Dynamic Sky Simulation with Scanning Sky Monitor on ASTROSAT using spherical trigonometry

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23rd Nov 2004

Abstract

This is a report on the dynamic sky simulation with the Scanning Sky Monitor (SSM) assembly aboard ASTROSAT implemented using spherical transformations. The canting of the slanted cameras also has been taken in to account.

Contents

Contact

The software has been generated by Ravi Shankar B T, Sushila R Mishra and Arun G under the supervision and guidance of Dipankar Bhattacharya.

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1 Introduction

The Scanning Sky Monitor assembly aboard ASTROSAT consists of three identical Coded Mask cameras mounted on a rotating boom as shown in figure 1. In the modified design the two slanted cameras will be canted towards the boom camera (instead of being perpendicular to the plane of the boom camera as shown in the figure).

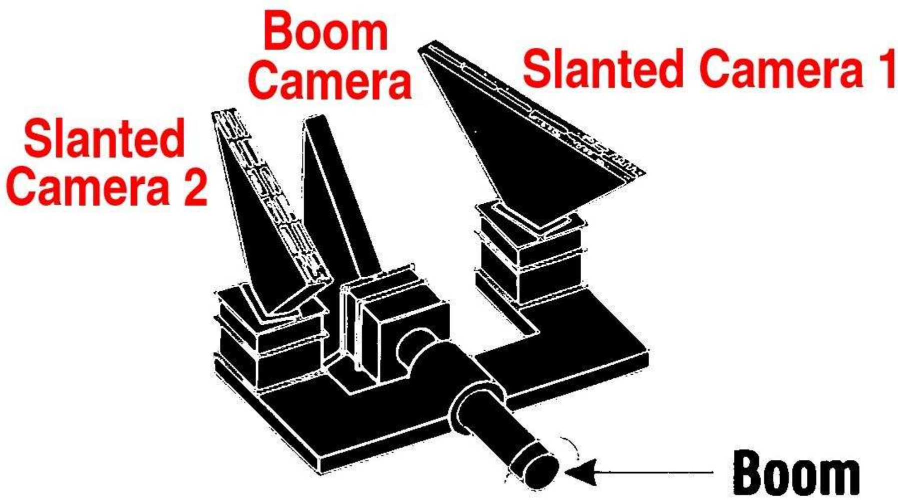

Figure 1: The SSM Assembly

The aim is to find transformation equations connecting several inclined co-ordinate systems on the celestial sphere (equatorial frame, boom camera frame and slanted camera frames). These can be obtained with the aid of spherical geometry, for example in the case of the relations between the equatorial and heliocentric co-ordinate systems (see pages 498 and 504 of book: 'Astrophysical Formulae', 2nd Edition, 1980, by Kenneth R. Lang). The graphical representation for such a system on the celestial sphere is shown in figure 2.

The transformation equations connecting the longitude, λ, and latitude, β, in the ecliptic to the right ascension, α, and declination, δ, are as follows:

$$\cos\beta\ \cos\lambda = \cos\delta\ \cos\alpha \tag{1}$$
$$\cos\beta\ \sin\lambda = \cos\delta\ \sin\alpha\ \cos\epsilon\ +\ \sin\delta\ \sin\epsilon \tag{2}$$
$$\sin\beta = \sin\delta\ \cos\epsilon\ -\ \cos\delta\ \sin\alpha\ \sin\epsilon \tag{3}$$

The inverse transformation equations are given by,

$$\cos\delta\ \cos\alpha = \cos\beta\ \cos\lambda \tag{4}$$
$$\cos\delta\ \sin\alpha = \cos\beta\ \sin\lambda\ \cos\epsilon\ -\ \sin\beta\ \sin\epsilon \tag{5}$$
$$\sin\delta = \cos\beta\ \sin\lambda\ \sin\epsilon\ +\ \sin\beta\ \cos\epsilon \tag{6}$$

2 Boom Camera Frame

Supposing the boom is pointing towards (α_B, δ_B) (which is also the center coordinates of the Field of View, FOV of the boom camera) as shown in figure 3. The position (α, δ) is that of a celestial source and to find its co-ordinates (λ, β) in the boom camera FOV frame, the equatorial plane is tilted in such a way that (α_B, δ_B) appears to be at the pole, shown as axes (X_1, Y_1, Z_1)

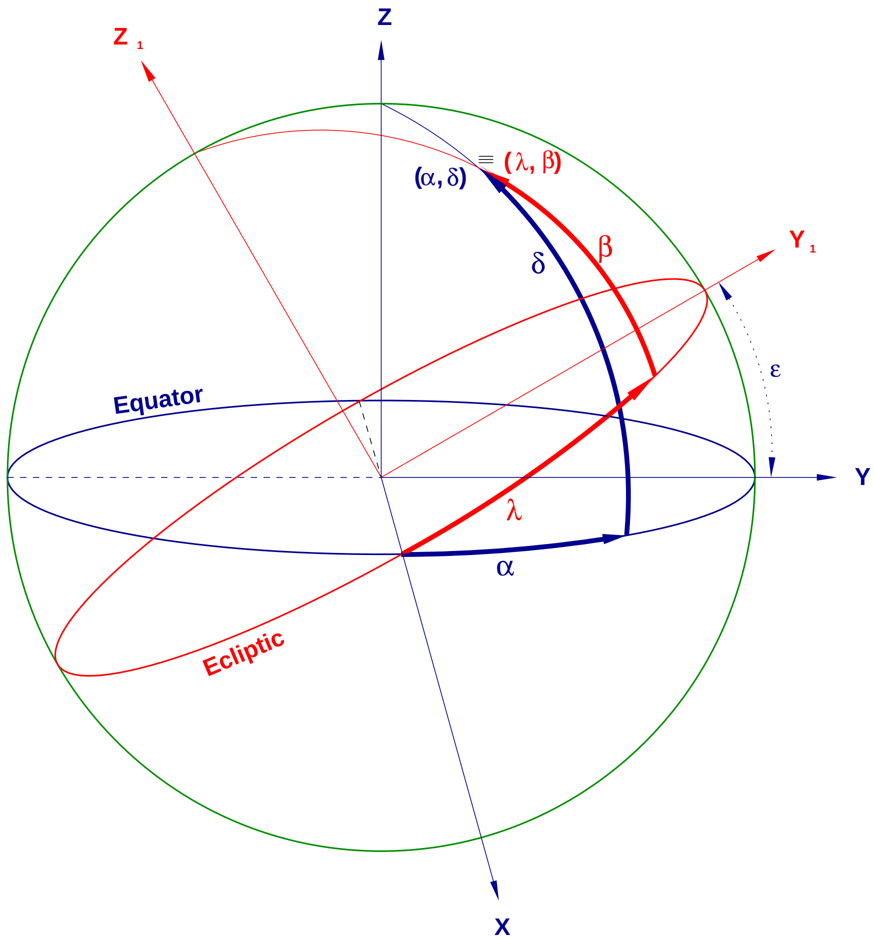

Figure 2: Equatorial and Heliocentric coordinates

in the figure. The co-ordinates of the node (one of the points of intersection of the two inclined planes) are α_N and λ_N in equatorial and inclined planes respectively.

Comparing the two geometries in figures 2 and 3,

- $(\alpha)_{fig\,2} \rightarrow (\alpha - \alpha_N)_{fig\,3}$
  - From figure 3,

$$\alpha_B = \alpha_N + \frac{3 \times \pi}{2}$$

$$\Rightarrow \alpha_N = \alpha_B - \frac{3 \times \pi}{2}(+\ 2 \times \pi) = \alpha_B + \frac{\pi}{2}$$

hence,

$$\alpha - \alpha_N = -\frac{\pi}{2} + (\alpha - \alpha_B) \tag{7}$$

- $(\lambda)_{fig\,2} \rightarrow (\lambda - \lambda_N)_{fig\,3}$
  - (with reference to figure 4, an excerpt of figure 3),

$$\phi_B - \frac{\pi}{2} = \frac{\pi}{2} - \lambda_N$$

$$\Rightarrow \lambda_N = \pi - \phi_B$$

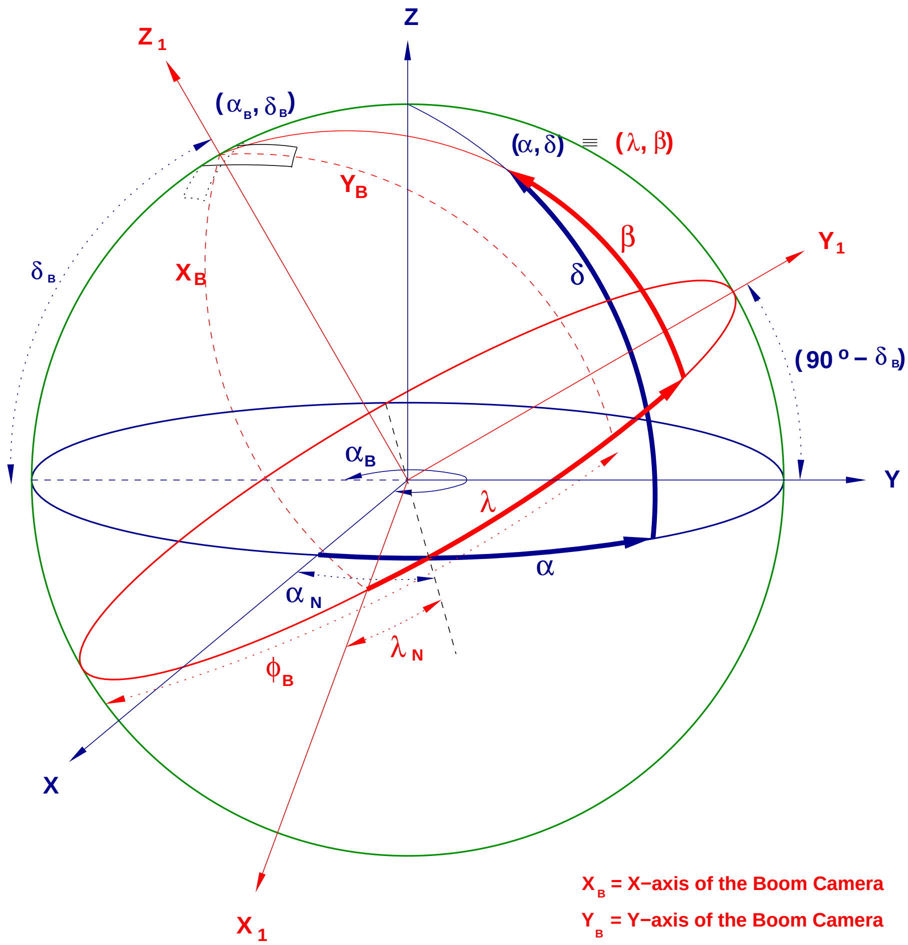

Figure 3: Boom Camera Frame

hence,

$$\lambda - \lambda_N = -\pi + (\lambda + \phi_B) \tag{8}$$

•

$$(\epsilon)_{fig\,2} = \left(\frac{\pi}{2} - \delta_B\right)_{fig\,3} \tag{9}$$

Replacing α by $(\alpha - \alpha_N)$ (& using equation 7), λ by $(\lambda - \lambda_N)$ (& using equation 8), ϵ by $(\frac{\pi}{2} - \delta_B)$ (equation 9) in the transformation equations 1, 2 and 3, the following transformation equations are obtained, linking the RA and declination (α, δ) of a celestial source with its coordinates (λ, β) in the boom camera FOV frame:

$$\cos\beta\ \cos(\lambda + \phi_B) = -\cos\delta\ \sin(\alpha - \alpha_B) \tag{10}$$

$$\cos\beta\ \sin(\lambda + \phi_B) = \cos\delta\ \cos(\alpha - \alpha_B)\ \sin\delta_B\ -\ \sin\delta\ \cos\delta_B \tag{11}$$

$$\sin\beta = \sin\delta\ \sin\delta_B\ +\ \cos\delta\ \cos(\alpha - \alpha_B)\ \cos\delta_B \tag{12}$$

Making similar modifications in the inverse transformation equations 4, 5 and 6, the following equations are obtained:

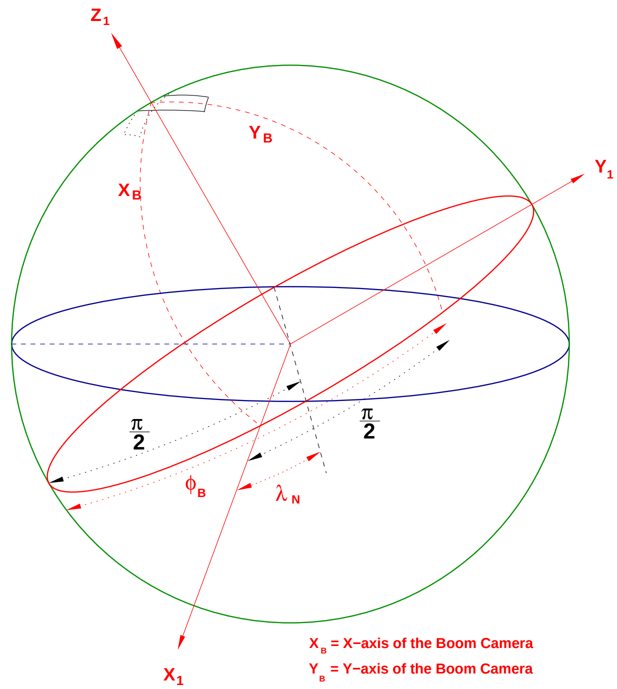

Figure 4: Excerpt from figure 3

$$\cos\delta\ \sin(\alpha-\alpha_B) = -\cos\beta\ \cos(\lambda+\phi_B) \tag{13}$$

$$\cos\delta\ \cos(\alpha-\alpha_B) = \cos\beta\ \sin(\lambda+\phi_B)\ \sin\delta_B\ +\ \sin\beta\ \cos\delta_B \tag{14}$$

$$\sin\delta = -\cos\beta\ \sin(\lambda+\phi_B)\ \cos\delta_B\ +\ \sin\beta\ \sin\delta_B \tag{15}$$

3 Slanted Camera frame

The common center co-ordinates of the slanted cameras lie on the Y-axis of the FOV of the boom camera. The boom camera frame in the figure 3 is positioned in such a way that the boom appears to be pointing towards the pole and the Y-axis of FOV of the boom camera coincides with the meridian passing through RA = 12 hour. Then the common center co-ordinates of the slanted cameras will be lying on this RA = 12 hour line, the declination being equal to 90° minus θ_C (where θ_C is the cant-angle, the angle between the center of the boom camera FOV and common center of the fields of view of the slanted cameras). The reference point (axis: X_2) is chosen such that the X-axes of the two slanted cameras lie on its either sides, each away by an angle equal to that of the inclination angle i, as shown in the figure 5. The resultant geometry is shown in the figure 6.

Comparing the figures 2 and 6,

- $$(\alpha)_{fig\,2} \rightarrow (\lambda)_{fig\,6} \tag{16}$$

- $$(\delta)_{fig\,2} \rightarrow (\beta)_{fig\,6} \tag{17}$$

- $$(\lambda)_{fig\,2} \rightarrow (\lambda_S \pm i)_{fig\,6} \tag{18}$$

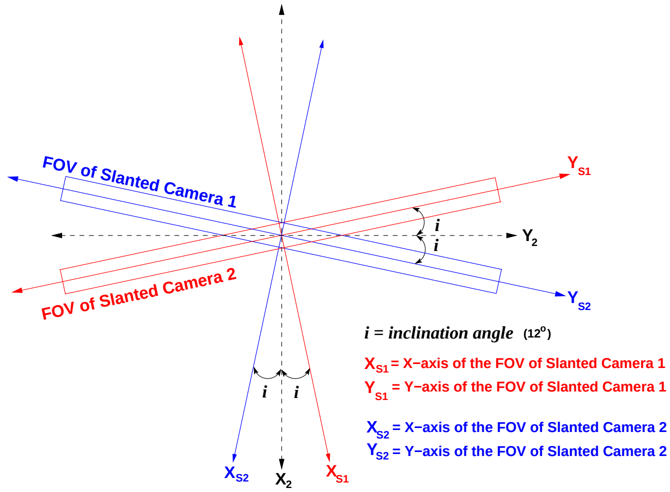

Figure 5: Slanted Camera Frame

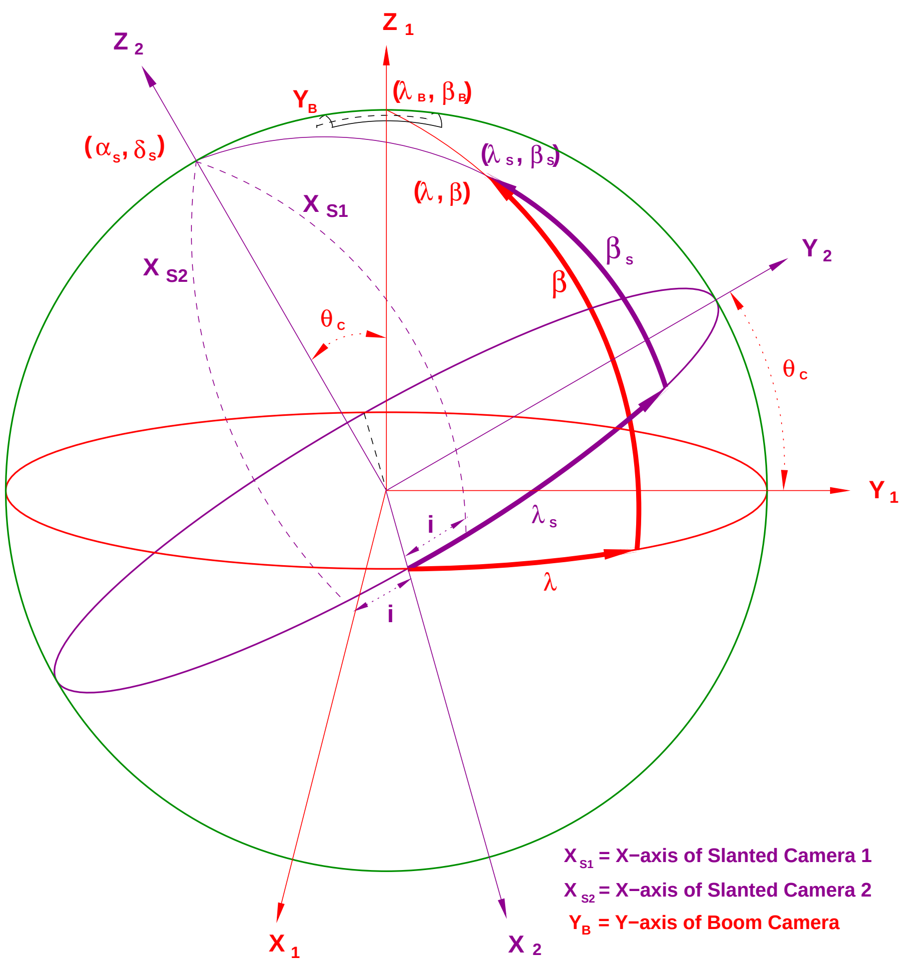

Figure 6: Slanted Camera Frame

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$$(\beta)_{fig\,2} \rightarrow (\beta_S)_{fig\,6} \tag{19}$$

Using equations 16, 17, 18 and 19, the equations 1, 2 and 3 are modified to represent the co-ordinates of the celestial source in the frame of the fields of view of the slanted cameras (λ_S, β_S) in terms of its co-ordinates in the boom camera FOV frame (λ,β). These transformation equations for the geometry in the figure 6 are:

$$\cos\beta_S\ \cos(\lambda_S \pm i) = \cos\beta\ \cos\lambda \tag{20}$$

$$\cos\beta_S\ \sin(\lambda_S \pm i) = \cos\beta\ \sin\lambda\ \cos\theta_C\ +\ \sin\beta\ \sin\theta_C \tag{21}$$

$$\sin\beta_S = \sin\beta\ \cos\theta_C\ -\ \cos\beta\ \sin\lambda\ \sin\theta_C \tag{22}$$

Making similar modifications to the inverse transformation equations 4, 5 and 6, the following are obtained:

$$\cos\beta\ \cos\lambda = \cos\beta_S\ \cos(\lambda_S \pm i) \tag{23}$$

$$\cos\beta\ \sin\lambda = \cos\beta_S\ \sin(\lambda_S \pm i)\ \cos\theta_C\ -\ \sin\beta_S\ \sin\theta_C \tag{24}$$

$$\sin\beta = \cos\beta_S\ \sin(\lambda_S \pm i)\ \sin\theta_C\ +\ \sin\beta_S\ \cos\theta_C \tag{25}$$

4 Obtaining (θ_x, θ_y) for a given (λ, β)

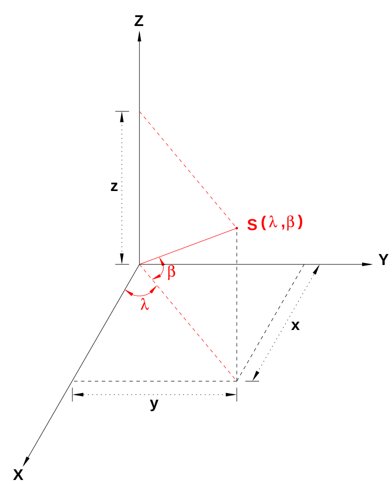


Figure 7:

With reference to figure 7,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\lambda\ \cos\beta \\ \sin\lambda\ \cos\beta \\ \sin\beta \end{pmatrix} \tag{26}$$

Further, x, y and z are related to θ_x and θ_y by the following relations:

$$\sin\theta_x = \frac{x}{\sqrt{x^2+z^2}} \qquad \cos\theta_x = \frac{z}{\sqrt{x^2+z^2}} \tag{27}$$

$$\Rightarrow \theta_x = \tan^{-1}\left(\frac{\sin\theta_x}{\cos\theta_x}\right) \tag{28}$$

$$\sin\theta_y = \frac{y}{\sqrt{y^2+z^2}} \qquad \cos\theta_y = \frac{z}{\sqrt{y^2+z^2}} \tag{29}$$

$$\Rightarrow \theta_y = \tan^{-1}\left(\frac{\sin\theta_y}{\cos\theta_y}\right) \tag{30}$$

5 Obtaining (λ, β) for a given (θ_x, θ_y)

From the equation 26,

$$\frac{y}{x} = \tan\lambda$$

Using equation 27, $\tan\theta_x = \frac{x}{z}$ and from equation 29, $\tan\theta_y = \frac{y}{z}$
Hence,

$$\tan\lambda = \frac{\tan\theta_y}{\tan\theta_x} \tag{31}$$

From the figure 7,

$$\begin{aligned}
\tan\left(\tfrac{\pi}{2} - \beta\right) &= \frac{\sqrt{x^2+y^2}}{z} \\
&= \sqrt{\left(\tfrac{x}{z}\right)^2 + \left(\tfrac{y}{z}\right)^2} \\
&= \sqrt{\tan^2\theta_x + \tan^2\theta_y} \qquad \text{(using equations 27 and 29)}
\end{aligned}$$

Hence,

$$\beta = \frac{\pi}{2} - \tan^{-1}\left(\sqrt{\tan^2\theta_x + \tan^2\theta_y}\right) \tag{32}$$

6 Hammer-Aitoff Projection

Hammer-Aitoff Projection is an equal area projection, showing the sky as an ellipse. The transformation equations connecting the x, y to RA,dec $= (\alpha, \delta)$ are as follows:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \dfrac{2 \times \cos\delta \times \sin\left(\frac{\alpha}{2}\right)}{\sqrt{1 + \cos\delta \times \cos\left(\frac{\alpha}{2}\right)}} \\ \\ \dfrac{\sin\delta}{\sqrt{1 + \cos\delta \times \cos\left(\frac{\alpha}{2}\right)}} \end{pmatrix} \tag{33}$$

To be able to use the Hammer-Aitoff transformation equations, it is necessary that the RA (α) is in the range $-180°$ to $+180°$. In our software, values of α in the range $180°$ to $360°$ are mapped to $-180°$ to $0°$, as shown in the figure 8.

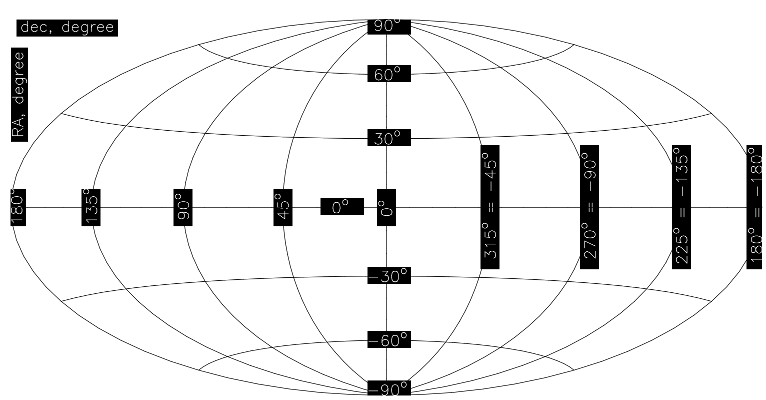


Figure 8: Projection of the sky using the Hammer-Aitoff Transformation

7 Conversion from Equatorial to Galactic Co-ordinates

The transformation equations connecting the equatorial co-ordinates, (Right Ascension, Declination) = (α, δ), and the galactic co-ordinates, (longitude, latitude) = (l, b), are given by the following:

$$\sin b = \sin\delta_{GP}\ \sin\delta\ +\ \cos\delta_{GP}\ \cos\delta\ \cos(\alpha - \alpha_{GP}) \tag{34}$$

$$\cos b\ \sin(l_{CP} - l) = \cos\delta\ \sin(\alpha - \alpha_{GP}) \tag{35}$$

$$\cos b\ \cos(l_{CP} - l) = \cos\delta_{GP}\ \sin\delta\ -\ \sin\delta_{GP}\ \cos\delta\ \cos(\alpha - \alpha_{GP}) \tag{36}$$

where,
$\alpha_{GP} \equiv$ Right Ascension of the (North) Galactic Pole (= 192.85948402°, J2000)
$\delta_{GP} \equiv$ Declination of the (North) Galactic Pole (= 27.128296370°, J2000)
$l_{CP} \equiv$ Longitude of the (North) Celestial Pole (= 122.93193212°)

8 The Software

The output display is divided in to four panels, as shown in figure 9.

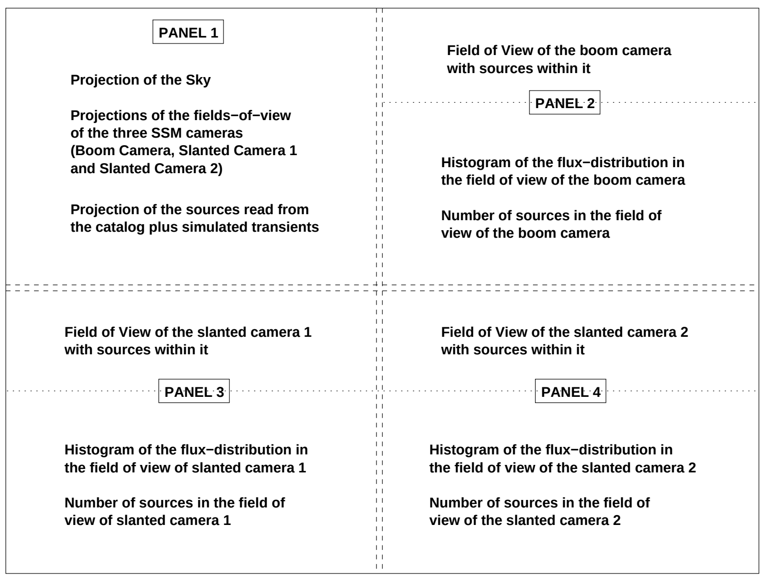
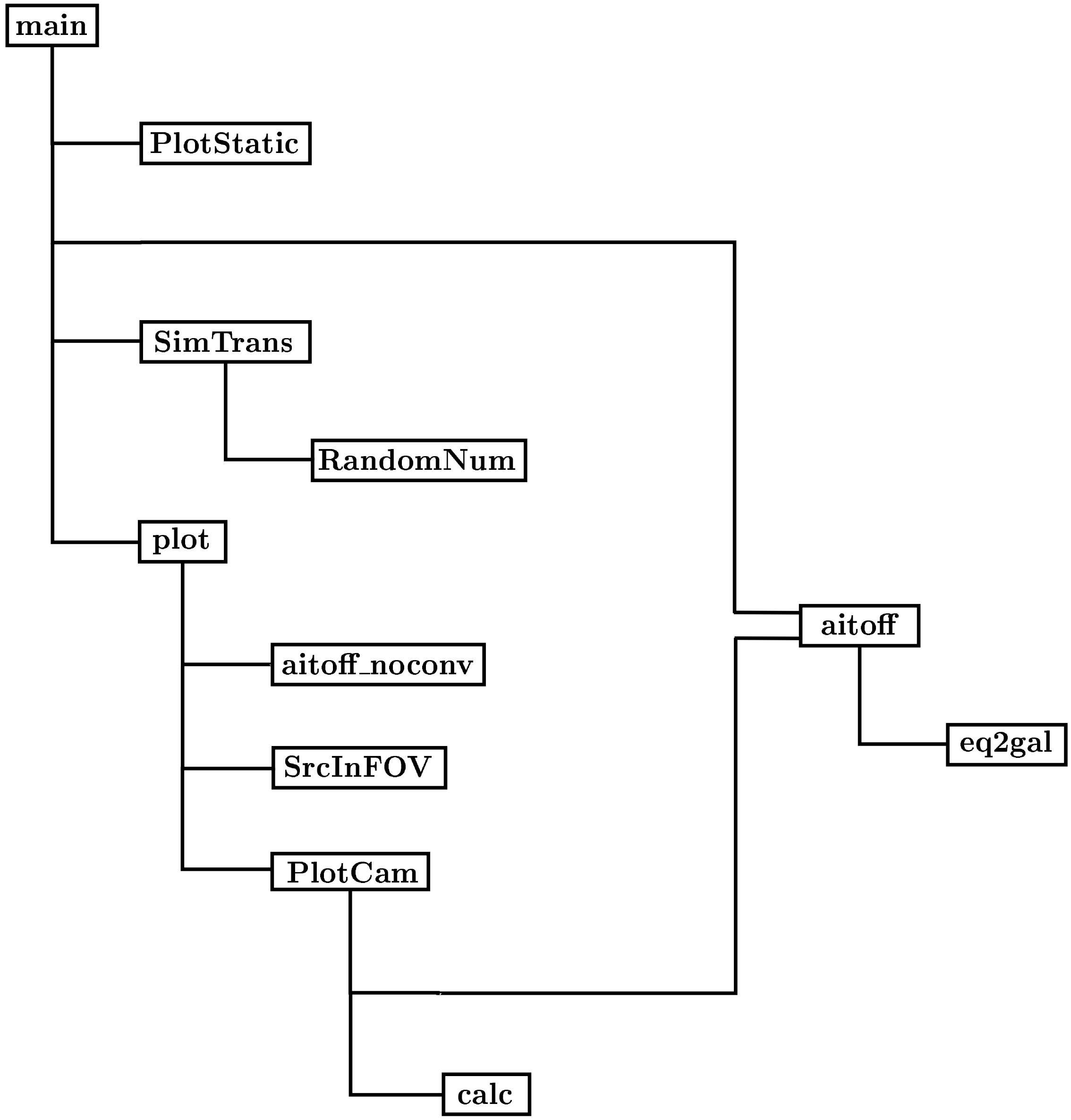
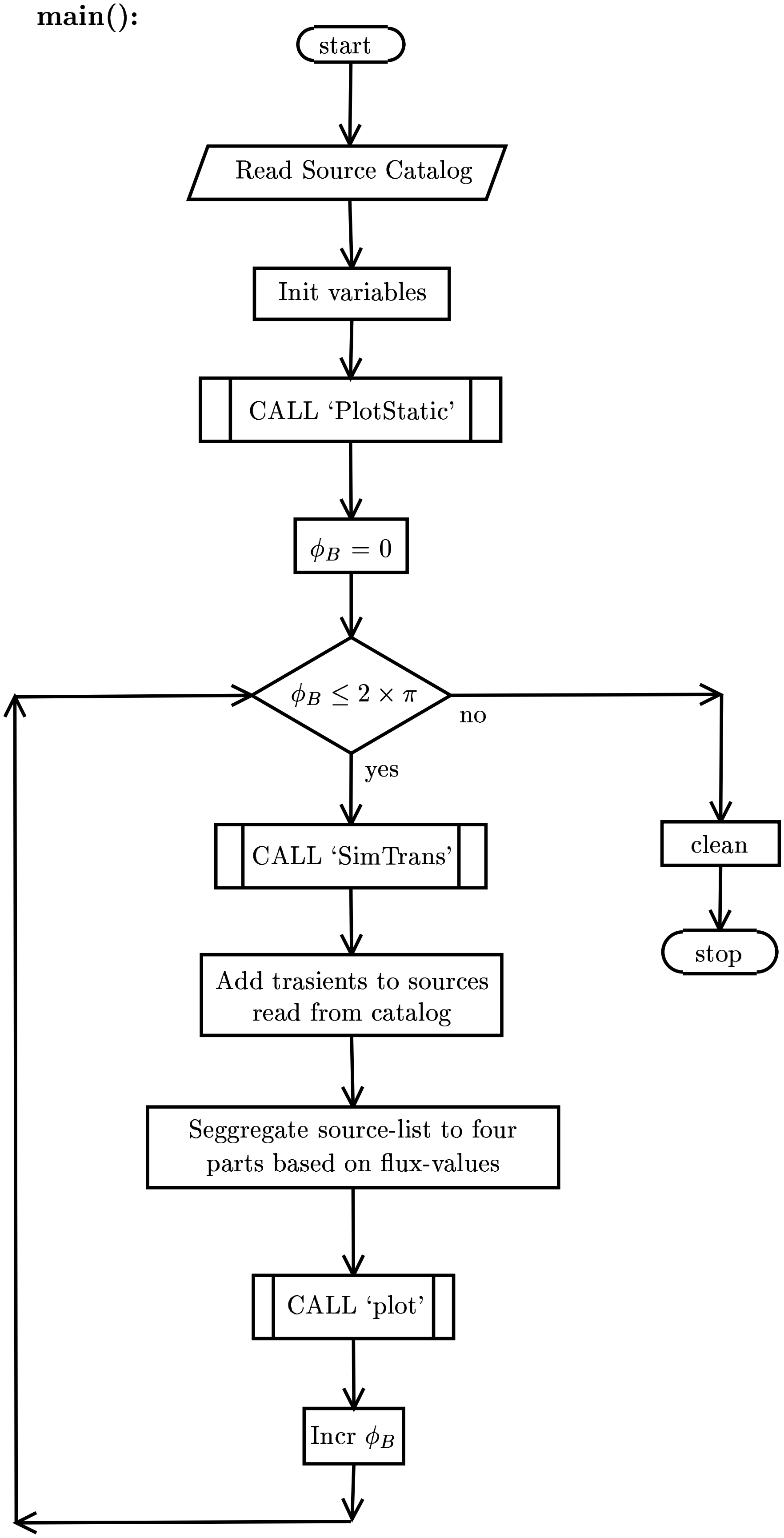


Figure 9: The output display

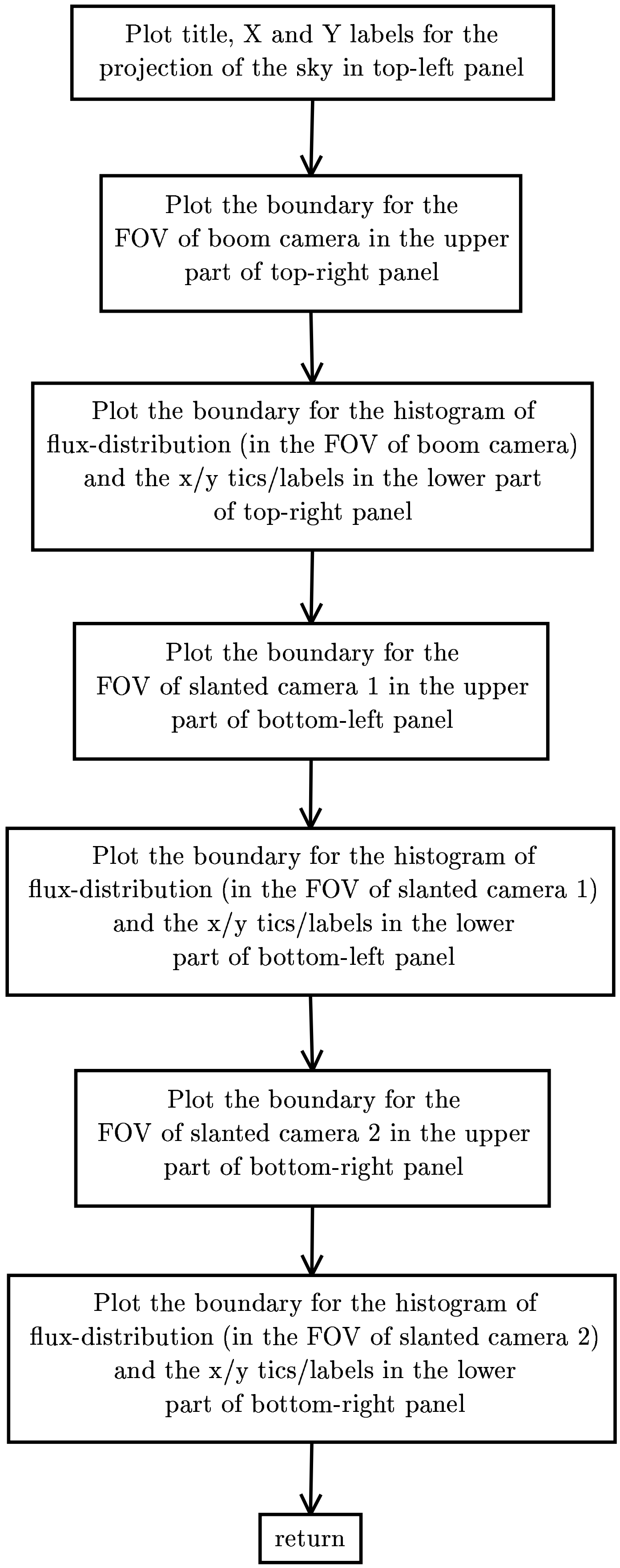
The following chart lists the procedures used in the software and also shows the dependencies. In the subsequent pages, the flowcharts for each of these procedures are displayed. The flowcharts describe the basic implementation only, the actual software has many more additional instructions, which, one should be able to learn from the documentation in the codes.


main
PlotStatic
SimTrans
RandomNum
plot
aitoff
aitoff_noconv
eq2gal
SrcInFOV
PlotCam
calc

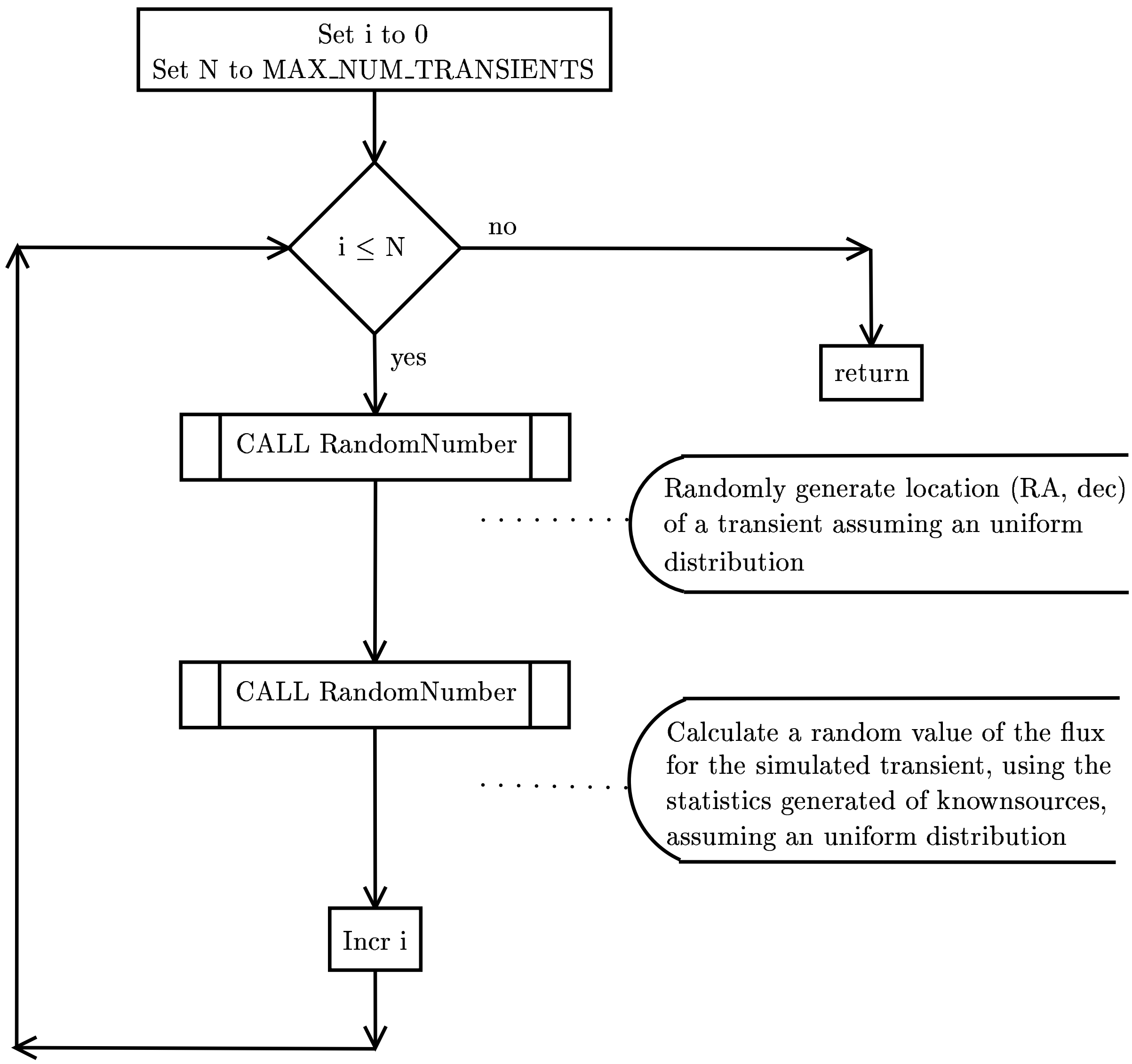
main():


start
Read Source Catalog
Init variables
CALL 'PlotStatic'
$\phi_B = 0$
$\phi_B \leq 2 \times \pi$
no
yes
CALL 'SimTrans'
clean
stop
Add trasients to sources read from catalog
Seggregate source-list to four parts based on flux-values
CALL 'plot'
Incr ϕ_B

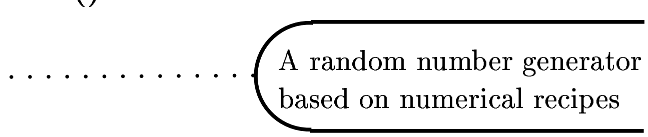
PlotStatic():


Plot title, X and Y labels for the projection of the sky in top-left panel
Plot the boundary for the FOV of boom camera in the upper part of top-right panel
Plot the boundary for the histogram of flux-distribution (in the FOV of boom camera) and the x/y tics/labels in the lower part of top-right panel
Plot the boundary for the FOV of slanted camera 1 in the upper part of bottom-left panel
Plot the boundary for the histogram of flux-distribution (in the FOV of slanted camera 1) and the x/y tics/labels in the lower part of bottom-left panel
Plot the boundary for the FOV of slanted camera 2 in the upper part of bottom-right panel
Plot the boundary for the histogram of flux-distribution (in the FOV of slanted camera 2) and the x/y tics/labels in the lower part of bottom-right panel
return

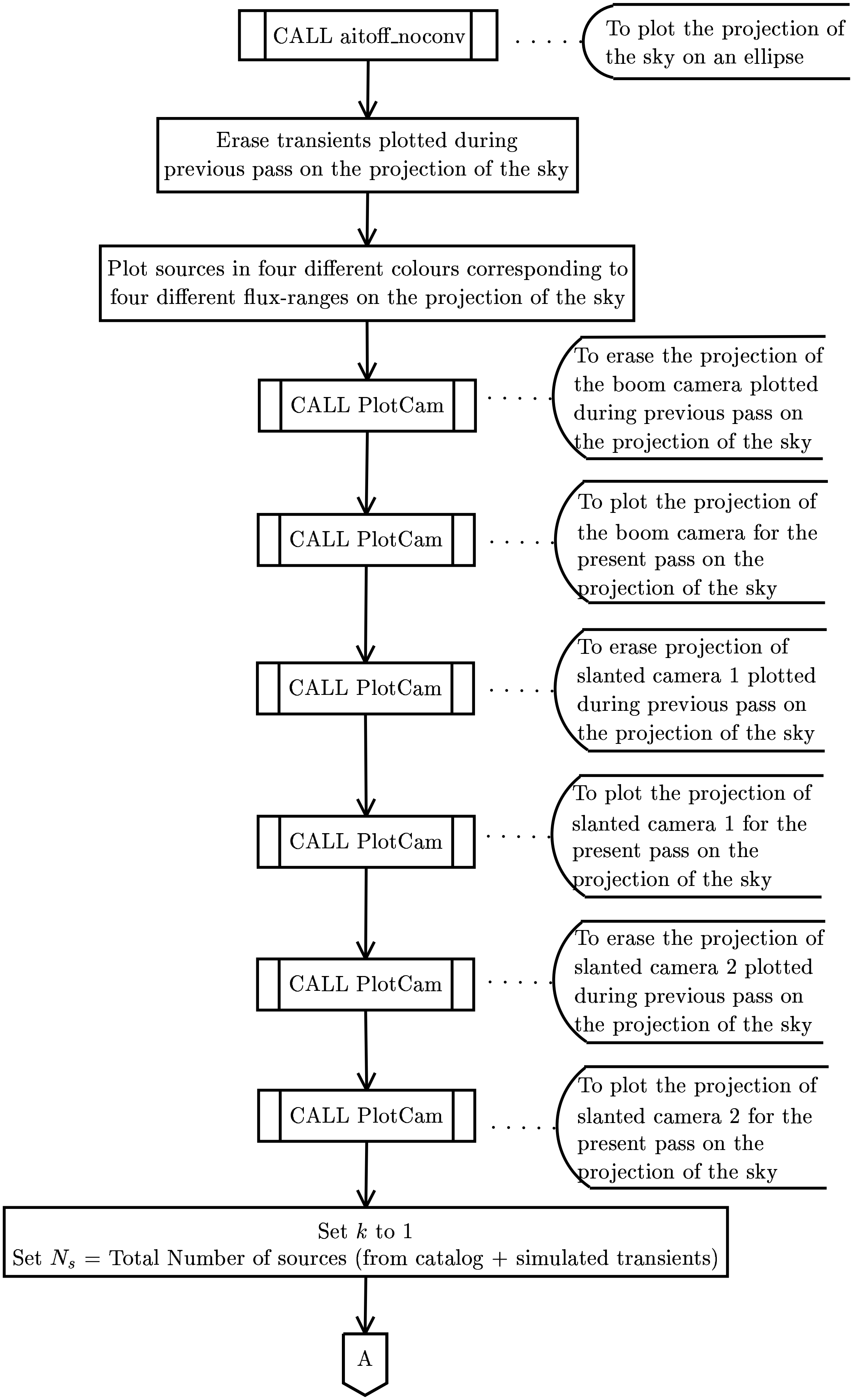
SimTrans():



RandomNumber():

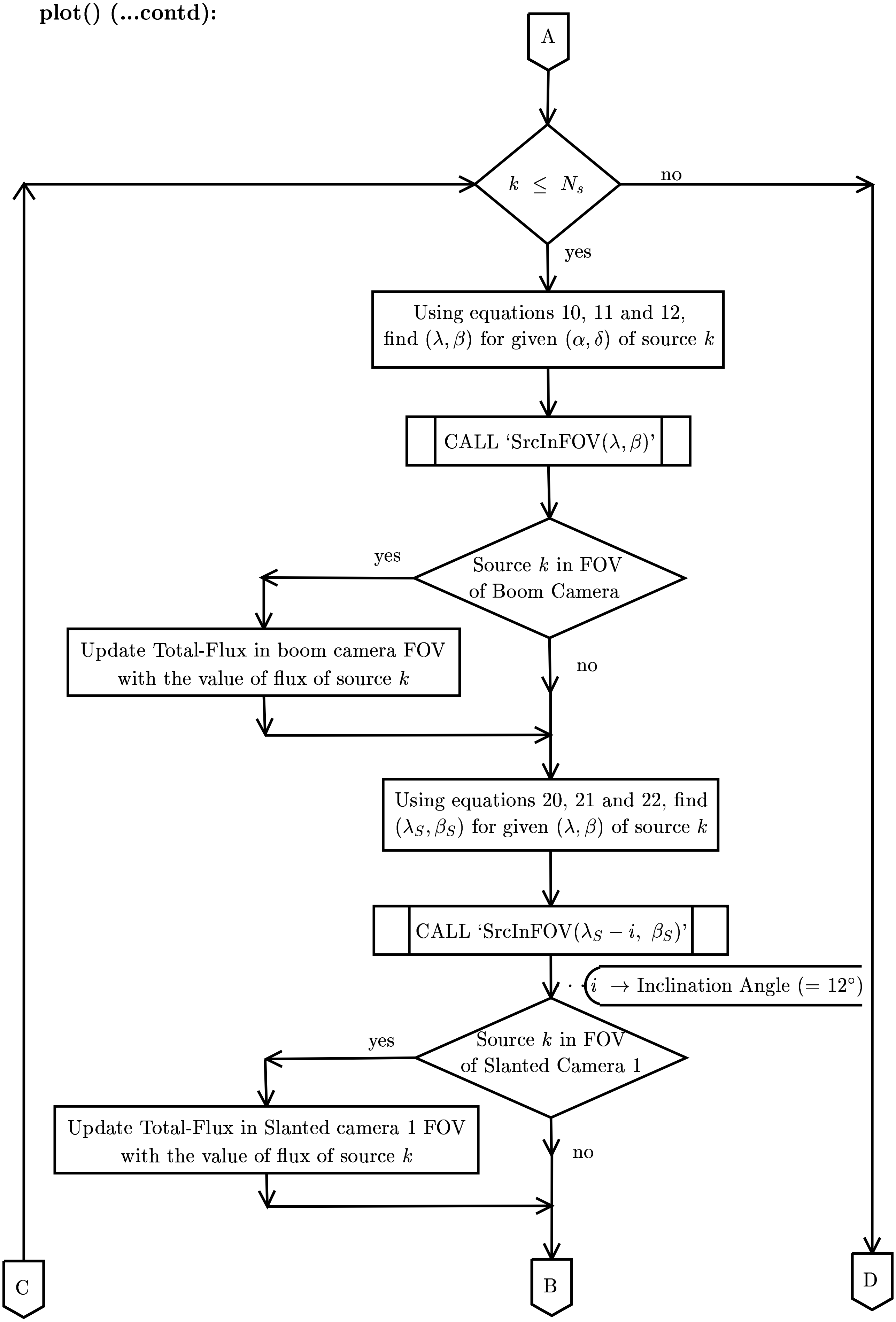


plot():

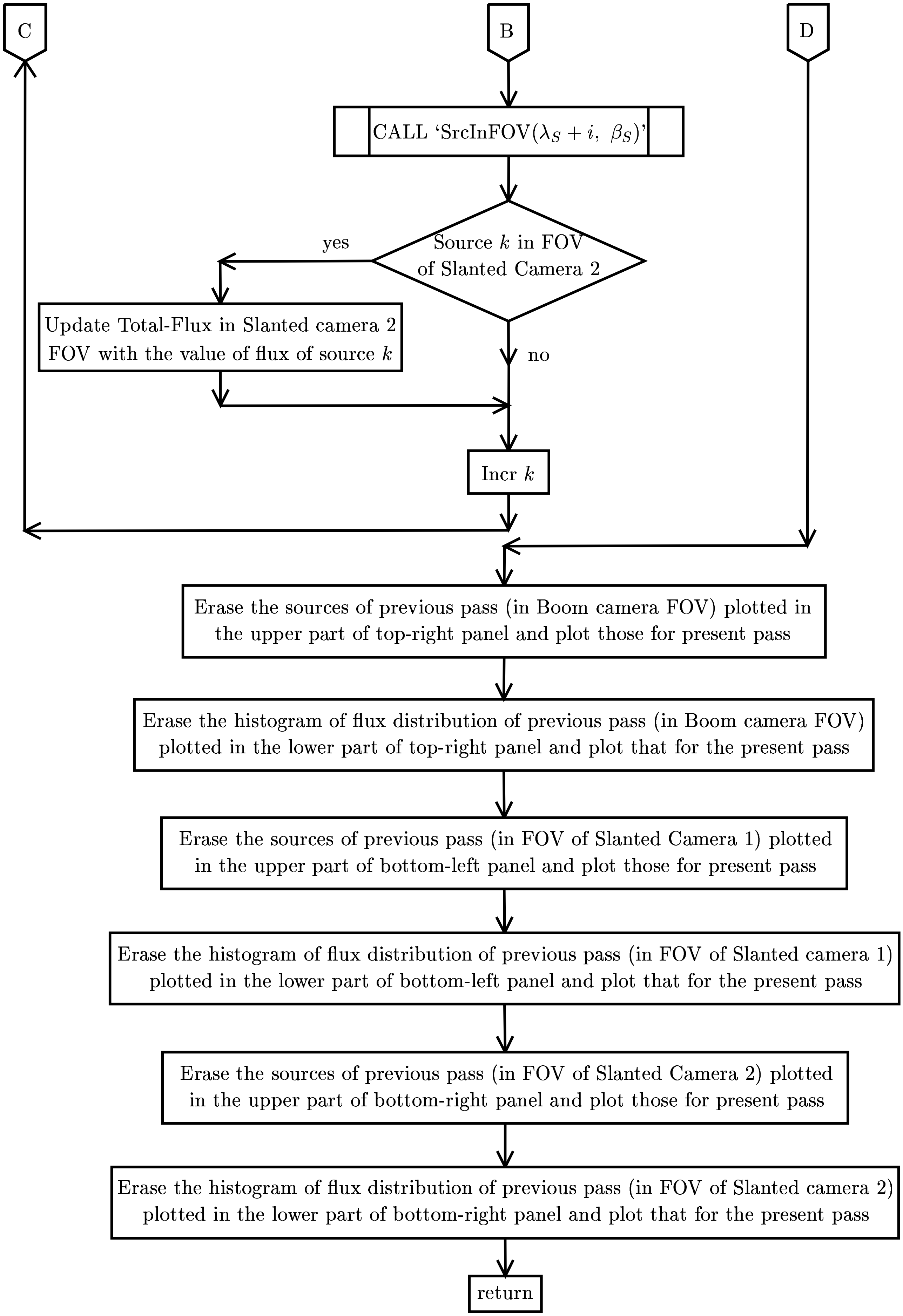


CALL aitoff_noconv
To plot the projection of the sky on an ellipse
Erase transients plotted during previous pass on the projection of the sky
Plot sources in four different colours corresponding to four different flux-ranges on the projection of the sky
CALL PlotCam
To erase the projection of the boom camera plotted during previous pass on the projection of the sky
CALL PlotCam
To plot the projection of the boom camera for the present pass on the projection of the sky
CALL PlotCam
To erase projection of slanted camera 1 plotted during previous pass on the projection of the sky
CALL PlotCam
To plot the projection of slanted camera 1 for the present pass on the projection of the sky
CALL PlotCam
To erase the projection of slanted camera 2 plotted during previous pass on the projection of the sky
CALL PlotCam
To plot the projection of slanted camera 2 for the present pass on the projection of the sky
Set k to 1
Set N_s = Total Number of sources (from catalog + simulated transients)
A

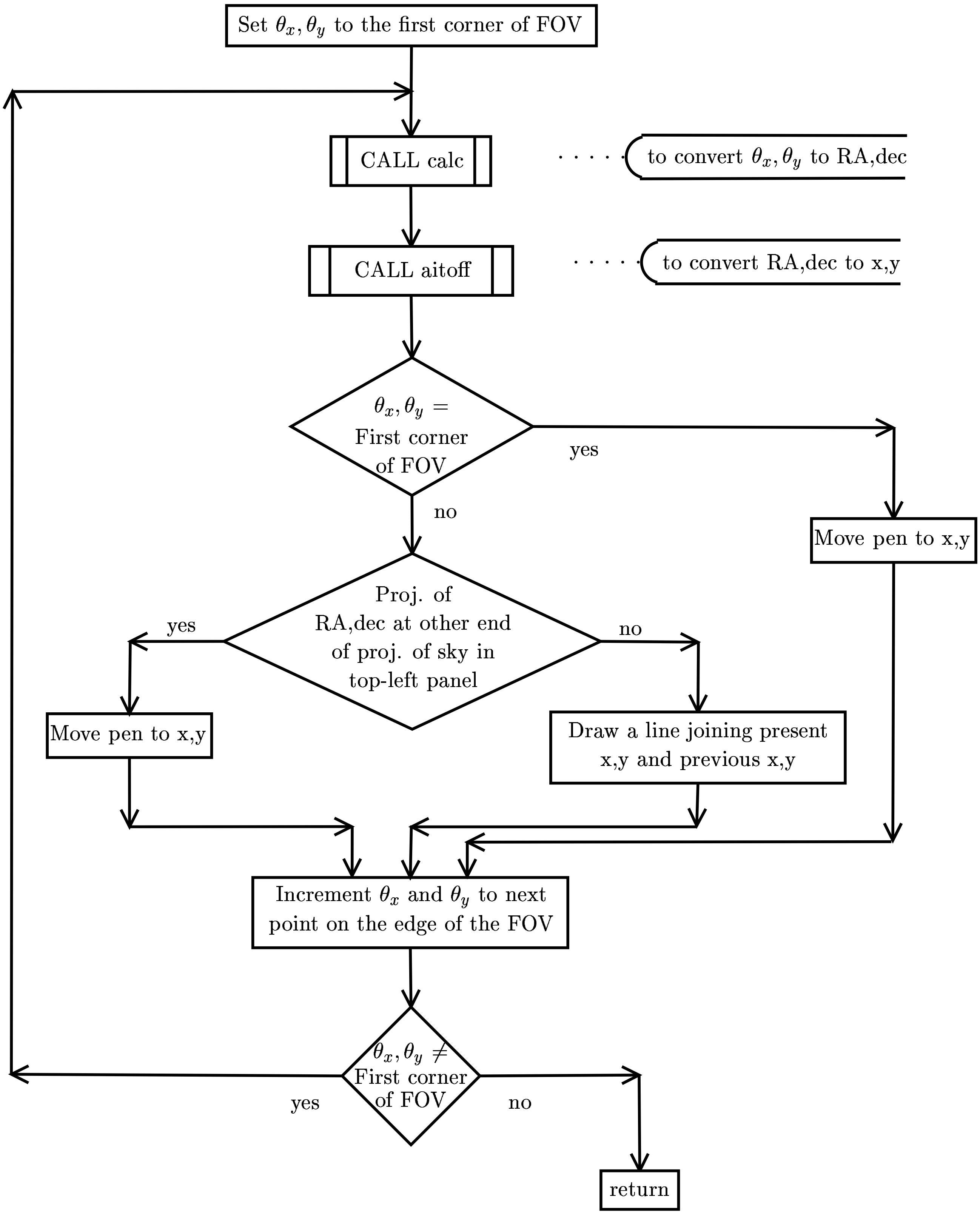
plot() (...contd):


A
$k \leq N_s$
no
yes
Using equations 10, 11 and 12, find (λ, β) for given (α, δ) of source k
CALL 'SrcInFOV(λ, β)'
yes
Source k in FOV of Boom Camera
Update Total-Flux in boom camera FOV with the value of flux of source k
no
Using equations 20, 21 and 22, find (λ_S, β_S) for given (λ, β) of source k
CALL 'SrcInFOV($\lambda_S - i,\ \beta_S$)'
$i \rightarrow$ Inclination Angle (= 12°)
yes
Source k in FOV of Slanted Camera 1
Update Total-Flux in Slanted camera 1 FOV with the value of flux of source k
no
C
B
D

plot() (...contd):

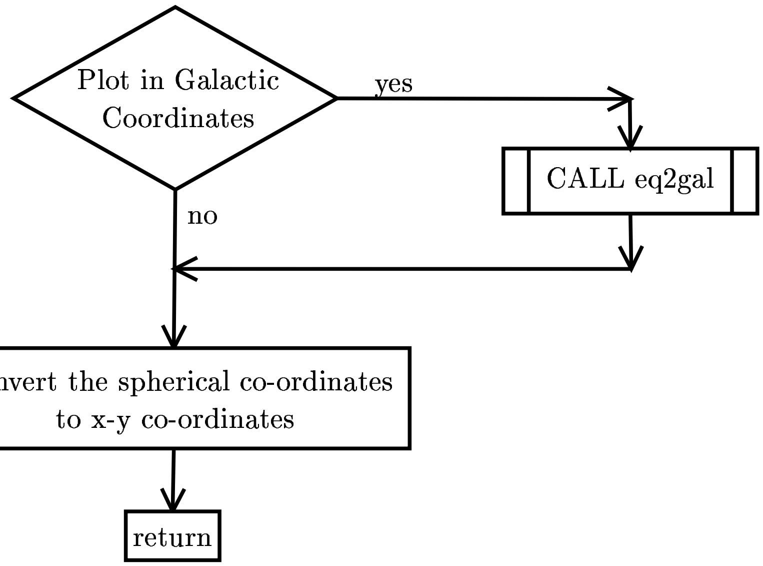

C
B
D
CALL 'SrcInFOV($\lambda_S + i,\ \beta_S$)'
Source k in FOV of Slanted Camera 2
yes
no
Update Total-Flux in Slanted camera 2 FOV with the value of flux of source k
Incr k
Erase the sources of previous pass (in Boom camera FOV) plotted in the upper part of top-right panel and plot those for present pass
Erase the histogram of flux distribution of previous pass (in Boom camera FOV) plotted in the lower part of top-right panel and plot that for the present pass
Erase the sources of previous pass (in FOV of Slanted Camera 1) plotted in the upper part of bottom-left panel and plot those for present pass
Erase the histogram of flux distribution of previous pass (in FOV of Slanted camera 1) plotted in the lower part of bottom-left panel and plot that for the present pass
Erase the sources of previous pass (in FOV of Slanted Camera 2) plotted in the upper part of bottom-right panel and plot those for present pass
Erase the histogram of flux distribution of previous pass (in FOV of Slanted camera 2) plotted in the lower part of bottom-right panel and plot that for the present pass
return

PlotCam():

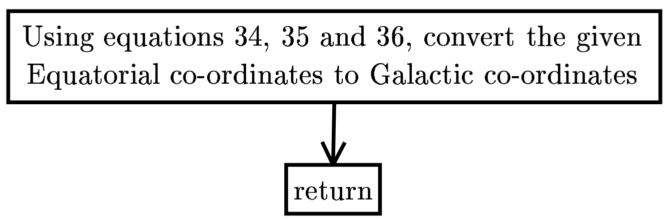


Set θ_x, θ_y to the first corner of FOV
CALL calc
to convert θ_x, θ_y to RA,dec
CALL aitoff
to convert RA,dec to x,y
$\theta_x, \theta_y =$ First corner of FOV
yes
no
Move pen to x,y
Proj. of RA,dec at other end of proj. of sky in top-left panel
yes
no
Move pen to x,y
Draw a line joining present x,y and previous x,y
Increment θ_x and θ_y to next point on the edge of the FOV
$\theta_x, \theta_y \neq$ First corner of FOV
yes
no
return

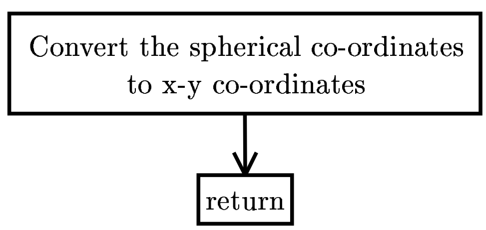
aitoff():



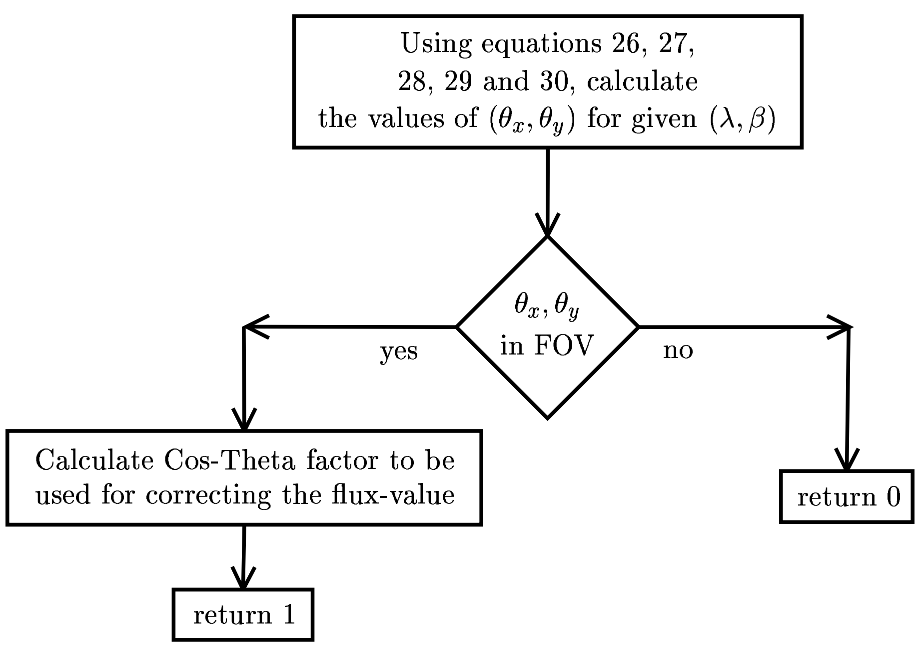
eq2gal():



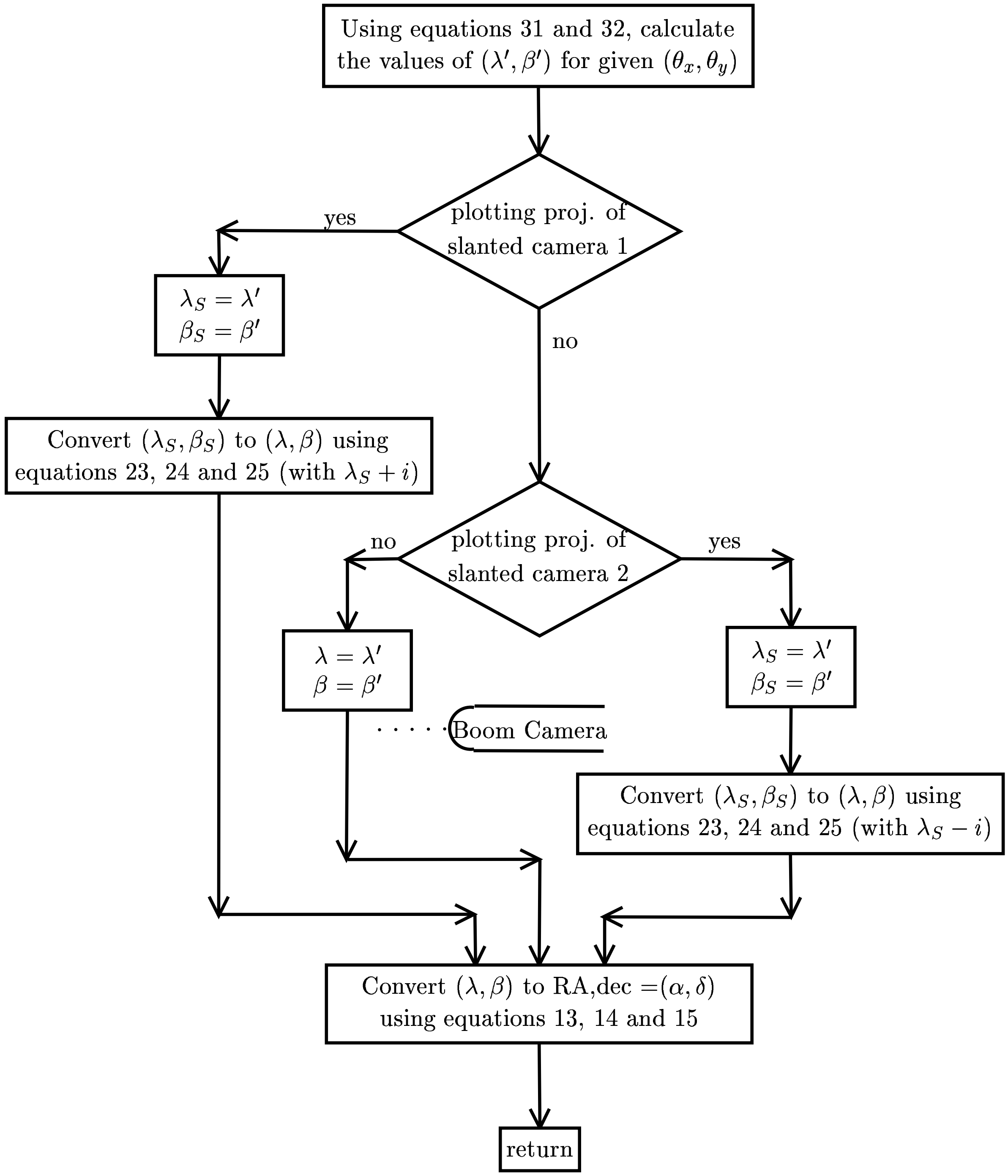
aitoff_noconv():



SrcInFOV():



calc():



Using equations 31 and 32, calculate the values of (λ', β') for given (θ_x, θ_y)
plotting proj. of slanted camera 1
yes
no
$\lambda_S = \lambda'$
$\beta_S = \beta'$
Convert (λ_S, β_S) to (λ, β) using equations 23, 24 and 25 (with $\lambda_S + i$)
plotting proj. of slanted camera 2
no
yes
$\lambda = \lambda'$
$\beta = \beta'$
Boom Camera
$\lambda_S = \lambda'$
$\beta_S = \beta'$
Convert (λ_S, β_S) to (λ, β) using equations 23, 24 and 25 (with $\lambda_S - i$)
Convert (λ, β) to RA,dec $=(\alpha, \delta)$ using equations 13, 14 and 15
return