Cross-correlation Analysis of γ-ray Burst Time Profiles

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Abstract: We develop the formalism to use cross-correlation techniques to identify gravitationally lensed burst events among dissimilar-looking, faint bursts occuring in the same error box in the sky. The technique is verified using actual burst time profiles.

1. Introduction

One of the major issues pertaining to Gamma-ray Bursters (GRBs) is the question whether they are Galactic or extragalactic in origin. Though several Galactic (Proc. of Los Alamos Workshop), cosmological (Usov, V. V. & Chibisov, G. V. 1975; Paczynski, B. 1986a) and mixed (Lingenfelter, R. E. & Higdon, J. C. 1992) models had been suggested, none of them have so far been able to establish successfully, the distance at which the bursts originate. A potential observation that can settle the controversy of burst locations, is that of detecting a gravitationally lensed event (Paczynski, B. 1986b). If the GRBs are extragalactic in origin, the probability that a burst is lensed by an intervening galaxy or clusters of galaxies increases with the distance to the event. Hence, the fainter bursts are statistically more likely to be lensed, in which case it is important to develop methods that will allow us to determine when two faint burst events seen in the same error box are lensed images of a single burst. Recently, J. Wambsganss (1993) has studied the complementary problem of developing a method to distinguish between two burst time profiles that appear similar.

In this paper, we develop a method that studies the time profiles corresponding to two burst events originating in the same error box, but separated in time, using cross-correlation techniques.

2. Formalism

The observed counts $C_1(i)$ and $C_2(j)$, corresponding to two burst events lying in the same error box, i and j being the time bin indices, also contain background noise so that

$$C_\alpha(i) = S_\alpha(i) + n_\alpha(i). \tag{1}$$

Hereafter $\alpha = 1, 2$ stands for the appropriate quantity corresponding to the first and second event respectively. In eq(1), S_α are the intrinsic time profiles of the two events, while n_α are the background noise counts that are mutually uncorrelated and are assumed to be stationary. Choosing equal lengths for the two data trains, we cross-correlate these to obtain

$$\big(O_1 o O_2\big)(i) = \sum_{j=1}^{N} O_1(j) O_2(j+i); \qquad \text{where} \qquad O_\alpha(i) = C_\alpha(i) - \frac{1}{N}\sum_{k=1}^{N} C_\alpha(k), \tag{2}$$

are the observed counts less their averages.

The background noise counts at different time bins are assumed to be uncorrelated so that

$$\big\langle n_\alpha(i) n_\alpha(j)\big\rangle = \sigma_\alpha^2 \delta_{ij} + \mu_\alpha^2, \tag{3}$$

where μ_α and σ_α^2 are the means and variances of n_α, with $\langle\ \ \rangle$ representing an ensemble average. The cross-correlation signal and noise are given by

$$\mathcal{S}(i) \equiv \big\langle \big(O_1 o O_2\big)(i)\big\rangle = \sum_{j=1}^{N} S_1'(j) S_2'(j+i) \tag{4}$$

and

$$\begin{aligned}\mathcal{N}(i) &\equiv \sqrt{\big\langle (O_1 o O_2)^2(i)\big\rangle - \big\langle (O_1 o O_2)(i)\big\rangle^2} \\ &= \sqrt{N\sigma_1^2\sigma_2^2 + \sigma_2^2 \sum_{j=1}^{N} S'^2_1(j) + \sigma_1^2 \sum_{j=1}^{N} S'^2_2(j+i)},\end{aligned} \tag{5}$$

respectively, where

$$S_\alpha'(i) \equiv S_\alpha(i) - \frac{1}{N}\sum_{j=1}^{N} S_\alpha(j). \tag{6}$$

The signal-to-noise ratio then is given by

$$\frac{\mathcal{S}(i)}{\mathcal{N}(i)} = \frac{\sum_{j=1}^{N} x_j y_{j+i}}{\sqrt{N + \sum_{j=1}^{N} x_j^2 + \sum_{j=1}^{N} y_{j+i}^2}}, \qquad \text{where} \qquad x_j = \frac{S_1'(j)}{\sigma_1} \quad \text{and} \quad y_j = \frac{S_2'(j)}{\sigma_2}. \tag{7}$$

Applying Schwarz inequality to the above expression, it is easy to see that the signal-to-noise ratio is maximized when $y_{j+i} \propto x_j$.

The above results tell us that if the two intrinsic time profiles are same except for an overall scalefactor and time lag (which will be the case if the burst is gravitationally lensed), then the cross-correlation will result in a sharp peak when these time profiles sit exactly on top of each other.

3. Results

We apply the method described in the previous section by simulating faint bursts. For this, we scale the time profile of the burst corresponding to trigger number 179, down by a factor 1.87, so that the peak counts are just 3σ above the mean background noise. The scaled time profile S_{179}, having length 17s is then assumed to be the intrinsic burst signal for our analysis. The burst counts are then obtained as

$$C_1(i) = n_1(i) + S_{179}(i) \qquad \text{and} \qquad C_2(i) = n_2(i) + A.S_{179}(i-L), \tag{8}$$

where A is a de-amplification factor lying in the range (0,1), L is the time lag index in number of bins, while n_1 and n_2 are statistically independent background noise counts. The question, then, is : How far can we decrease A and still maintain a sharp peak in $O_1 o O_2$ with a sufficiently high S/N ratio ?

The results we have obtained are shown in table I. The time lag index was taken to be L=300, which corresponds to a time delay of 17.216 s. From table I, it is clear that cross-correlation indeed show high peaks at or very close to 17.216 s. Therefore, this technique not only helps in identifying lensed time profiles but also in determining accurate value of the time-delay. Upto A $\sim$ 0.3, the cross-correlation shows a distinct peak with a full width at half maxima (FWHM) of 0.5 s and S/N ratio of 5. For A less than 0.2, $O_1 o O_2$ starts becoming noisy. What this implies is that in the case of a lensed event, if one of the images is detected with a burst peak at 3σ above the average background noise while the other is with a peak only at 1σ above the mean noise, it is still possible to say with high degree of confidence that both these events correspond to the same burst.

A similar analysis has also been carried out for the burst 451, for which we find that upto A $\sim$ 0.1 the cross-correlation peak is distinct with a S/N ratio over 3. This verifies the effectiveness of this technique in identifying faint lensed events. Since for the numerical evaluation of cross-correlations, standard algorithms use fast Fourier transforms, our method turns out also to be computationally efficient.

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Table I BURST 179_1 scaled down by 1.87; lag=300 bins

ampli. factor	Corr. peak	time of peak (s)	FWHM of peak (s)	noise N
1.0	3.45×10^7	17.216	1.024	2.12×10^6
0.5	1.63×10^7	17.216	1.04	1.8×10^6
0.25	7.77×10^6	17.024	0.576	1.71×10^6