halo models. The different commonly used source count methods are compared in the context of GRBs. A few results connected with using the $\frac{V}{V_{max}}$ test for GRBs has been derived. Finally, we have listed a few problems which the GRBs pose.

1.Introduction

Even though the first Gamma-ray Burst was discovered about three decades ago, a consistent theory of GRBs has not so far been developed. The reason for this is manifold. (1)The non-repeating and transient nature of the phenomena makes the real-time determination of intrinsic characteristics like luminosity, distance scale etc. very difficult. (2)The bursts have not so far been identified with any other known source population, neither has any other evidence of their existence, like radiation in other bands of the electro-magnetic spectrum, neutrinos, other particles, gravity-waves etc. been detected. (3)The bursts themselves show a wide range of structural and temporal characteristics. Having energies ranging from 60keV to 2MeV, and pulse durations from a few ms to several minutes, some of them show substructures in the millisecond scale, while others are just multi-peaked or even featureless. (4)The relation between the distribution of bursts and the distribution of their sources in the sky is not known.

But even so we have been able to gather substantial information about the nature of the bursts. If we trace the history of our understanding of the Gamma-ray Bursts we can see that it has gone through two phases — the earlier pre-BATSE one and the recent post-BATSE one. In the next section we take a brief glimpse of the pre-BATSE burster scenario. The third section depicts how the BATSE results came as a surprise to the general expectations based on the pre-BATSE results. A number of studies have been done to understand the origin of the Gamma-ray Bursts and their distribution in the sky. We briefly review some of the source models in the fourth section. In the fifth section we identify a few problems connected with spatial distribution studies.

2.pre-BATSE burster scenario

The GRBs were inadverantly discovered by the VELA series of military satellites deployed by the US to monitor Soviet compliance with outer-space nuclear ban treaty. But the discovery remained a secret for several years until Klebesadel, Strong and Orson described them in a seminal paper in 1973. The exciting news sent waves of activity among theorists and models ranging from comets to cosmic strings were suggested. Several balloon observations, space-crafts and satellites were used to study the bursters. The VELA satellites, Pioneer Venus Orbiter (PVO), Venera 11 and 12 (Konus experiment), High Energy Astronomical observatory (HEAO) 1 and 3, Solar Maximum Mission (SMM), Helios2, Ginga etc. are some examples. All these observations seemed to indicate a compact source such as a neutron star or a black hole. Attention of the scientists were focussed on neutron stars mainly because of the following reasons.

(1)Quite a few of the bursts showed millisecond risetimes which tell us that the sources cannot be larger than a few hundred kilometers in extent. [Higdon & Lingenfelter, 1990]

(2)Venera11 and 12 reported to have detected emission and absorption lines in the spectra of many Gamma-ray Bursts [Mazets et al, 1981]. These were interpreted as cyclotron lines in a 10^{12} G magnetic field which is found only in neutron stars. The possibility of a binary system of neutron stars as origin was also suggested.

(3)The non-thermal spectrum of the GRBs indicated that the sources are optically thin, since otherwise energy would be radiated in the X-ray, UV, or optical bands also. [Schmidt, 1978]

(4)Optical depth considerations seem to indicate also that the population is confined close to the Galactic disk.

(5)In the post-burst emission of the unique event GB790305b from a soft repeater, an 8.00±0.05s periodicity was observed for 8 cycles by the Konus experiment, and for 22 cycles in the Helios2 data. The double pulsed form of this emission was very similar to that of pulsars. It was suggested [Ramaty et al, 1980; Kazanas, 1980] that the extraordinary burst resulted from a core-quake that may have spun up the star. The position of the source has been identified to lie within the diffuse supernova remnant N49 in the LMC. [Higdon et al, 1990]

(6)The $\frac{V}{V_{max}}$ (see section 6) test done on the data obtained from these observations also seemed to indicate a Galactic disk origin of the bursts. The value of $\left\langle\frac{V}{V_{max}}\right\rangle$ obtained from the various experiments were 0.46±0.024(PVO), 0.43±0.024(Konus), 0.43±0.024 (Siane,Apex, Lilas) etc.

(7)The 244 bursts observed by Venera 13, Venera 14 and Phebus were reported [Atteia et al, 1991] to show concentration towards the Galactic plane.

Thus before the first announcement of BATSE results in September 1991, experts were fairly confident that the bursts were coming from neutron stars relatively nearby in the disk of our Galaxy. The models predicted that the bursts may be produced by a neutron star colliding with an asteroid, a thermo-nuclear runaway explosion of accreted material, a seismic adjustment of the neutron star crust etc. to name a few.

3.The BATSE results

The Burst & Transient Source Experiment (BATSE) on the Compton Gamma-ray Observatory (GRO) satellite, started detecting GRBs from April 1991 onwards with unprecedented sensitivity to weak bursts (flux threshold of 10^{-7} $ergscm^{-2}s^{-1}$). It contains eight detectors, sensitive to gamma-rays in the energy range of 20keV to 2MeV. The count rates are tested on three independent timescales of 64ms, 256ms and 1024ms.

Analysis of the BATSE data shows that the burst distribution gives $\left\langle\frac{V}{V_{max}}\right\rangle$ =0.32$\pm$ 0.02(0.5 for homogeneous distribution), dipole anisotropy with respect to the Galactic centre $\langle cos\theta\rangle = 0.04 \pm 0.03$(0.0 for isotropic distribution) and quadrupole anisotropy with respect to the Galactic plane $\langle sin^2 b\rangle$=0.33(0.333 for isotropic distribution). Also, a full sky burst rate of about 800 per year has been calculated. These results contradict some of the conclusions reached on the basis of pre-BATSE results.

If the population of the bursters were actually confined to the Galactic disk, their observed angular distribution in the sky would look isotropic only as long as the detector sensitivity limits the maximum observable distance, (D_{max}) to much less than the scale size of the disk population. It can be shown that (section 6) as D_{max} increases, $\left\langle\frac{V}{V_{max}}\right\rangle$ of the observed distribution begins to fall from 0.5, and when D_{max} is much greater than the scale size, $\left\langle\frac{V}{V_{max}}\right\rangle$ approaches 0.4. Mao & Paczynski [1992], after considering a wide variety of theoretical source populations, have demonstrated that this is indeed the case. The values of $\left\langle\frac{V}{V_{max}}\right\rangle$ obtained from BATSE's predecessors seem to indicate that one was beginning to probe the outer limits of a disk population. Thus it was expected that

BATSE with sensitivity much more than its predecessors would produce a $\left\langle \frac{V}{V_{max}} \right\rangle$ much closer to 0.4. But the fact that the value which BATSE produced was much lower than 0.4 rules out the disk population models, unless there is some systematic error in the calculation of $\left\langle \frac{V}{V_{max}} \right\rangle$.

A flaw in the above argument is that we do not know the value of the distance range D_{max}, or even the ratio of the D_{max} to the scale-size of the distribution. Hence we do not know if the observed fall in the value of $\left\langle \frac{V}{V_{max}} \right\rangle$ is actually due to the above effect. Besides, it is known [B. Paczynsky, 1991] that $\left\langle \frac{V}{V_{max}} \right\rangle$ is not very sensitive to different disk models. More preferable quantities to look at will be the dipole and quadrupole anisotropies of the distribution. It is seen that for any disk model, the concentration of the sources towards the Galactic plane as given by $\left\langle sin^2 b \right\rangle$ will be more readily detectable than the deviation from radial anisotropy as measured by $\left\langle \frac{V}{V_{max}} \right\rangle$. But the BATSE results have already shown inhomogenity in radial distribution while the $\left\langle sin^2 b \right\rangle$ which it gives is quite consistent at 3σ level with isotropy. This indicates very strongly against the plausibility of disk models.

A study of the burst duration data immediately reveals that the straight-forward neutron star merger models, which were the favourite before BATSE, could not be used as such. These simple-minded mergers will be over in a few milliseconds, while the typical bursts last about half a minute and often shows several millisecond substructures [B. Schwarzschild, 1992]. Besides, the energy spectrum of the bursts are much broader than the approximate black body spectrum one would expect from a simple merger.

Binary neutron star merger models can no longer go along with a disk population of bursts since it has been calculated [B. Pazcynsky, 1990] that the rate at which these mergers occur in a galaxy like ours is $\sim 10^{-6}$ per year per galaxy, which is nowhere near the observed burst rates.

With the announcement of BATSE results, which clearly shows the high improbability of a Galactic disk burst source population, attention got diverted to other possibilities, which could explain the new results as well.

4.On the Origin of Gamma-ray Bursts

Of the wide variety of source models, that had been suggested, the most popular ones are those which associate GRBs with neutron stars. While the earlier trend was to use phenomena on single neutron stars to explain the bursts, the more recent attempts are based on neutron star–neutron star, and black hole–neutron star binaries. These models can be broadly classified into three types, namely the nuclear burning models, glitch models and accretion models.

In the thermonuclear models, a strongly magnetized, old neutron star slowly accretes matter which accumulates at its polar caps. If the accretion rate is greater than a critical value, which depends on the neutron star mass and temperature, the star will undergo a hydrogen flash which in turn will ignite a helium flash releasing an energy of about 10^{38} ergs [J. P. Lasota, 1990]. The suggested sources of accreting matter were inter-stellar medium, a companion star, or a cold circum-stellar disk.

Models requiring accretion from ISM are highly improbable, since such sources have to be close to the Galactic disk and hence would produce very strong dipole anisotropies which are not consistent with observations. Two other problems associated with these models, in general, are the low efficiency of conversion of thermal energy released by thermo-nuclear runaway at large depths and the lack of observed persistent soft X-ray emission, as predicted by these models.

In the second type, glitches which occur in the neutron stars are used to power the bursts. The processes that spin up the crust of a star during a glitch also excite stellar oscillations [R. I. Epstein, 1990]. Toroidal stellar oscillations which can persist for several seconds, generate electric fields partially aligned with the surface magnetic field. The curvature radiation photons start electron-positron cascades which are accelerated by the electric field. The photons emitted at the stellar surface are Compton and/or synchrotron scattered to produce the continuum spectrum of GRBs [I. A. Smith, 1990]. A feature of such an emission is that it may be highly beamed.

Calculations have shown that the Compton upscattering process is a promising mechanism for explaining GRBs with energies from several keV to several MeV [Ho et al, 1989; Dermer, 1990]. A problem with this cascade model is that the energy which the positrons carry towards the stellar surface may be comparable to the energy radiated as GRBs. If all this energy is radiated as photons, the X-ray emission could exceed the observed limits. Hence a proper mechanism for removing this excess energy in some other form has to be deviced.

The accretion models depend on the release of gravitational potential energy by the sudden infall of matter onto the neutron star, from the ISM, a comet or a pre-existing circum-stellar disk. Accretion from the ISM has the same problem as in the case of neuclear burning models. The cometary models envisage a number of comet–neutron star encounter scenarios such as collision with the comets bound in the Oort cloud of the star, random encounters between neutron stars and comet clouds, accretion from inter-stellar comets etc. [Pineault, 1990]. No proper emission mechanisms have been rigourously worked out so far for such models.

Among the models which predict accretion from a circum-stellar disk around neutron stars as the source of GRBs, the most promising one involves NS-NS or BH-NS binaries. Speculations were triggered off, after the discovery of the first binary pulsar by Hulse and Taylor in 1975. Since then four more of them have been discovered so far and several studies have been undertaken to estimate the frequency of such systems among neutron stars. The results are not consistent with the observed GRB rate, but several loose ends still remain to be tied together. An interesting merger model due to Paczynski, predicts that a gravitationally decaying binary pair of compact objects can often merge to form a black hole by way of an intermediate phase. Most of the mass will already be in the pre-existing or newly formed black hole. But a surrounding accretion disk of neutron star material will survive in orbit around the black hole for tens of seconds to generate the extended GRB. The millisecond substructure of the burst would be indicative of the orbital period of the disk.

But neither the details of such a model is available, nor a mechanism for the conversion of the released gravitational energy into gamma ray photons have been suggested.

Another interesting model involves the creation of an opaque, relativistic, electron-positron pair plasma usually called a fireball [Goodman, 1990]. It has been shown that as long as the fireball is optically thick, the radiation-pair plasma mixture will behave like a fluid. The fireball will expand and cool until the electron-positron pairs annihilate and the radiation escapes. The fluid which will attain relativistic velocities in the meantime, would be blue-shifted to much higher energies.

A major problem with these models was that, the presence of even a little baryonic contamination of the fireball, would prevent efficient generation of gamma rays. Several methods had been suggested [Meszaros & Rees, 1992] to reconvert the energy back into gamma rays, while at the same time preserving the observed non-thermal spectrum [Narayan et al, 1991]. Tsvi Piran[1992] has shown that the physical conditions implied by the fireball models favour a cosmological distribution of sources, rather than a Galactic one.

Thus we see that all the burst source models, involve two physical aspects, namely, the generation of energy required for the bursts and the conversion of the energy into the spectral form which is observed. Constraints on the second of these have been obtained from the analysis of the spectral features observed by the various detectors.

A detailed temporal analysis of the data obtained by SMM, HEAO and Ginga has clearly demonstrated the presence of absorption and emission features superimposed on the continuum spectra of many bursts. Mazets et al [1981] first demonstrated that the absorption lines seen in the energy range from 20keV to 70keV resulted from absorption of continuum emission at the cyclotron frequency by surrounding colder plasma in a very strong magnetic field. Wang et al [1989] have shown that these scattering lines could be formed in a thin, relatively cool plasma sheet surrounding a neutron star. Mazets et al have found absorption features in the spectra of 19 out of the $\sim$60 bursts observed by Konus on Venera 11 and 12.

Emission features also, at energies of about 350 to 500 keV, have been observed simultaneously by detectors on more than one space-craft. These are generally interpreted as hot electron-positron annihilation radiation, gravitationally redshifted by neutron star fields.

The strong connection of the GRBs with neutron stars indicated by these features has become somewhat doubtful, since no such features have been reported in the BATSE data, though BATSE has already detected more number of bursts than all the previous detectors put together.

GRBs also display very diverse temporal structures and hence no typical burst light curve has been identified. Another feature of interest that has been observed is the spectral variability displayed by many of the bursts [Mazets et al, 1982]. With respect to the instant at which the trigger occured, the different energy ranges peak at different times. Such variability is complex and no single pattern describes the bulk of the bursts. However, bursts are, in general, hardest initially and soften with time [Teegarden, 1982]. Similar hard to soft evolution has been observed within individual pulses of complex, multi-peaked bursts.

5.On the spatial distribution of GRBs

In general, four types of models have been suggested to explain the observed distribution of GRBs, namely, solar system models, Galactic disk models, Galactic halo models, and cosmological models.

The models which use phenomena in the solar system to explain the GRBs have not recieved much acceptance, neither has much work been done in this direction.

We have already seen how the BATSE results have diminished the credibility of the Galactic disk models. Here, we discuss a test which is often used to study the distribution of sources in the sky.

The logN-logS test:

Let $N(> S_{min})$ denote the number of sources observed with fluence, S (flux integrated over some selected integration time scale) greater than some S_{min}. We denote

$N(> S_{min})$ by the power-law relation,

$$N(> S_{min}) = AS_{min}^{-\beta}, \tag{1}$$

where A and β are constants. Then

$$logN(> S_{min}) = -\beta log S_{min} + log A. \tag{2}$$

Thus if we plot the logN-logS curve, the slope of the curve which is equal to β, will depend on the distribution of the sources. In particular, if the sources are distributed uniformally in Euclidean space, β will be equal to 1.5 as shown below.

Let n_0 denote the number density of sources in the sky. Then the number of sources within a shell between radial distances r and r+dr, is given by

$$n_0 dV = n_0 4\pi r^2 dr. \tag{3}$$

For a given luminosity L and minimum fluence S_{min}, the sources can be detected upto a maximum distance given by

$$r_{max} = [\frac{L}{4\pi S_{min}}]^{1/2}. \tag{4}$$

If $\phi(L)$ denote the luminosity function of the sources, we see that

$$\begin{aligned} N(> S_{min}) &= \int_{L_{min}}^{L_{max}} dL \phi(L) \int_0^{r_{max}} n_0 4\pi r^2 dr \\ &= n_0 \int_{L_{min}}^{L_{max}} dL \phi(L) \ \frac{4\pi}{3} r_{max}^3. \end{aligned} \tag{5}$$

Substituting for r_{max} from eqn.(4), we get

$$N(> S_{min}) = \frac{4\pi}{3} n_0 \int_{L_{min}}^{L_{max}} dL \phi(L) [\frac{L}{4\pi}]^{3/2} \times S_{min}^{-3/2}. \tag{6}$$

Therefore

$$N(> S_{min}) = AS_{min}^{-3/2}, \tag{7}$$

where

$$A = \frac{4\pi}{3} n_0 \int_{L_{min}}^{L_{max}} dL \phi(L) \left[\frac{L}{4\pi}\right]^{3/2}.$$

Comparing eqn.(7) with eqn.(1), we see that β=1.5 for a homogeneous distribution in Euclidean space.

The logN-logS analysis of the burst data show that a relation $N \sim S^{-1.5}$ seems to hold for the strongest bursts, but it rapidly flattens out at low values of fluence (S). This flattening was interpreted as evidence for the observations reaching the limiting distance in the distribution of bursters, and it was suggested [Jennings,1982; Shklowski & Mitrofanov,1985] that the GRBs may be distributed in the halo of the Galaxy.

However, Higdon & Lingenfelter [1984] pointed out that the flattening at low fluences mostly resulted from incomplete sampling due to instrumental selection biases arising because typical detectors trigger not on fluence or flux but on the photon count rate in some energy band and integration time scale. They noticed that a logN-logP relation was easier to analyse.

The reason for using detector count rate to define the threshold is the following. If all the bursts had identical spectra, it would be easy to define a threshold in terms of peak energy flux, $P \sim \frac{nE_{\gamma,av}}{\Delta t}$, where n is the photon count rate per unit time bin, $E_{\gamma,av}$ is the average photon energy, and Δt is the duration of the time bin over which the counts are integrated. If all the bursts had identical spectra and identical time duration, it would be equally easy to define the detection threshold in terms of their fluence, $S \sim \frac{nE_{\gamma,av}}{\Delta t} t_\gamma$, where t_γ is the total duration of the burst. But since neither of these two conditions are satisfied in the case of GRBs, it is impossible to define a meaningful threshold for any Gamma-ray Burst detector in terms of P or S. Hence the detection threshold is usually defined in terms of counts per unit time bin.

Mazets [1986] proposed that the logN-logn_{max} curves should be used for the analysis of source counts,where N is the number of bursts with photon count rates greater than n_{max}. He illustrated how logN-logS is most strongly affected by instrument selection effects, the log N-logP relation is less biased and the logN-logn_{max} relation allows the most direct interpretation of the data [Paczynski & Long, 1988].

However, it should be noted that the spatial non-uniformities doubtless account for the major portion of the deviations from a -1.5 slope at the very low fluences. This was made clear by a still better test suggested by Schmidt et al [1988], called the $\frac{V}{V_{max}}$ test. Unlike the other cumulative distributions, the $\frac{V}{V_{max}}$ test was considered to be independent of threshold variations and instrument sensitivities.

Though the original motivation for the Galactic halo bursts no longer exists, the halo models are not completely inconsistent with the observations. They could to some extent explain the isotropy and inhomogeneity of the distribution, which has been observed. Moreover, the distance scales suggested by these models imply burst energies in the range of 10^{41} to 10^{42} ergs which is quite consistent with the NS-NS, NS-BH merger models.

It seems ironical that the lack of anisotropy itself is now being used as an evidence against the Galactic halo models. The argument is that since we are about 8.5 kpc away from the centre of the Galaxy, any Galactic halo population to appear isotropic, should have a sufficiently large core radius within which the sources are uniformally distributed. An analytical expression connecting the statistical deviation of the dipole anisotropy, $(\langle cos\theta\rangle_{max})$ to the corresponding core radius has been derived, [Mao & Paczynsky, 1992] almost independent of the luminosity function.

$$\langle cos\theta\rangle_{max} = \frac{2R_0}{3R_c}g, \qquad \text{for } \frac{R_c}{R_o} >> 1, \qquad g = 0.46 \tag{8}$$

where R_c = core radius and $R_o = 8.5$kpc, the distance to Galactic centre.

For a random distribution of N sources in the sky we have

$$\langle cos\theta\rangle = 0.0 \pm (3N)^{-1/2}. \tag{9}$$

This formula can be used to place a rough 3σ limit, $\langle cos\theta\rangle_{max} < 0.0748$ on the currently observed 536 bursts. This combined with eqn. (8), gives a value of $R_c \sim 35$ kpc, which is needed to explain the observed isotropy. A halo distribution with such a large uniform core radius is not known, for any established class of objects. Objects ejected from the Galactic disk, would not produce a uniform density over the inner 35 kpc, no matter what the ejection velocity is. Thus the halo models call for a new type of theoretical model.

Using this result we show that even if such a hitherto unknown population, which is bound to the Galaxy exists, the maximum allowable uniform core radius is about 50 kpc. Any source beyond this radius will be strongly affected by the presence of the Megallenic clouds and even M31, which would manifest as strong anisotropies in the burst distribution towards these directions. Even speculative assumptions such as the bursts might be in the dark matter halo around our Galaxy, will not help the matters much since it has been shown that [T. Padmanabhan & M. Vasanthi, 1985] the halo of our Galaxy cannot extend significantly beyond $\sim$ 60 kpc. Using R_c=50 kpc, we can obtain a value of 0.05213 for $\langle cos\theta \rangle_{max}$ from eqn.(8). The number of bursts which should be discovered to put this value as a 3σ limit on $\langle cos\theta \rangle$ is about 1100. Thus we see that, if even after detecting 1100 bursts , we do not begin to observe any anisotropy at 3σ level in the burst distribution, the Galactic halo models can be ruled out. At the current rate of detection of bursts, it may take another year to detect this many number of bursts.

The observed isotropy can be explained also if we assume that we are located at the centre of a spherical distribution of bursters. Since we are at the centre of the observed universe, which appears to be isotropic, this immediately suggests the possibility of a cosmological distribution of bursters.

A burst distribution at a redshit of $\sim$ 1, would imply intrinsic burst energies of about 10^{51} ergs per second. In such a cosmological scenario also, relativistic fireballs and coelescing binaries are the major contenders as burst sources.

There are a few points which we would like to emphasize in this context.

The so called $\frac{V}{V_{max}}$ test has been used extensively to study the distribution of bursts in the sky.

The $\frac{V}{V_{max}}$ test:

The idea of the $\frac{V}{V_{max}}$ test is to evaluate for each source, its radial location within the volume available to it as determined by the detector threshold, C_{min}. [Schmidt,1988]. The value of $\frac{V}{V_{max}}$ for each source is the ratio of the spherical volume V, defined by the radial distance to the burst and the maximum volume V_{max} from which the burst could have been detected. For a given luminosity L,

$$r = [\frac{L}{4\pi C}]^{1/2}, \tag{10}$$

and r_{max} is as given by eqn.(4) with S_{min} replaced by C_{min}. Therefore

$$\frac{V}{V_{max}} = \frac{\int_0^r 4\pi r^2 dr}{\int_0^{r_{max}} 4\pi r^2 dr} = \frac{r^3}{r_{max}^3}. \tag{11}$$

Also

$$\langle \frac{V}{V_{max}} \rangle = \frac{\int_0^{r_{max}} n(r) \frac{V}{V_{max}} 4\pi r^2 dr}{\int_0^{r_{max}} n(r) 4\pi r^2 dr}, \tag{12}$$

where n(r) is the number density of the sources at a distance r. For a uniform distribution of sources,we have $n(r) = n_0$, a constant. Therefore, substituting for $\frac{V}{V_{max}}$ from eqn.(11),

$$\langle \frac{V}{V_{max}} \rangle = \frac{\int_0^{r_{max}} \frac{r^3}{r_{max}^3} r^2 dr}{\int_0^{r_{max}} r^2 dr}, \tag{13}$$

Therefore

$$\langle \frac{V}{V_{max}} \rangle = 0.5. \tag{14}$$

In deriving the above result we had assumed that the sources were distributed uniformly in a 3-dimensional Euclidean space. However, this result can be generalised to a distribution in k-dimensions, where k=2 or 3, depending on whether the sources are distributed in 2-dimensions (as in a disk) or in 3-dimensions. Indepedent of the type distribution, the photons emitted by a source will spread uniformally over a spherical surface centred at the source. Hence eqn.(10) still remains valid in the general case also. However, the number of sources within a shell between r and r+dr will now be given by

$$n(r) dV(r) = constant \times n(r) k r^{k-1} dr. \tag{15}$$

Therefore eqn.(13) gets modified as

$$\left\langle\frac{V}{V_{max}}\right\rangle = \frac{\int_0^{r_{max}} \frac{r^3}{r_{max}^3} k r^{k-1} dr}{\int_0^{r_{max}} k r^{k-1} dr}$$
$$= \frac{k}{k+3}. \tag{16}$$

Thus we see that for a uniform disk-like distribution of sources (k=2), $\left\langle\frac{V}{V_{max}}\right\rangle$ =0.4. We also see that for k=3, $\left\langle\frac{V}{V_{max}}\right\rangle$ =0.5, as required.

But $\frac{V}{V_{max}}$ is not a variable which can be directly observed. It has to calculated from more fundamental variables which can be observed. From eqn.(10) & eqn.(11) we have

$$C \propto r^{-2} \qquad \text{and} \qquad V \propto r^3. \tag{17}$$

Therefore we get

$$V \propto C^{-3/2} \tag{18}$$

and hence

$$\frac{V}{V_{max}} = [\frac{C}{C_{min}}]^{-3/2}. \tag{19}$$

Thus we get $\frac{V}{V_{max}}$ in terms of the observed variables C and C_{min}, where C is the integrated photon count obtained from a source and C_{min} is the detection threshold. It is customary to use the variable C_{max} in place of C, where C_{max} is the peak photon fluence obtained from the source. Then we have

$$\frac{V}{V_{max}} = [\frac{C_{max}}{C_{min}}]^{-3/2}. \tag{20}$$

In the general case,

$$V \propto r^k \qquad \text{and hence} \qquad \frac{V}{V_{max}} = [\frac{C_{max}}{C_{min}}]^{-k/2}. \tag{21}$$

But once we assume a cosmological distribution of sources, $\frac{V}{V_{max}}$ cannot be calculated from the observables unless we have additional information about the redshift or the luminosity ranges involved. In the case of GRBs no such information is available. On the other hand, $\left[\frac{C_{max}}{C_{min}}\right]^{-3/2}$, can still be calculated from the observed values of C_{max} and C_{min}. But $\left[\frac{C_{max}}{C_{min}}\right]^{-3/2}$ is equal to $\frac{V}{V_{max}}$ only in a Euclidean space. The two variables are related in a cosmological context through the redshift ranges involved, as shown below. Let the spectrum of a typical source be represented by the power-law relation,

$$L_\nu d\nu = k\nu^\alpha, \tag{22}$$

where k is a constant and α is the spectral index. Let us for simplicity take $\alpha = 1$. It can be very easily shown that the following result is true for any α. Now, the total intrinsic luminosity of the source is given by

$$L(z) = \int_{\nu_{e_1}}^{\nu_{e_2}} L_\nu d\nu. \tag{23}$$

Therefore

$$L(z) = \int_{\nu_{o_1}(1+z)}^{\nu_{o_2}(1+z)} L_\nu d\nu = L_0(1+z) \qquad \text{where} \qquad L_0 = \int_{\nu_{o_1}}^{\nu_{o_2}} L_\nu d\nu. \tag{24}$$

Hence we find that

$$C_{max} = \frac{L_0(1+z)}{4\pi r_z^2(1+z)^2} = \frac{L_0}{4\pi r_z^2(1+z)} \quad \text{and} \quad C_{min} = \frac{L_0}{4\pi r_{z_{max}}^2(1+z_{max})}. \tag{25}$$

Therefore

$$\left[\frac{C_{max}}{C_{min}}\right]^{-3/2} = \frac{(1+z)^{3/2}}{(1+z_{max})^{-3/2}}\frac{r_z^3}{r_{z_{max}}^3} = \frac{(1+z)^{3/2}}{(1+z_{max})^{3/2}}\frac{V}{V_{max}}, \tag{26}$$

where r_z is the proper distance to the sources. Thus we find that $\left[\frac{C_{max}}{Cmin}\right]^{-3/2}$ is not equal to $\frac{V}{V_{max}}$ for a cosmological distribution of sources. In the Euclidean limit, when z and z_{max} tends to zero, the r.h.s. of eqn.(26) becomes equal to $\frac{V}{V_{max}}$. It is be to noted that the volumes used in this case are the comoving volumes. We also show that $\left\langle \frac{V}{V_{max}} \right\rangle$ =0.5 in this case. Let

$$\frac{V}{V_{max}} = \frac{\int_0^z 4\pi r_z^2 \frac{dr_z}{dz} dz}{\int_0^{z_{max}} 4\pi r_z^2 \frac{dr_z}{dz} dz} = \frac{r_z^3}{r_{z_{max}}^3} \tag{27}$$

be the ratio of the comoving volumes.Then

$$\left\langle \frac{V}{V_{max}} \right\rangle = \frac{\int_0^{z_{max}} n(z) \frac{V}{V_{max}} 4\pi r_z^2 \frac{dr_z}{dz} dz}{\int_0^{z_{max}} n(z) 4\pi r_z^2 \frac{dr_z}{dz} dz}. \tag{28}$$

For a uniform distribution of sources $n(z) = n_0$.Therefore

$$\left\langle \frac{V}{V_{max}} \right\rangle = \frac{\int_0^{z_{max}} \frac{V}{V_{max}} r_z^2 \frac{dr_z}{dz} dz}{\int_0^{z_{max}} r_z^2 \frac{dr_z}{dz} dz}. \tag{29}$$

Put

$$\int_0^z r_z^2 \frac{dr_z}{dz} dz = X(z). \tag{30}$$

Then

$$\frac{dX(z)}{dz} = r_z^2 \frac{dr_z}{dz}. \tag{31}$$

Therefore from eqn.(27)

$$\frac{V}{V_{max}} = \frac{X(z)}{X(z_{max})} \tag{32}$$

Substituting eqns.(31) & (32) in eqn.(29),

$$\begin{aligned} \left\langle \frac{V}{V_{max}} \right\rangle &= \frac{\int_0^{z_{max}} X(z) \frac{dX(z)}{dz} dz}{X^2(z_{max})} \\ &= \frac{X^2(z_{max}) - \frac{X^2(z_{max})}{2}}{X^2(z_{max})} \\ &= 0.5, \qquad \text{as required.} \end{aligned} \tag{33}$$

It is generally stated that the average value of $\frac{V}{V_{max}}$ will be 0.5 for a homogeneous ditribution of sources. There are two aspects to this statement. The first one is that $\left\langle \frac{V}{V_{max}} \right\rangle$ =0.5 is only a necessary condition for homogeneity, but is not sufficient. The more important aspect is that $\left[\frac{C_{max}}{C_{min}}\right]^{-3/2}$, which is not equal to $\frac{V}{V_{max}}$ for a cosmological distribution of sources, will not give an average value of 0.5, even for a uniform distribution of sources. $\left\langle \left[\frac{C_{max}}{C_{min}}\right]^{-3/2} \right\rangle$ is equal to 0.5 only for a distribution in Euclidean space.This result is proved below.

If n_0 is the number of sources per unit comoving volume in a cosmological distribution of sources, the total number of sources within z_{max} is

$$N(z_{max}) = \frac{4}{3}\pi r^3_{z_{max}} n_0. \tag{34}$$

Therefore

$$\left\langle \left(\frac{C_{max}}{C_{min}}\right)^{-3/2} \right\rangle = \frac{1}{N(z_{max})} \int_0^{r_{z_{max}}} n_0 \left[\frac{C_{max}}{C_{min}}\right]^{-3/2} 4\pi r_z^2 dr_z \tag{35}$$

$$= \frac{1}{N(z_{max})} \int_0^{r_{z_{max}}} n_0 \frac{r_z^3(1+z)^{3/2}}{r^3_{z_{max}}(1+z_{max})^{3/2}} 4\pi r_z^2 dr_z \tag{36}$$

$$= \frac{4\pi n_0}{N(z_{max}) r^3_{z_{max}}(1+z_{max})^{3/2}} \int_0^{r_{z_{max}}} r_z^5 (1+z)^{3/2} dr_z. \tag{37}$$

For a Friedmann cosmology with $q_0 = 0.5$ and $\Lambda = 0$,

$$r_z = \frac{2c}{H_0}\left[1 - \frac{1}{\sqrt{1+z}}\right] = 2R_0\left[1 - \frac{1}{\sqrt{1+z}}\right]. \tag{38}$$

Therefore

$$\frac{1}{\sqrt{1+z}} = \left[1 - \frac{r_z}{2R_0}\right]. \tag{39}$$

Putting eqn.(39) into eqn.(37) we get,

$$\left\langle \left(\frac{C_{max}}{C_{min}}\right)^{-3/2} \right\rangle = \frac{4\pi n_0 \left[1 - \frac{r_{z_{max}}}{2R_0}\right]^3}{\frac{4}{3}\pi r^3_{z_{max}} n_0 r^3_{z_{max}}} \int_0^{r_{z_{max}}} r_z^5 \left[1 - \frac{r_z}{2R_0}\right]^{-3} dr_z \tag{40}$$

$$= \frac{3\left[1 - \frac{r_{z_{max}}}{2R_0}\right]^3}{r^6_{z_{max}}} \int_0^{r_{z_{max}}} r_z^5 \left[1 + 3\frac{r_z}{2R_0} + 6\frac{r_z^2}{4R_0^2} + \cdots \right] dr_z \tag{41}$$

$$= \frac{3\left[1 - \frac{r_{z_{max}}}{2R_0}\right]^3}{r^6_{z_{max}}} \left[\frac{1}{6} r^6_{z_{max}} + \frac{3}{7}\frac{r^7_{z_{max}}}{2R_0} + \cdots \right]. \tag{42}$$

Therefore

$$\left\langle \left(\frac{C_{max}}{C_{min}}\right)^{-3/2} \right\rangle = 3\left[1 - \frac{r_{z_{max}}}{2R_0}\right]^3 \left[\frac{1}{6} + \frac{3}{7}\frac{r_{z_{max}}}{2R_0} + \cdots \right]. \tag{43}$$

Thus we find that in the Euclidean limit, when $r_{z_{max}}$ tends to zero, $\left\langle \left(\frac{C_{max}}{C_{min}}\right)^{-3/2} \right\rangle = 0.5$, as required.

It has been shown [M. Schmidt,1968; M .S. Longair et al] that in the case of steady sources, if more than one value of $\frac{V}{V_{max}}$ is available, say from two surveys using different bands, on combining the data, the largest value of $\frac{V}{V_{max}}$, available for each source should chosen in calculating $\langle \frac{V}{V_{max}} \rangle$. We show that this is true for the $[\frac{C_{max}}{C_{min}}]^{-3/2}$ variable also.

Consider a distribution of sources, for each of which two values of $[\frac{C_{max}}{C_{min}}]^{-3/2}$ are available, due to two threshold values C_{min_1} and C_{min_2} used. From the sample, select a source for which

$$[\frac{C_{max}}{C_{min_1}}]^{-3/2} < [\frac{C_{max}}{C_{min_2}}]^{-3/2}. \tag{44}$$

For this source $C_{min_1} < C_{min_2}$. Since the threshold defines the maximum distance at which one could have seen the sources, this particular source will be within a radial distance defined by C_{min_2}. This is true for all the sources in a class of sources which satisfies eqn.(44). Thus we see that the above distribution can be analysed as if it comprises of two classes, one for which eqn.(44) is valid and $[\frac{C_{max}}{C_{min_1}}]^{-3/2}$ is the corresponding variable and a second one which satisfies $[\frac{C_{max}}{C_{min_1}}]^{-3/2} > [\frac{C_{max}}{C_{min_2}}]^{-3/2}$ for which C_{min_1} is the corresponding variable. Now, if the distribution is truly random, both these classes will give the same distriution. Hence we can study the distribution of $[\frac{C_{max}}{C_{min}}]^{-3/2}$ for the above distribution of sources by choosing the larger of the two values of $[\frac{C_{max}}{C_{min}}]^{-3/2}$ available for each source. This result is particularly important in the case of GRBs , since here, the same detector arrangement may use more than one photon integration time scale and hence give more than one $[\frac{C_{max}}{C_{min}}]^{-3/2}$ per burst.

It has been shown that [David Band, 1992], variations in the detection threshold smooth the actual $\frac{V}{V_{max}}$ distribution and the one we observe is this smoothed version. Hence models should be fit to the $\frac{V}{V_{max}}$ distribution only after deconvolving it of the effect of detector threshold variations. Band has demonstrated that a simple but unphysical source distribution and a distribution of detector thresholds, results in a $\frac{V}{V_{max}}$ distribution similar to that observed by BATSE . We have extended this work to study the effect of detector threshold variations in the observed $[\frac{C_{max}}{C_{min}}]^{-3/2}$ distribution in a cosmological scenario. A matter-dominated Friedmann cosmological model with $q_o = 0.5$ and $\Lambda = 0$

which is valid upto $z \leq 100$ is assumed. In this model, the luminosity distance is related to the red-shift by the relation

$$D_L(z) = \frac{2c}{H_o}\left[1 + z - \sqrt{1+z}\right], \tag{45}$$

and the proper motion distance r_z is given by

$$r_z = \frac{D_L(z)}{1+z}. \tag{46}$$

Hence the comoving volume of the shell between redshifts z and z+dz is

$$dV(z) = 4\pi r_z^2 \frac{dr_z}{dz} dz. \tag{47}$$

For a detector with a finite bandwidth given by $\nu_1 \leq \nu \leq \nu_2$, the total photon luminosity of a source at redshift z, observable in the given band is

$$L(z) = \int_{\nu_{e_1}=\nu_1(1+z)}^{\nu_{e_2}=\nu_2(1+z)} L_{\nu_e} d\nu_e. \tag{48}$$

The observed photon count is given by

$$C = \frac{L(z)}{4\pi D_L^2(z)}. \tag{49}$$

A given detector threshold C_{min}, corresponds to a luminosity distance $D_L(z_{max})$ for a given luminosity L(z), through the relation

$$D_L(z_{max}) = \left[\frac{L(z_{max})}{4\pi C_{min}}\right]^{1/2}. \tag{50}$$

Let

$$x = \left[\frac{C_{min}}{C}\right]^{3/2}; \quad 0 \leq x \leq 1. \tag{51}$$

Given the distribution of C and a fixed detector threshold C_{min}, the normalized probability distribution of x is obtained as

$$p(x)dx = \frac{\text{number of bursts with values of } x \text{ in } [x, x+dx]}{\text{total number of bursts}}$$

$$= \frac{\text{number of bursts with values of } C \text{ in } [C, C+dC]}{\text{total number of bursts}}.$$

Therefore

$$p(x)dx = \frac{-N(C)dC}{\int_{C_{min}}^{\infty} N(C')dC'} ; \qquad C_{min} = constant. \tag{52}$$

Here N(C) is the number of bursts having photon count per unit time in the observer's rest frame equal to C; the negative sign arises because x decreases as C increases. We can now write

$$p(x)dx = \frac{-N(C)}{N(>C_{min})} \frac{dC}{dx} dx, \tag{53}$$

where

$$N(>C_{min}) = \int_{C_{min}}^{\infty} N(C')dC' \tag{54}$$

is the number of bursts having photon counts in excess of C_{min}, which is equal to the total number of bursts. From eqn.(51)

$$C = C_{min} x^{-2/3}, \qquad \frac{dC}{dx} = -\frac{2}{3} C_{min} x^{-5/3} \tag{55}$$

and therefore

$$p(x)dx = \frac{2}{3} \frac{N(C)}{N(>C_{min})} C_{min} x^{-5/3} dx. \tag{56}$$

Let $\phi(L, z)$ be the luminosity function of the source population which harbours Gamma-ray Bursts, and let R(L,z) be the fraction of the sources which produce bursts per unit time in the rest frame of the burst producing sources. The number of bursts with luminosity between L and L+dL and redshift in the range z and z+dz occuring per unit time in the observer's rest frame is

$$\frac{\phi(L, z)R(L, z)}{1+z} dV(z)dL$$

The observed number of bursts as a function of photon count C in the observer's rest frame is

$$N(C)dC = dC \int_{L_{min}}^{L_{max}} dL \int_0^{\infty} dz \frac{R(L, z)}{1+z} \phi(L, z) \delta \Big[C - \frac{L(z)}{4\pi r_z^2 (1+z)^2}\Big] 4\pi r_z^2 \frac{dr_z}{dz} \tag{57}$$

The Dirac delta function in eqn.(57) ensures that sources in a given luminosity– redshift bin make a contribution to N(C) only when they produce the photon count rate C. Using eqn.(54), the integral count is

$$N(>C_{min}) = \int_{C_{min}}^{\infty} dC \int_{L_{min}}^{L_{max}} dL \int_0^{\infty} \phi(L,z)dz \frac{R(L,z)}{1+z}\delta\Big[C - \frac{L(z)}{4\pi r_z^2(1+z)^2}\Big] 4\pi r_z^2 \frac{dr_z}{dz}$$

$$= \int_{L_{min}}^{L_{max}} dL \int_0^{z_{max}(C_{min})} dz\phi(L,z)\frac{R(L,z)}{1+z} 4\pi r_z^2 \frac{dr_z}{dz}. \tag{58}$$

The above relations have been derived assuming a fixed threshold of detection. Such a threshold is possible in optical and radio astronomy. In the case of Gamma-ray Bursts however, the detection threshold varies from burst to burst. If $f(C_{min})$ denotes the normalized distribution of detection thresholds, the distribution of x for all C_{min} is given by

$$p(x)dx = \frac{-N(C)dC \int_{C_{min}=C}^{\infty} dC'_{min} f(C'_{min})}{\int_0^{\infty} dC'_{min} f(C'_{min}) \int_{C=C_{min}}^{\infty} dC N(C)}. \tag{59}$$

Therefore the source count is given by

$$N(C)dC = dC \int_{L_{min}}^{L_{max}} dL \int_{C'_{min}=C}^{\infty} dC'_{min} f(C'_{min}) \int_0^{\infty} dz \frac{R(L,z)}{1+z}\phi(L,z)$$

$$\times\delta\Big[C - \frac{L(z)}{4\pi r_z^2(1+z)^2}\Big] 4\pi r_z^2 \frac{dr_z}{dz} \tag{60}$$

and

$$N(>C_{min}) = \int_{L_{min}}^{L_{max}} dL \int_{C'_{min}=C_{min}}^{\infty} dC'_{min} f(C'_{min})$$

$$\times \int_0^{z_{max}(C_{min})} dz\phi(L,z)\frac{R(L,z)}{1+z} 4\pi r_z^2 \frac{dr_z}{dz}. \tag{61}$$

Substituting for C from eqn.(55) we get

$$N(C) = \int_{L_{min}}^{L_{max}} dL \int_{C'_{min}=C_{min}}^{\infty} dC'_{min} f(C'_{min}) \int_0^{\infty} dz \frac{R(L,z)}{1+z}$$

$$\times\phi(L,z)\delta\Big[C_{min}x^{-2/3} - \frac{L(z)}{4\pi r_z^2(1+z)^2}\Big] 4\pi r_z^2 \frac{dr_z}{dz} \tag{62}$$

and

$$N(>C_{min}) = \int_{L_{min}}^{L_{max}} dL \int_{C'_{min}=C}^{\infty} dC'_{min} f(C'_{min}) \times \int_{0}^{z_{max}(C_{min})} dz \phi(L,z) \frac{R(L,z)}{1+z} 4\pi r_z^2 \frac{dr_z}{dz}. \qquad (63)$$

Substituting eqns.(62) & (63) in eqn.(55) we get the observable distribution of x.

6.Some problems to be addressed

(1)The spectra of the bursts have been observed to peak at different times after the trigger occurs. Assuming that all photons are emitted at the source at the same time, it is possible to use this spectral variability to put constraints on the distance scales of the burst distribution.

(2)All studies have been undertaken assuming an isotropic emission of gamma rays at the sources. If we consider beaming effects, the actual number of bursts may be much higher than what we see. Beaming also reduce the required intrinsic burst luminosities. This is particularly important, since the observed burst substructures with extremely sharp rise-times can be the effect of beaming.

(3) Various tests can be deviced to study the nature of the burst distribution.

For example, the hypothesis of a cosmological distribution can be verified by looking out for gravitational lensing effects by the galaxies lying along the line of sight to the GRBs. This may manifest as two bursts, very similar in structure, but separated by a few days, and coming approximately from the same direction in the sky. A study for determining the probability that a galaxy will be present along the line of sight to the bursters, also can be undertaken.

Also in the case of a cosmological distribution of sources, if we assume that the range of redshifts dominates over the range of intrinsic burst luminosities, it should manifest as relative spectral softness of weaker bursts compared to stronger ones. A study verifying the presence of such a correlation in the burst data will reveal the cosmological nature of the ditribution.

(4)The plausibility of solar-system phenomena as GRB sources may be verified.

(5)The astounding diversity of burst features like spectra, durations, and time variability has prompted people to think about two population models of GRB distributions. But so far no consistent model has been evolved in this type also. Work on evolving a consistent two population model may be undertaken.

Speculation

Correlation studies of the observed angular distribution of the Gamma-ray Bursts and the anisotropy of the CMBR may be undertaken.

Conclusion

Though a compact source origin of the GRBs is strongly suggested by various lines of evidence, the origin of the bursts still remain a mystery.

From the current results it is apparant that the observed isotropy cannot be explained in terms of a Galactic disk model. The plausibility of even a Galactic halo distribution is very doubtful. We have shown that even after BATSE has detected about 1100 bursts, if we do not see any anisotropy in the distribution of the sources, the Galactic halo models can be ruled out.

Though a cosmological origin of the sources has been suggested, no strong supporting evidence has so far been detected.

It was also seen that the logN-logS and logN-logP tests are affected by instrumental selection biases and hence the use of $\frac{V}{V_{max}}$ test has been suggested. We have shown the following features of the $\frac{V}{V_{max}}$ test.

$\left\langle\frac{V}{V_{max}}\right\rangle$ =0.5 for a uniform distribution of sources.

$\frac{V}{V_{max}} = \left[\frac{C_{max}}{C_{min}}\right]^{-3/2}$ only for a distribution of sources in the Euclidean space. In a cosmological context these two variables are related through the redshifts involved.

$\left\langle\frac{V}{V_{max}}\right\rangle$ =0.5 is only a necessary condition for homogeneity, but it is not sufficient.

For the more fundamental variable $\left[\frac{C_{max}}{C_{min}}\right]^{-3/2}$, the average value is not equal to 0.5 even for a uniform cosmological distribution.

The observed distribution of the variable $\left[\frac{C_{max}}{C_{min}}\right]^{-3/2}$, is affected by the variation of the detection threshold, which is a result contrary to the beliefs hitherto.

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