Coning in the Boom Camera axis of the SSM assembly

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Abstract

In this report we discuss the procedure of simulation of the coning of the boom camera axis, that is the boom camera axis not being aligned with the rotation axis of the Scanning Sky Monitor assembly. We show that it will be possible to derive the coning angle by registering the trajectory of a source present near the centre of the field of view of the boom camera.

Introduction

The Scanning Sky Monitor (SSM) aboard ASTROSAT is a system of three identical coded mask cameras mounted on a rotatable platform.The rotation axis of the SSM assembly is intended to be aligned with the direction of the centre of the field of view (FOV) of the boom camera (the latter is referred to as the boom camera axis). Due to mechanical constraints, the boom camera axis may not be exactly aligned with the rotation axis. With such an arrangement, when the SSM assembly is rotated, the boom camera axis will describe a circular trajectory around the rotation axis (this is referred to as the *coning of the boom camera axis*). The trajectories of the sources in the fields of view of each of the three SSM cameras will alter as a function of the coning angle. The knowledge of the trajectory of the sources is needed to be able to do a better localisation. In this report we discuss a method to simulate the coning and show how the coning angle can be derived from observations of celestial sources.

Spherical Transformations

We utilise the spherical transformations between inclined co-ordinate systems, similar to the one used earlier for the dynamic sky simulation (report RRI_SSM_0010) to transform the co-ordinates in the frame of the rotation axis to the equatorial frame.

The transformation equations connecting the longitude, λ, and latitude, β, in the ecliptic to the equatorial co-ordinates of right ascension, α, and declination, δ, are as follows (figure 1):

$$\cos\beta\ \cos\lambda = \cos\delta\ \cos\alpha \tag{1}$$

$$\cos\beta\ \sin\lambda = \cos\delta\ \sin\alpha\ \cos\epsilon\ +\ \sin\delta\ \sin\epsilon \tag{2}$$

$$\sin\beta = \sin\delta\ \cos\epsilon\ -\ \cos\delta\ \sin\alpha\ \sin\epsilon \tag{3}$$

The inverse transformation equations are given by,

$$\cos\delta\ \cos\alpha = \cos\beta\ \cos\lambda \tag{4}$$

$$\cos\delta\ \sin\alpha = \cos\beta\ \sin\lambda\ \cos\epsilon\ -\ \sin\beta\ \sin\epsilon \tag{5}$$

$$\sin\delta = \cos\beta\ \sin\lambda\ \sin\epsilon\ +\ \sin\beta\ \cos\epsilon \tag{6}$$

Let the rotation axis co-ordinates be (α_d, δ_d) and that of the boom camera axis be (α_B, δ_B) as shown in the figure 2. θ_T is the Coning angle between the rotation axis and the boom camera axis and ϕ_B is the inclination angle of the FOV of the boom camera with respect to the meridian passing through the centre of the boom camera (see figure 3 of the report RRI_SSM_0010). α_N and Δ_ϕ are the co-ordinates of the reference node (one of the points of intersection of the two inclined planes). Δ_ϕ is the offset between the reference from which the inclination angle of the FOV of the boom camera is measured and the reference node. If we consider an inclined co-ordinate system with the pole at (α_d, δ_d) and axes $(\mathrm{X}_1, \mathrm{Y}_1, \mathrm{Z}_1)$, the equatorial co-ordinates, (α_B, δ_B) will be equivalent to (ϕ_B, $90^\circ - \theta_T$) in the frame of the rotation axis.

Comparing the two geometries in figures 1 and 2,

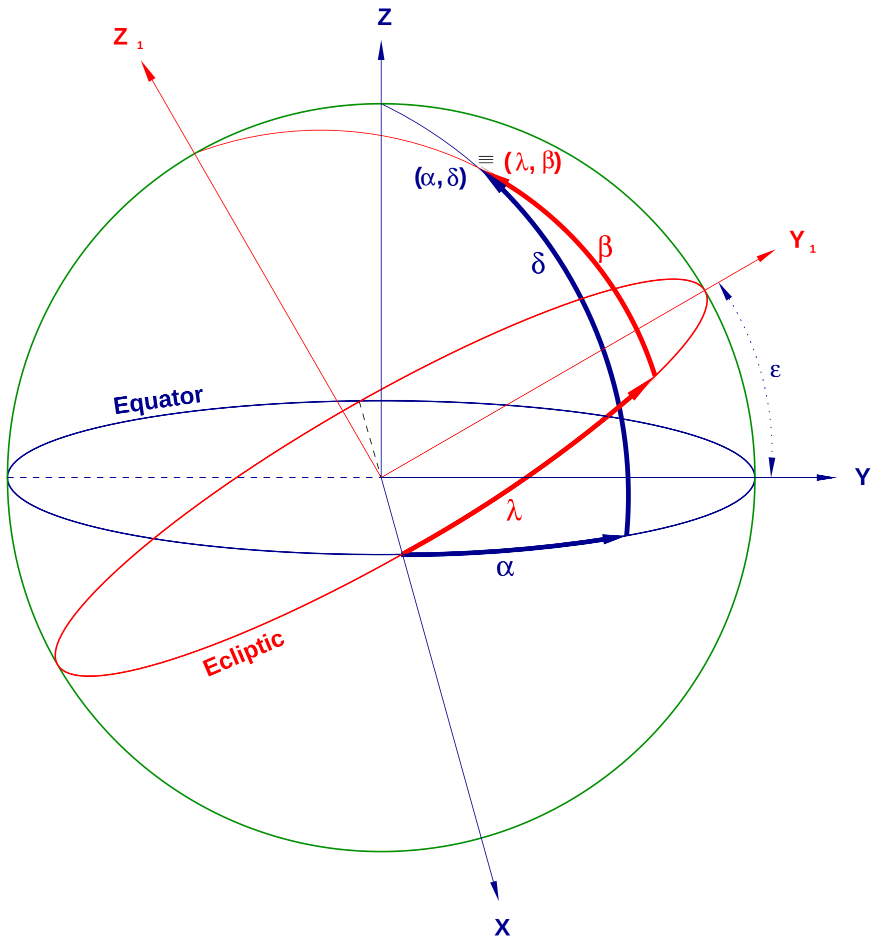

Figure 1: Equatorial and Heliocentric coordinates

- $(\alpha)_{fig\,1} \rightarrow (\alpha_B - \alpha_N)_{fig\,2}$
  - From figure 2,

$$\alpha_d = \alpha_N + \frac{3\pi}{2}$$

$$\Rightarrow \alpha_N = \alpha_d - \frac{3\pi}{2}(+\ 2\pi) = \alpha_d + \frac{\pi}{2}$$

hence,

$$\alpha_B - \alpha_N = -\frac{\pi}{2} + (\alpha_B - \alpha_d) \tag{7}$$

- $$(\delta)_{fig\,1} = (\delta_B)_{fig\,2} \tag{8}$$

- $$(\lambda)_{fig\,1} = (\phi_B - \Delta_\phi)_{fig\,2} \tag{9}$$

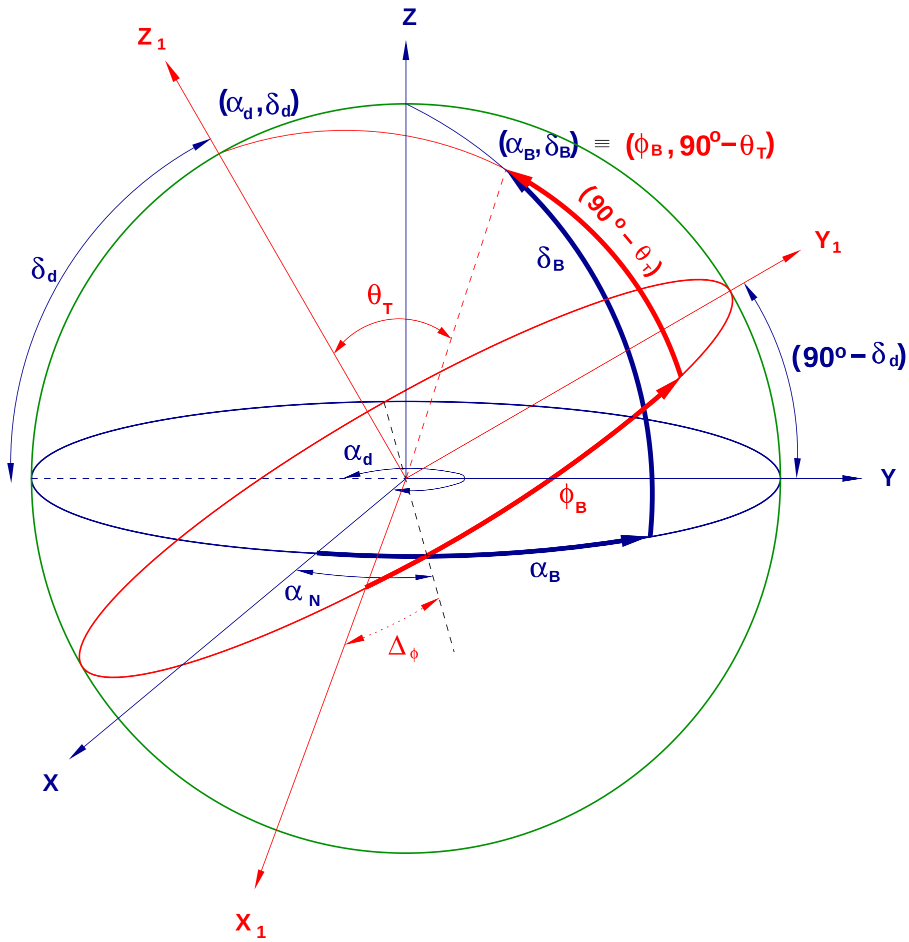

Figure 2: The geometry of the mounted and the boom camera axes with the coning angle, θ_T

- $$(\beta)_{fig\,1} = \left(\frac{\pi}{2} - \theta_T\right)_{fig\,2} \tag{10}$$

- $$(\epsilon)_{fig\,1} = \left(\frac{\pi}{2} - \delta_d\right)_{fig\,2} \tag{11}$$

Replacing α by $(\alpha - \alpha_N)$ (& using equation 7), and also using the relationships 8, 9, 10, and 11 in the transformation equations 1, 2 and 3, we get the following equations,

$$\sin\theta_T\ \cos(\phi_B - \Delta_\phi) = \cos\delta_B\ \sin(\alpha_B - \alpha_d) \tag{12}$$

$$\sin\theta_T\ \sin(\phi_B - \Delta_\phi) = -\cos\delta_B\ \cos(\alpha_B - \alpha_d)\ \sin\delta_d\ +\ \sin\delta_B\ \cos\delta_d \tag{13}$$

$$\cos\theta_T = \sin\delta_B\ \sin\delta_d\ +\ \cos\delta_B\ \cos(\alpha_B - \alpha_d)\ \cos\delta_d \tag{14}$$

Making similar modifications in the inverse transformation equations 4, 5 and 6, the following equations are obtained:

$$\cos\delta_B \ \sin(\alpha_B - \alpha_d) = \sin\theta_T \ \cos(\phi_B - \Delta_\phi) \tag{15}$$

$$\cos\delta_B \ \cos(\alpha_B - \alpha_d) = -\sin\theta_T \ \sin(\phi_B - \Delta_\phi) \ \sin\delta_d \ + \ \cos\theta_T \ \cos\delta_d \tag{16}$$

$$\sin\delta_B = \sin\theta_T \ \sin(\phi_B - \Delta_\phi) \ \cos\delta_d \ + \ \cos\theta_T \ \sin\delta_d \tag{17}$$

We use these inverse transformation equations 15, 16 and 17 to derive the co-ordinates of the boom camera axis (α_B, δ_B), for the specified co-ordinates of the rotation axis (α_d, δ_d), Coning angle θ_T, inclination angle of the boom camera FOV ϕ_B and the offset angle Δ_ϕ.

The co-ordinates of the boom camera axis (α_B, δ_B) thus obtained are used to transform the equatorial co-ordinates (α, δ) into the boom camera FOV frame co-ordinates (λ, β) (see equations 10, 11, 12 in the report RRI_SSM_0010 and the rest of the procedure of obtained the co-ordinates in the slanted camera fields of view frames follows from the transformations discussed in that report).

To derive the coning angle

We selected sources such that they trace a complete circle in the FOV of the boom camera for one full 360° rotation of the SSM assembly. As a function of the offset angle Δ_ϕ, the trajectories traverse through different regions in the FOV with their centroids [1] always shifted away by an amount equal to the coning angle (figure 3 and its caption).

Depending on the location of the source in the FOV of the boom camera (one which is present throughout a full 360° rotation of the SSM assembly), the extent of its trajectory will be different. With a fixed offset angle Δ_ϕ in the operational stages in orbit, the trajectory will be similar to one of those thinner (grey) circles of figure 3 with the coning angle θ_T being equal to the angular separation between centroid of the trajectory and (θ_x, θ_y) = (0°, 0°).

In order to derive the coning angle it is desirable to operate the SSM assembly in a continuous rotation mode recording the trajectory of a source of relatively strong flux.

[1]For a closed polygon of N vertices, the centroid (c_x, c_y) is given by (adopted from http://astronomy.swin.edu.au/~pbourke/geometry/polyarea/),

$$c_x = \frac{1}{6A}\sum_{i=0}^{N-1}(x_i \ + \ x_{i+1})\ (x_i \ y_{i+1} \ - \ x_{i+1} \ y_i)$$

$$c_y = \frac{1}{6A}\sum_{i=0}^{N-1}(y_i \ + \ y_{i+1})\ (x_i \ y_{i+1} \ - \ x_{i+1} \ y_i)$$

where A is the area of the polygon given by,

$$A = \frac{1}{2}\sum_{i=0}^{N-1}(x_i \ y_{i+1} \ - \ x_{i+1} \ y_i)$$

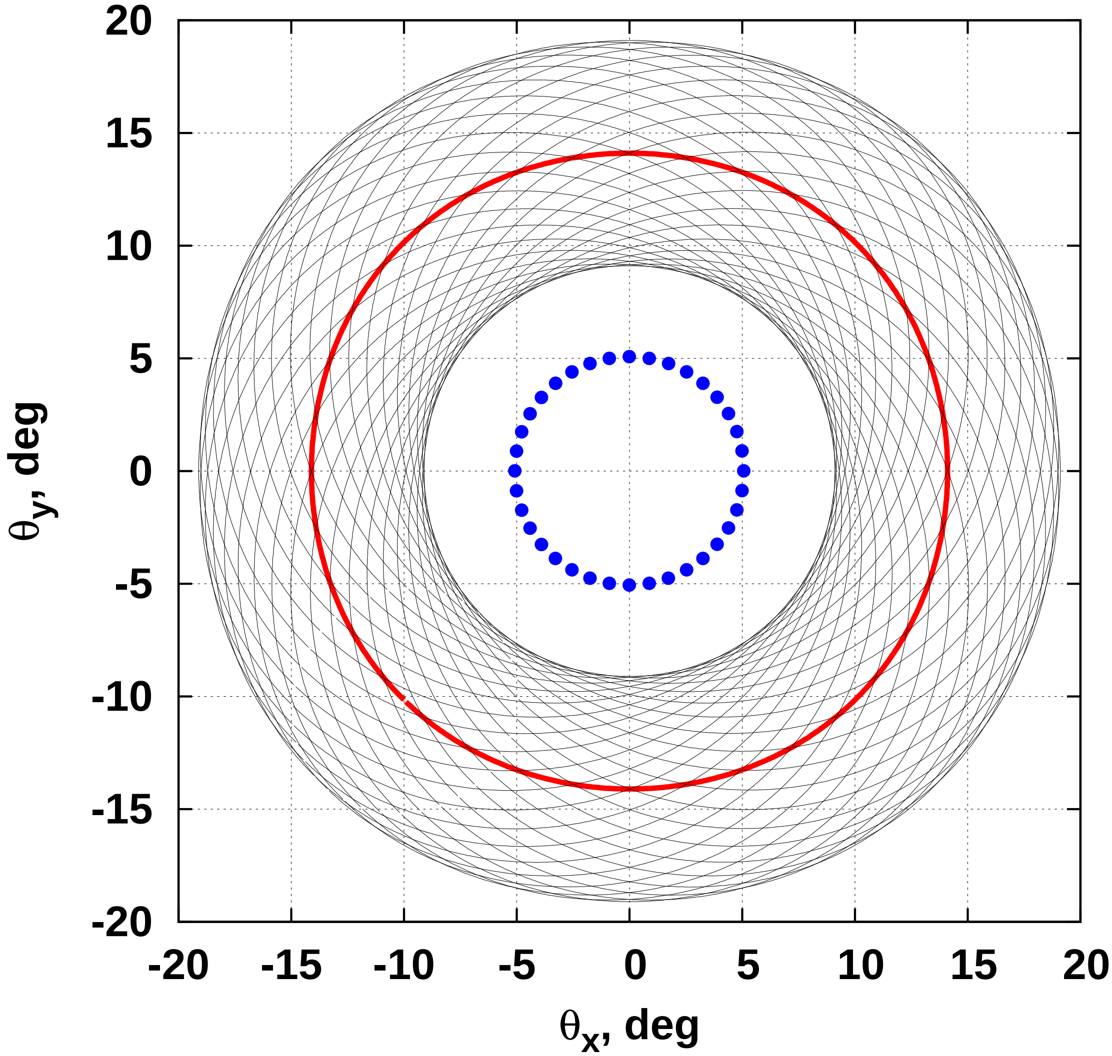

Figure 3: Trajectories of a source (co-ordinates: $(\alpha_{src} = 10°, \delta_{src} = 10°)$) in the FOV of the boom camera without coning and with coning for different values of the offset angle Δ_ϕ. Rotation axis: $(\alpha_d = 0°, \delta_d = 0°)$. The thick (red) line spanning between $\sim -14°$ and $+14°$ in both θ_x and θ_y is the trajectory of the source when the coning angle, $\theta_T = 0°$. The thinner (grey) lines are the trajectories of the same source in the presence of coning $(\theta_T = 5°)$ traced for different values of Δ_ϕ, with their centroids falling on the circle made of large (blue) dots, the radius of this circle being 5°, same as θ_T.

The poor resolution ($\sim 2^\circ\!.5$) in the cross coding direction will, however, limit the accuracy of the determination of the coning angle (θ_T).