Fluids in Astrophysics

Astrophysical fluids are mainly in gaseous and plasma form

The cosmos is pervaded by gas – fluids are everywhere

Average matter density of the universe: 1.5×10^{-27} kg / m^3

Most (>85%) of this is dark matter. The rest is ordinary matter

Mass density in ordinary matter (baryons) is 2×10^{-28} kg / m^3
≈ 0.1 baryons / m^3 (number density)

Dark matter is cold and pressureless – behaves like "dust"

Fluid behaviour is displayed by the baryonic component

Baryonic fluid in the present-day universe

Average number density ~ 0.1 atom / m^3, average temperature ~ 3 K
Composition (mass fraction) ~ 71% H, ~ 27% He, ~ 2% "metals"

=> In atom count, ~ 90% is Hydrogen

~75% (mass fraction) Hydrogen and ~25% Helium was synthesised in the early universe, within ~3 minutes of the Big Bang. Metals, and more helium, have been synthesised later in stars.

In the beginning, gas distribution was very smooth and uniform. Today the distribution is highly inhomogeneous and non-uniform in small scale. Physical conditions span a very wide range.

Collapsed structures: Planets, Stars, Galaxies, Clusters

Diffuse gas fills the space between collapsed structures

Stars and planets: Self-gravitating gas globes:

The Sun: central no. density ~ 10^{32} baryons/m^3, temperature ~ 10^7 K
average density ~ 10^{30} m^{-3}, surface temperature ~ 6000 K
compare: air on earth: ~ 10^{25} atoms / m^3

White Dwarf: no. density ~ 10^{36} m^{-3}

Neutron Star: no. density ~ 10^{45} m^{-3}

Jupiter: average density ~ 10^{23} m^{-3}

Diffuse matter:

Interplanetary medium: $n \sim 10^7$ m^{-3}, $T \sim 10^5$ K

Material between stars in our galaxy (ISM): $n \sim 10^6$ m^{-3}, $T \sim 10^4$ K

Material between galaxies (IGM): $n < 0.1$ m^{-3} - $n \sim 10^4$ m^{-3}
$T \sim 10^5 - 10^8$ K
Hotter and denser IGM in clusters

Character of Astrophysical Fluids

Collision between particles is rare in the diffuse gases encountered in astrophysics

e.g. ISM: ~ 10^6 atoms / m^3 , T ~ 10^4 K

Collisional mean free path:

neutral: ~ 10^{14} m ≈ 1000 times the Earth-Sun distance

ionized: ~ 10^{11} m ≈ Earth-Sun distance

Can this be considered a fluid?

In a fluid, the length scale of variation of physical quantities (density, pressure, velocity) must be much larger than the mean free path.

Often not satisfied for collisional mean free path in diffuse astrophysical fluids.
Momentum transport via magnetic field very important

Magnetic fields

Magnetic fields are ubiquitous in the cosmos

ISM: $B \sim 10^{-6}$ G.

Larmor radius: electrons $\sim 10^3$ m, protons $\sim 10^7$ m

much smaller than collisional mean free path

Interplanetary medium near the Earth: $B \sim 10^{-5}$ G

IGM: $B \sim 10^{-9}$ G

Magnetic field in these diffuse media are usually quite tangled

Jupiter: ~10 G; Sun: ~1G dipole, $\sim 10^3$ G sunspots

Magnetic stars: $\sim 10^3$ G dipole

White Dwarfs: up to $\sim 10^6$ G; Neutron Stars: 10^8 - 10^{15} G

Magnetic fields and ionised plasma make MHD the appropriate description for the dynamics of Astrophysical Fluids.

Ionisation of hydrogen in astrophysical conditions

Ionisation potential E_i = 13.6 eV ; $T_i = E_i / k_B = 1.6 \times 10^5$ K

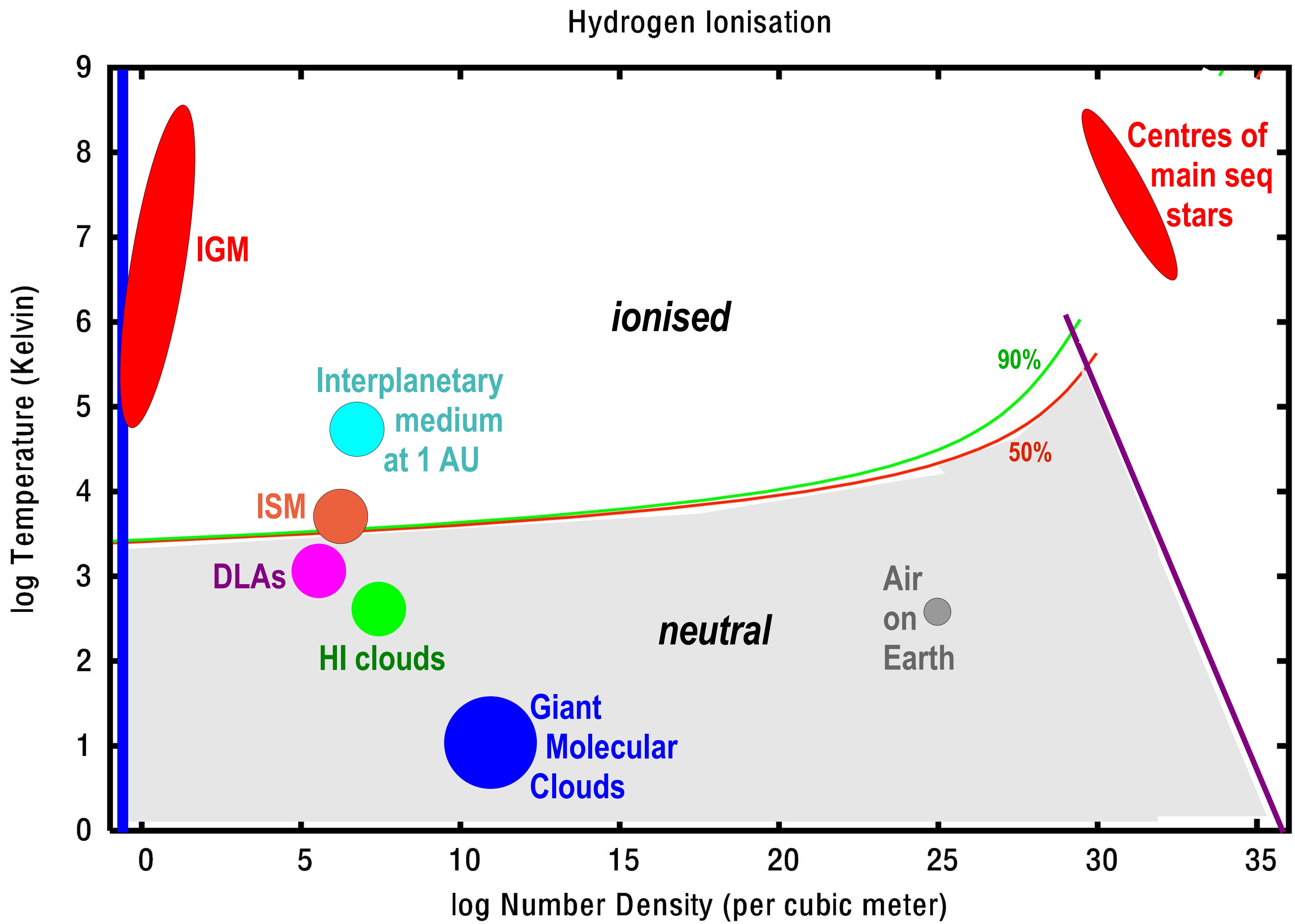
Ionisation equilibrium dictated by the Saha ionisation equation

$$\frac{n_e n_p}{n_H} = \left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} \exp(-T_i/T)$$

Gives the remarkable result that hydrogen plasma is fully ionised at $T \sim 0.1\, T_i$ for a very wide range of densities.

If $n > 10^{30}\ m^{-3}$ then pressure ionisation dominates.

Most of the diffuse astrophysical gas we encounter is thus ionised. Notable exceptions are HI clouds and molecular clouds in the ISM.



Hydrogen Ionisation
log Temperature (Kelvin)
log Number Density (per cubic meter)
0
1
2
3
4
5
6
7
8
9
0
5
10
15
20
25
30
35
IGM
Centres of main seq stars
ionised
90%
50%
Interplanetary medium at 1 AU
ISM
DLAs
HI clouds
neutral
Air on Earth
Giant Molecular Clouds

Evolution of cosmic gas

We live in an expanding universe. In the past, the universe was smaller and denser. $a(t)$ = the scale factor of the universe; redshift $z = a(t_0)/a(t) - 1$ is a measure of look-back time.

Density $\rho \propto (1+z)^3$; $T_{rad} \propto 1/a(t) \propto (1+z)$

At large z, matter distribution was very uniform.

Structures (e.g. stars, galaxies, clusters) have formed due to growth, via gravitational instability, of very small initial density perturbations.

Dark matter and baryonic matter behave differently. Dynamics of dark matter governed only by gravity and cosmological expansion. Baryonic component influenced by pressure and interaction with radiation.

Formation of Dark Matter halos

Dark matter overdensity $\delta\rho/\rho$ grows initially as $\propto$ a

Due to self gravity, the overdense region expands progressively slower than Hubble flow

At some point, the expansion of the overdense region stops completely. It turns around and collapses when the average density of the region reaches ~ 6 times the background density. It then settles down to a virialized structure (halo) with density ~ 200 times the background density at the time of collapse.

Structures form hierarchically. Small scale halos form earlier than larger ones. Large halos result both from late turnaround of large length-scale perturbations and from merger of smaller halos.

Dark matter halos provide potential wells for baryons to fall into

Baryonic Fluid

At very large scales dark matter and baryonic matter follow each other. Perturbations grow the same way.

At scales below acoustic horizon pressure matters.
Tight coupling with background radiation prevents infall.

Radiation decouples when matter becomes neutral at $z \sim 1000$.

Baryonic fluid then falls into dark matter halos. Gets hot, radiates, loses energy and condenses even further to form galaxies. Further cooling, fragmentation and collapse forms stars within the galaxy. The first galaxies form around $z \sim 10$.

Radiation generated by stars and galaxies re-ionize the Intergalactic Medium.

In general, the time to reach thermal equilibrium is very long in the diffuse media encountered in astrophysics. Regions of different density and temperature may therefore co-exist. Pressure equilibrium is established much more quickly.

ISM in our galaxy has multiple phases in pressure equilibrium:

Molecular clouds:	$n \sim 10^{8}\,m^{-3}$; $T \sim 10$ K
HI clouds:	$n \sim 10^{7}\,m^{-3}$; $T \sim 100$ K
Warm Neutral Medium:	$n \sim 10^{6}\,m^{-3}$; $T \sim 1000$ K
Warm Ionized Medium:	$n \sim 10^{5}\,m^{-3}$; $T \sim 10^{4}$ K
Coronal gas:	$n \sim 10^{3}\,m^{-3}$; $T \sim 10^{6}$ K

Expanding overpressure regions are produced by
- Hot stars – ionisation, winds
- Explosions – novae, supernovae, GRBs

Heating and cooling of cosmic gas

Important heating sources:
• Ionising radiation field
• Cosmic rays
• Mechanical energy input (e.g. Winds, Supernovae, Jets, Bubbles)
• Gravitational infall
• Nuclear energy release

Important cooling processes
• Radiation – bremsstrahlung, line cooling
• Scattering – Compton cooling
• Mechanical – expansion cooling

Astrophysical Fluids

Hydrostatics

By setting time derivatives to zero in the Euler equation, one finds to condition of Hydrostatic Equilibrium. In a spherically symmetric case of fluid in gravitational field this looks like

$$\frac{dP}{dr} = -\frac{d\Phi(r)}{dr}\rho(r) = -\frac{GM(r)\rho(r)}{r^2}$$

Where $\Phi(r)$ is the gravitational potential and $M(r)$ is the mass included within radius r. Either $\Phi(r)$ or $M(r)$ needs to be specified: if the fluid's self gravity is dominant then

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

for example in the case of baryonic fluid in stars. On the other hand, if the gravitational potential is determined by a different source (e.g. Dark Matter, in case of baryonic fluid falling into a cluster halo), appropriate specification for $\Phi(r)$ has to be given. If the Dark Matter density distribution $\rho_{\rm DM}(r)$ is known, then

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho_{\rm DM}(r)$$

To obtain a solution one needs to also specify a relation between Pressure and Density: $P(\rho)$. This is called the "equation of state". In some cases this relation will involve other variables (e.g. thermal pressure depends both on temperature and density). One additional equation for each such variable is needed to solve the Hydrostatic Equilibrium equation.

A simple example is that of a highly degenerate white dwarf. In this case the temperature is unimportant, the pressure-density relation goes from

$$P \propto \rho^{5/3}$$

at low density, to

$$P \propto \rho^{4/3}$$

at high density where electrons become relativistic. Hydrostatic equilibrium for such ”polytropic” equations of state:

$$P \propto \rho^{\gamma}$$

can be rewritten in the form

$$\frac{1}{z^2}\frac{d}{dz}\left(z^2\frac{dw}{dz}\right) + w^n = 0$$

where $n = 1/(\gamma - 1)$, z is a scaled (dimensionless) radius and w is the gravitational potential in units of that at $r = 0$. One finds that the radius R of the configuration is finite if $n < 5$, and the mass-radius scaling goes as $M \propto R^{(n-3)/(n-1)}$. For a white dwarf, the solution gives $R \propto M^{-1/3}$ for low mass ($\gamma = 5/3$), but as $\gamma \to 4/3$, the Mass of the configuration approaches a unique value

$$M_{\rm crit} = \frac{5.836}{\mu_e^2} M_{\odot}$$

which is known as the Chandrasekhar limit. Here μ_e is the mean molecular weight per electron.

Another special case of a polytropic equation of state is that of an ”isothermal sphere”: T = constant, $P \propto \rho$, for which the solution turns out to be

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

which is often used as a simple model for the density profile of Dark Matter halos, star clusters etc. The mass in this configuration increases linearly with radius:

$$M(r) = \frac{2\sigma^2}{G} r$$

If the pressure depends on temperature, then the temperature stratification needs to be simultaneously solved for. Information about this comes from energy transport. Energy flows down a temperature gradient, so one can relate the luminosity of the object to the temperature profile. In a star, where the energy is generated by nuclear burning in a small core, The amount of energy crossing any spherical shell outside the core per unit time is the same, and is equal to the total luminosity L.

Much of the heat transport in stars is radiative. The radiation flux interacts with matter and exerts a force which equals the radiation pressure gradient

$$\frac{L}{4\pi r^2 c}\kappa\rho = -\frac{d}{dr}\left(\frac{1}{3}aT^4\right) = -\frac{4}{3}aT^3\frac{dT}{dr}$$

where κ is the opacity which gives

$$\frac{dT}{dr} = -\frac{3\kappa\rho}{4acT^3}\frac{L}{4\pi r^2}$$

as the temperature gradient necessary to transport the flux radiatively. Combining this with the Hydrostatic equilibrium equation, one may write

$$\nabla \equiv \frac{d\ln T}{d\ln P}$$

$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{\kappa LP}{M(r)T^4}$$

When the luminosity or the opacity is large, however, the necessary temperature gradient becomes so large that convection can set in.

Convection

The criterion for convective instability can be worked out as follows.

Let us consider a matter element at a radius r in the star, and displace it upwards to $r + dr$. The element would come to pressure equilibrium with the new surroundings, but its density and temperature would not necessarily be the same as those of the surrounding material (see fig. 1). If its density is smaller than the surrounding material then the element

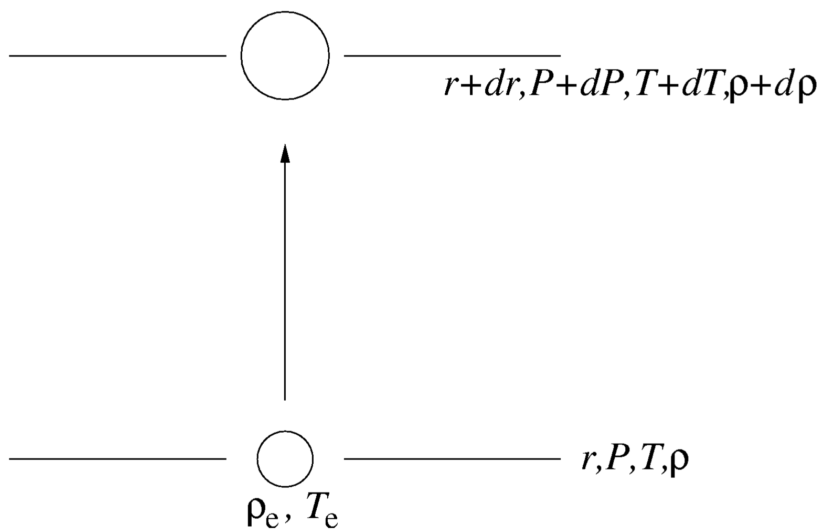


Figure 1: Perturbation of matter element to test for convective instability

would rise due to buoyancy. If the density is higher than the surroundings, it would sink back. The situation will be stable against convection if

$$\left(\frac{d\rho}{dr}\right)_{\rm e} - \left(\frac{d\rho}{dr}\right)_{\rm s} > 0 \qquad (1)$$

where the subscript 'e' refers to the element and 's' to the surroundings. Since $P = \rho kT/\mu m_p$, one has

$$\frac{d\rho}{\rho} = \frac{dP}{P} - \frac{dT}{T} + \frac{d\mu}{\mu}$$

Ignoring composition gradient for the time being, we can rewrite eq. (1) as

$$\left(\frac{1}{P}\frac{dP}{dr}\right)_{\rm e} - \left(\frac{1}{T}\frac{dT}{dr}\right)_{\rm e} - \left(\frac{1}{P}\frac{dP}{dr}\right)_{\rm s} + \left(\frac{1}{T}\frac{dT}{dr}\right)_{\rm s} > 0$$

The terms containing pressure gradient cancel due to the pressure equilibrium established between the element and the surroundings, leaving a stability condition in terms of the temperature gradients:

$$-\left(\frac{d\ln T}{dr}\right)_{\rm e} > -\left(\frac{d\ln T}{dr}\right)_{\rm s}$$

Writing in terms of derivatives w.r.t. pressure P instead of r,

$$\left(\frac{d\ln T}{d\ln P}\right)_{\mathrm{s}} < \left(\frac{d\ln T}{d\ln P}\right)_{\mathrm{e}}$$

or

$$\nabla < \nabla_{\mathrm{e}}$$

since $d\ln P/dr < 0$. If the element evolves adiabatically then

$$\nabla_{\mathrm{e}} = \nabla_{\mathrm{ad}} = \frac{\gamma - 1}{\gamma}$$

where γ is the ratio of specific heats. For monatomic gases with $\gamma = 5/3$ the value of ∇_{ad} is 0.4, except in regions of partial ionisation where addition of energy causes an increase in number of particles and hence temperature increases slower than normal, depressing ∇_{ad} below its standard value of 0.4.

If indeed all the transport does take place via radiation then

$$\nabla = \nabla_{\mathrm{rad}}$$

We can then write the condition for stability against convection as

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} \tag{2}$$

This is called the *Schwarzschild criterion* for dynamical stability. If a composition gradient is present, then the stablility criterion is modified to

$$\nabla_{\mathrm{rad}} < \nabla_{\mathrm{ad}} + \nabla_{\mu} \tag{3}$$

where $\nabla_{\mu} = (d\ln\mu/d\ln P)_{\mathrm{s}}$. This is called the *Ledoux criterion* for dynamical stability. If these conditions are violated then convection sets in to transport energy and the temperature gradient ∇ is no longer given by ∇_{rad}. ∇_{rad} now stands for the temperature gradient that would have been necessary to transport the whole flux by radiation.

Convective motion present in the outer layers of solar-type stars is the main contributor to the generation of strong magnetic fields, sunspots (and starspots), and mechanical injection of energy into the atmosphere, producing hot coronae.

Stellar Wind

The hot corona of the Sun tends to expand and gives rise to the solar wind. Parker (1958) constructed a model for the solar wind assuming that it is steady, spherically symmetric and isothermal. The basic equations then are:

$$\dot{M} = -4\pi r^2 \rho v$$

giving

$$\frac{1}{\rho}\frac{\partial \rho}{\partial r} = -\frac{1}{vr^2}\frac{\partial}{\partial r}(vr^2)$$

The Euler equation gives

$$\rho v \frac{\partial v}{\partial r} = -\frac{\partial P}{\partial r} - \frac{GM\rho}{r^2}$$

Writing $c_s^2 = \partial P/\partial \rho$, one can rearrange this to obtain

$$v\frac{\partial v}{\partial r} - \frac{c_s^2}{vr^2}\frac{\partial}{\partial r}(vr^2) + \frac{GM}{r^2} = 0$$

or

$$\frac{1}{2}\left(1 - \frac{c_s^2}{v^2}\right)\frac{\partial (v^2)}{\partial r} = -\frac{GM}{r^2}\left(1 - \frac{2c_s^2 r}{GM}\right)$$

we will see later that this equation has use beyond the solar wind.

Introducing a critical radius $r_c = GM/2c_s^2$, we see that at $r = r_c$ either $v = c_s$ or $dv/dr = 0$. On the other hand if $v = c_s$ then either $r = r_c$ or $dv/dr = \infty$

This allows five different types of solution of the differential equation. One can integrate the equation to give

$$\left(\frac{v}{c_s}\right)^2 - \ln\left(\frac{v}{c_s}\right)^2 = 4\frac{r_c}{r} + 4\ln\left(\frac{r}{r_c}\right) + C$$

where C is an integration constant. Different values of C may choose different branches of solutions.

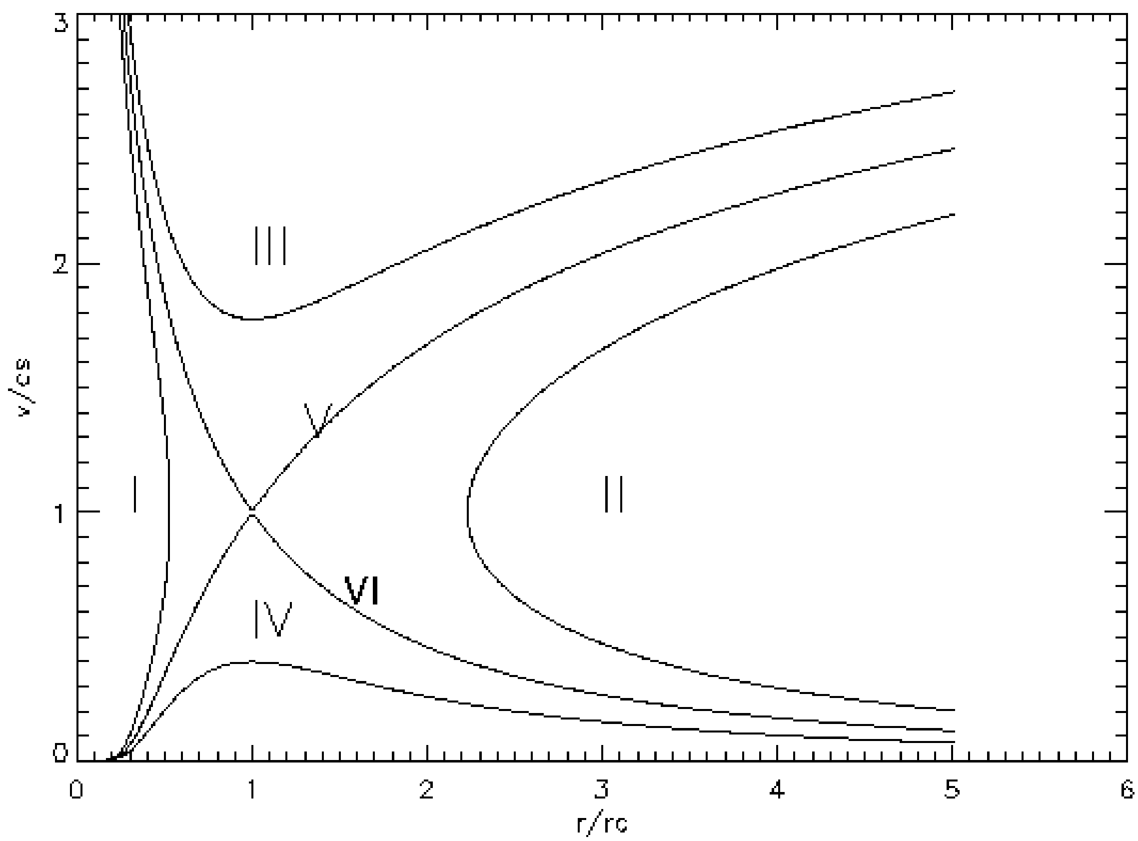


Figure 2: Different solution classes for the Parker wind equation

For solar wind, the velocity is finite at large r, while the wind is subsonic at small r. So the flow is transonic, i.e. $v = c_s$ at $r = r_c$. This is realised for C = -3 and corresponds to the solution V drawn in figure 2. At large r this gives

$$\frac{v}{c_s} \approx 2\left(\ln\frac{r}{r_c}\right)^2$$

and hence

$$\rho \propto \frac{1}{r^2\sqrt{\ln(r/r_c)}}$$

The other solution (VI) passing though the critical point has, at large r, $v \propto 1/r^2$, suggesting a constant density (and hence constant pressure). This can not be contained by ISM pressure and is thus considered unphysical for the solar wind. Also this solution would need the wind to start with a very large speed at the solar surface, which is unlikely.

The time-reversed version of solution VI is, however, reasonable and corresponds to spherical accretion onto a central mass.

Solution IV is referred to as the "Solar Breeze" solution, which would correspond to a subsonic wind from the sun. Satellite measurements, however, reveal that the solar wind at large r is indeed supersonic and hence solution V provides the appropriate escription.

Shock Waves

A shock wave is a surface of discontinuity moving through a medium at a speed larger than the speed of sound upstream. The change in the fluid properties upon passing the shock can be investigated using simple conservation laws.

In a fluid flow, conservation laws are usually expressed in the form of continuity equations. Instead of the usual differential form, the continuity equations can also be expressed as relations between quantities on two sides of an arbitrary stationary surface: the mass flux, the momentum flux and the energy flux must be continuous through the surface. For a surface normal to the local fluid velocity $\vec{\mathrm{v}}$, these relations can be expressed as:

$$\begin{aligned} [\rho \mathrm{v}] &= 0 \quad \text{(mass conservation)} \\ \left[P + \rho \mathrm{v}^2\right] &= 0 \quad \text{(momentum conservation)} \\ \left[\mathrm{v}\left(u + P + \rho \mathrm{v}^2/2\right)\right] &= 0 \quad \text{(energy conservation)} \end{aligned}$$

Here v= $|\vec{\mathrm{v}}|$, P is the local pressure and u is the internal energy per unit volume. $u + P$ is the enthalpy, or *heat function* per unit volume of the fluid. The square brackets represent the difference between the enclosed quantity evaluated on two sides of the stationary surface. For an adiabatic index (ratio of specific heats) γ, $u = P/(\gamma - 1)$, and hence the energy relation can be written as

$$\left[\mathrm{v}\left(\gamma P/(\gamma - 1) + \rho \mathrm{v}^2/2\right)\right] = 0$$

Using the mass conservation equation, this can be further simplified to

$$\left[\gamma P/(\gamma - 1)\rho + \mathrm{v}^2/2\right] = 0$$

We can apply these relations to the quantities upstream and downstream of a shock wave, once we move to a reference frame in which the surface of discontinuity is at rest (fig. 1). In this frame, the upstream fluid approaches the discontinuity at a speed v_1 and the downstream fluid leaves the shock with a speed v_2. Let ρ_1 and ρ_2 be the densities on the two sides respectively, and P_1 and P_2 the corresponding pressures. In this frame, we can then write

$$\rho_1 \mathrm{v}_1 = \rho_2 \mathrm{v}_2$$

$$\begin{aligned} P_1 + \rho_1 \mathrm{v}_1^2 &= P_2 + \rho_2 \mathrm{v}_2^2 \\ \frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1} + \frac{1}{2}\mathrm{v}_1^2 &= \frac{\gamma}{\gamma-1}\frac{P_2}{\rho_2} + \frac{1}{2}\mathrm{v}_2^2 \end{aligned}$$

These are known as the shock *jump conditions* or *Rankine-Hugoniot* relations. Note that in the rest frame of the unshocked fluid, v_1 is the speed of propagation of the shock, and $\mathrm{v}_1 - \mathrm{v}_2$ is the speed with which the shocked fluid is seen to move forward.

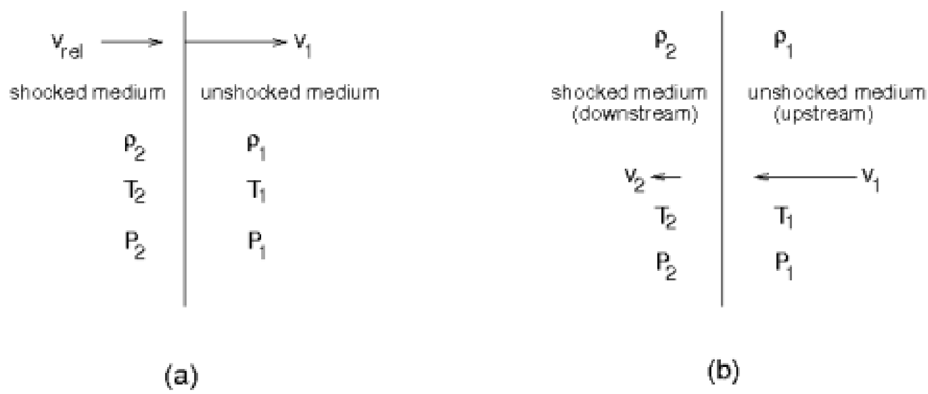


Figure 1: A shock wave discontinuity in (a) the reference frame of the unshocked medium and (b) in a reference frame where the surface of discontinuity is at rest. The shock advances into the unshocked fluid at speed v_1. In the rest frame of the shock, the upstream medium approaches it at speed v_1. The shocked fluid moves away from the discontinuity at speed $\mathrm{v}_2 = \rho_1 \mathrm{v}_1/\rho_2$. The shocked fluid therefore approaches the unshocked fluid at a speed $\mathrm{v}_{\mathrm{rel}} = \mathrm{v}_1 - \mathrm{v}_2$.

We will be interested in very strong shocks, for which the upstream medium can be approximated as a cold ($T \sim 0$) fluid. The upstream pressure can therefore be neglected in comparison with all other quantities appearing in the jump conditions. Setting $P_1 = 0$ in the above equations, and defining the *Compression Ratio*

$$R \equiv \frac{\mathrm{v}_1}{\mathrm{v}_2}$$

one obtains from the above:

$$\begin{aligned}\frac{\rho_2}{\rho_1} &= R \\ \frac{P_2}{\rho_2} &= v_1^2\frac{R-1}{R^2} \\ 1 &= \frac{2\gamma}{\gamma-1}\frac{R-1}{R^2}+\frac{1}{R^2}\end{aligned}$$

The third equation is a quadratic in R and can be easily solved to yield

$$R = \frac{\gamma+1}{\gamma-1}$$

where we have ignored the trivial solution $R = 1$ which is valid in the case of no discontinuity in flow. For a monatomic gas with three degrees of freedom, $\gamma = 5/3$ and hence $R = 4$. This shows that on crossing a strong shock the density of the fluid jumps by a factor of 4. Noting that $P_2/\rho_2 = kT_2/(\mu m_p)$, one can estimate the postshock temperature:

$$T_2 = \frac{\mu m_p}{k}v_1^2\frac{R-1}{R^2}$$

for $R = 4$,

$$T_2 = \frac{3}{16}\frac{\mu m_p}{k}v_1^2$$

which gives $T_2 \sim 10^7$ K for v = 1000 km/s. Clearly, gas shocked in a young supernova remnant, where the expansion speed of the blast wave is several thousand km/s, is hot enough to produce X-rays. X-ray spectroscopy of this hot gas is now the subject of detailed study by contemporary X-ray satellites Chandra and XMM-Newton. One of the important results emerging from such studies is the composition of the matter ejected in the supernova explosion.

As a supernova remnant (SNR) expands, it sweeps up more and more matter. The kinetic and thermal energy is shared with all the swept-up matter and hence the expansion slows down. When the swept-up matter dominates the dynamics and radiation losses from the material are small, the dynamics of the blast wave is governed by only two dimensional parameters: the energy of the blast wave E_0 and the external density ρ. In such a situation the expansion would have a self-similar solution, with the radius of the blast wave proportional to the only possible combination of these parameters that yields a dimension of length:

$$R \propto \left(\frac{E_0}{\rho}\right)^{1/5} t^{2/5}$$

This is known as the "Sedov-Taylor" expansion law, originally derived to explain the behaviour of the expanding blast waves of atmospheric nuclear detonations. The speed of the blast wave then falls as

$$\mathrm{v} \propto t^{-3/5}$$

This power-law behaviour is applicable only at times $t > t_0$ when the swept-up mass $4\pi R^3\rho/3$ exceeds the originally ejected mass M_{ej} in the explosion. Until $t = t_0$ the expansion is practically uniform, at a constant speed equal to the initial speed v_0. The value of t_0 is then given by

$$t_0 = \left(\frac{3M_{\mathrm{ej}}}{4\pi\rho}\right)^{1/3} \frac{1}{\mathrm{v}_0}$$

which evaluates to about 250 y for $M_{\mathrm{ej}} = 2M_\odot$, $\mathrm{v}_0 = 1000$ km/s and a typical interstellar density of 1 atom per cm^3. We have then the scaling law

$$\mathrm{v} = \mathrm{v}_0 \left(\frac{t}{t_0}\right)^{-3/5}$$

and the evolution of the postshock temperature, for typical parameters adopted above,

$$T_2 \approx 10^7\ \mathrm{K} \left(\frac{t}{250\ \mathrm{y}}\right)^{-6/5}$$

As the temperature falls to $T < 10^4$ K, line radiation becomes copious, causing a serious drain of blast wave energy. The blast wave stagnates and eventually disperses into the interstellar medium. The typical lifetime of a supernova remnant can be estimated from the above, as the time required for the postshock temperature to fall to $\sim 10^4$ K. This works out to be a few times 10^5 y.

Shock Acceleration of Cosmic Rays

Shock waves in astrophysical situations are thought to be the main contributors to the acceleration process of the very high energy electrons and nuclei that are observed as the so-called "Cosmic Rays". The mechanism, called *Diffusive Shock Acceleration*, proceeds as follows.

Astrophysical shocks are mostly "collisionless". The momentum exchange between the shocked matter and newly swept-up matter is normally accomplished by the local magnetic field. Magnetic fields are ubiquitous in astrophysics, at any point there are relatively large scale components along with irregularities of small scale. Small-scale, tangled fields may have a substantial strength and act as "magnetic mirrors" for even high energy particles. In postshock regions, magnetic fields can be amplified by turbulent motions.

Let us consider a non-relativistic shock, with speed $\mathrm{v_{sh}}$, and relativistic particles, with energy $E = pc$ (p=momentum), present in a given region. The speed of these particles are much higher than the shock speed, and they hardly notice the shock as a discontinuity. However, they do scatter from the magnetic irregularities on both sides of the shock.

Scattering with a magnetic irregularity is energy-conserving in the rest frame of the irregularity. Repeated scatterings of this kind with many such irregularities in a medium randomize the momentum of the particle, and establish an isotropic distribution in the rest frame of the medium. Around the shock, then, we have such isotropic distribution of the high-energy particles established, separately, both upstream and downstream of the shock. These two distributions then approach each other with a speed

$$\mathrm{v_{rel}} = \mathrm{v_{sh}} - \frac{\mathrm{v_{sh}}}{R} = \frac{R-1}{R}\mathrm{v_{sh}}$$

where R is the compression ratio.

In either distribution of the high energy particles, the average velocity of the whole distribution of particles is equal to the velocity of the local medium, but the speed of random motion is very nearly c, as the particles are relativistic. Since this is much larger than the shock speed, these particles can easily cross the shock front from either side, namely, from upstream to downstream and vice versa. Since the two media approach each other at $\mathrm{v_{rel}}$, upon crossing the shock the particles will find their momentum isotropized by scattering with magnetic irregularities approaching at speed $\mathrm{v_{rel}}$ with respect to the rest frame of the previous distribution. This process increases the average energy of the particles, with a particle gaining energy every time a shock crossing occurs.

If the particle has an energy E in the local rest frame before shock crossing, in the frame of the medium encountered after shock crossing its energy is seen to be, by Lorentz transformation,

$$E' = \gamma_{\rm rel}(E + \mathrm{v_{rel}} p_x)$$

where p_x is the x-component of the momentum and the x-axis is chosen to be along $\mathrm{v_{rel}}$. We note that the Lorentz factor $\gamma_{\rm rel} \equiv (1 - \mathrm{v_{rel}}^2/c^2)^{-1/2}$ is very close to 1 for the non-relativistic shocks under discussion. Writing $p_x = p\cos\theta = E\cos\theta/c$, we find that the gain in energy in each shock crossing is

$$\Delta E = E' - E = E\frac{\mathrm{v_{rel}}}{c}\cos\theta$$

i.e.

$$\frac{\Delta E}{E} = \frac{\mathrm{v_{rel}}}{c}\cos\theta = \frac{R-1}{R}\frac{\mathrm{v_{sh}}}{c}\cos\theta$$

Noting that the flux of particles arriving at an angle θ is proportional to $c\cos\theta\sin\theta d\theta$, we find a flux-weighted fractional energy gain per shock crossing:

$$\left\langle\frac{\Delta E}{E}\right\rangle = \frac{R-1}{R}\frac{\mathrm{v_{sh}}}{c}\frac{\int_0^{\pi/2}\cos^2\theta\sin\theta d\theta}{\int_0^{\pi/2}\cos\theta\sin\theta d\theta} = \frac{2}{3}\frac{R-1}{R}\frac{\mathrm{v_{sh}}}{c}$$

Defining a "cycle" as two crossings of the shock, downstream to upstream and back to downstream, we find the fractional energy gain per cycle

$$\eta \equiv \left\langle\frac{\Delta E}{E}\right\rangle_{\rm cycle} = \frac{4}{3}\frac{R-1}{R}\frac{\mathrm{v_{sh}}}{c}$$

All particles which cross into the downstream region will, however, not be able to cross the shock back to upstream, since the downstream medium moves away from the shock front at a speed $\mathrm{v}_2 = \mathrm{v}_{\rm sh}/R$. Since the average speed of the particles in the distribution is very close to c, the flux of particles crossing into the downstream region, according to standard kinetic theory, is $nc/4$ where n is the number density of the particles. In the downstream medium, the particles are advected away from the shock front with a flux $n\mathrm{v}_2$. Hence the probability of a particle escaping the acceleration zone in any cycle is given by

$$P_{\rm esc} = \frac{n\mathrm{v}_2}{nc/4} = \frac{4}{R}\frac{\mathrm{v}_{\rm sh}}{c}$$

Starting with N_0 particles at energy E_0, after n cycles the energy of the particles will become

$$E_n = E_0(1+\eta)^n$$

And the number of particles to survive n cycles would be

$$N_n = N_0(1-P_{\rm esc})^n$$

Some of these particles will go on for more cycles and gain more energy. The number N_n thus stands for the cumulative number of particles with energy $E > E_n$. Taking logarithm and dividing the above two expressions one can eliminate n, and the resulting distribution can be written as

$$N(>E) = E^{-x} \;\; \text{where } x = -\frac{\ln(1-P_{\rm esc})}{\ln(1+\eta)}$$

Since both $P_{\rm esc}$ and η are much smaller than unity, we could expand the logarithms and keep only the first order term, giving

$$x = \frac{P_{\rm esc}}{\eta} = \frac{3}{R-1}$$

which depends only on the compression ratio of the shock.

The differential energy distribution of these particles can be written from the above as

$$N(E)dE = E^{-p}dE$$

where

$$p = 1 + x = \frac{R+2}{R-1}$$

for a non-relativistic strong shock $R = 4$, giving $p = 2$, close to the observed power-law energy spectrum of Cosmic Rays, as well as of non-thermal synchrotron-emitting ultrarelativistic electrons in a variety of astrophysical situations. Interstellar shocks generated by supernovae are thought to be the prime acceleration sites for Cosmic rays. The fact that supernova remnants can accelerate particles to very high energies and produce power-law energy distributions is evident from the strong non-thermal radio emission seen in them. Recently, the non-thermal component of the X-ray emission has also been discovered in supernova remnants, and indeed the ultrarelativistic particles responsible for this emission are seen to lie very close to the advancing shock.

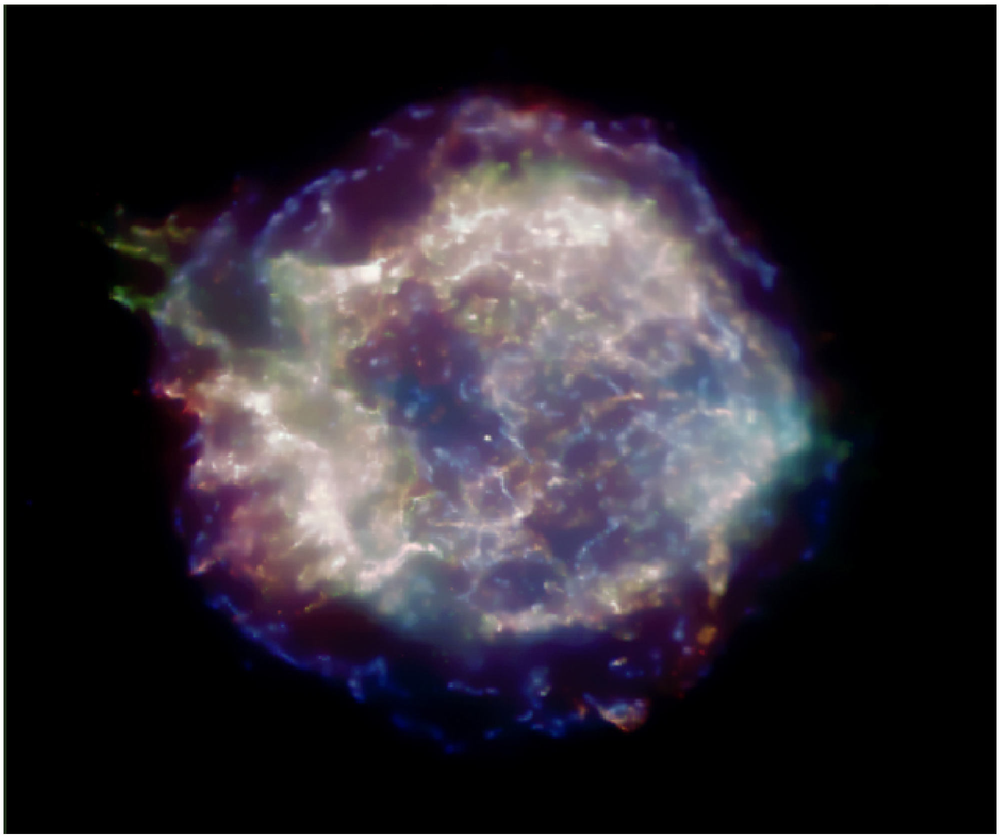


Figure 2: X-ray image of a 300 year-old supernova remnant Cas A taken with the Chandra observatory. The image is taken in three different X-ray energy bands and superposed in false colour. The outer red filaments are rich in Iron. (Credit: John Hughes et al, Rutgers University)

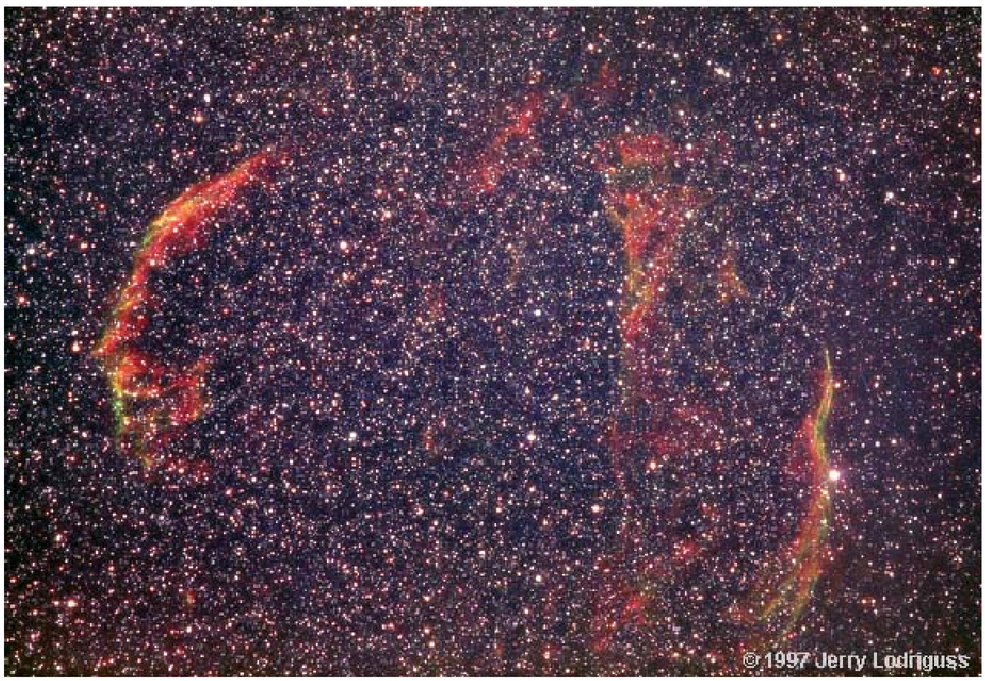


Figure 3: A large, 20,000 year-old supernova remnant Cygnus Loop imaged in optical light. The filaments produce strong emission in optical recombination lines such as $H\alpha$ (red) and forbidden metal lines (green)

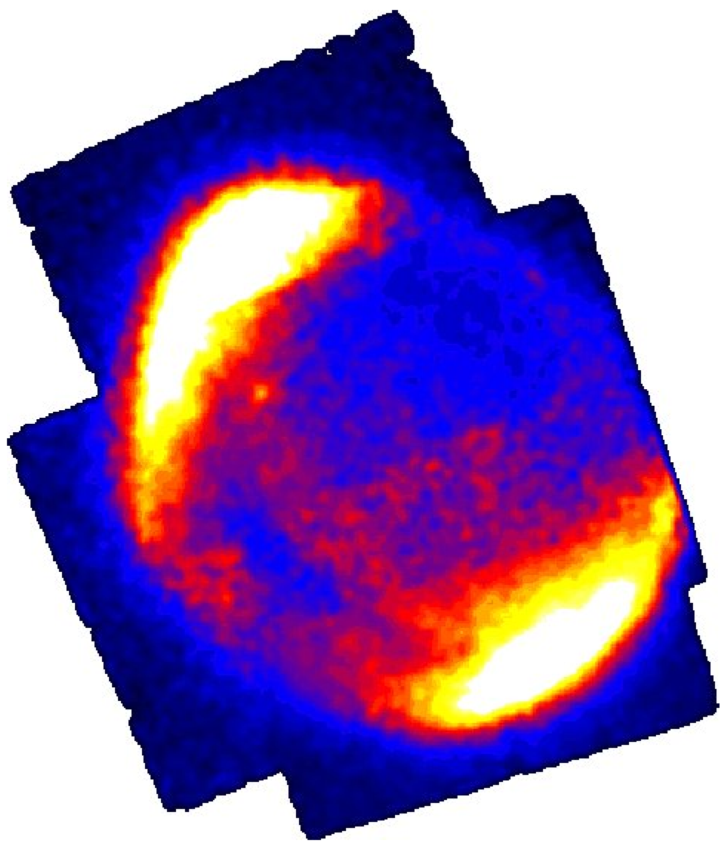


Figure 4: X-ray image of the remnant of a supernova seen to have exploded in the year 1006AD. The blue rims of the SNR in this false-colour image indicate non-thermal X-ray emission, locating the acceleration sites of the ultrarelativistic particles.

Gravitational collapse

Jeans' instability

Jeans' instability of a self-gravitating, thermally supported interstellar cloud is thought to be responsible for the collapse of parts of the cloud larger than a scale size that goes unstable, eventually fragmenting and forming stars.

The dynamics of gas in the cloud is controlled by two fluid equations:

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}.\vec{\nabla})\vec{v}\right] = -\vec{\nabla}P - \rho\nabla\Phi \quad \text{(Force Balance)} \tag{1}$$

and

$$\frac{\partial\rho}{\partial t} + \vec{v}.(\vec{\nabla}\rho) = -\rho\vec{\nabla}.\vec{v} \quad \text{(Continuity)} \tag{2}$$

where $\vec{v}$ is the gas velocity field, ρ is the mass density, P is the gas pressure and Φ is the local gravitational potential. Using the sound speed c_s one may write

$$\vec{\nabla}P = c_\mathrm{s}^2\vec{\nabla}\rho$$

We now split the velocity and density into two parts, spatially uniform (subscript 0) and spatially varying (subscript 1):

$$\vec{v} = \vec{v}_0 + \vec{v}_1$$

$$\rho = \rho_0 + \rho_1$$

We also assume that the uniform components are stationary, i.e.

$$\frac{\partial\vec{v}_0}{\partial t} = \frac{\partial\rho_0}{\partial t} = 0$$

We can then write the linear equations in spatially varying quantities as

$$\frac{\partial\vec{v}_1}{\partial t} + \vec{v}_0.\vec{\nabla}\vec{v}_1 = -\vec{\nabla}\Phi_1 - c_\mathrm{s}^2\vec{\nabla}\left(\frac{\rho_1}{\rho_0}\right) \tag{3}$$

$$\frac{\partial\rho_1}{\partial t} + \vec{v}_0.\vec{\nabla}\rho_1 = -\rho_0\vec{\nabla}.\vec{v}_1 \tag{4}$$

where we have kept as gravitational potential only that produced by the spatially varying part of the density distribution, since the gravitational force produced by a spatially uniform, infinite density distribution vanishes.

Transforming to a frame in which $\vec{v}_0 = 0$, we have

$$\frac{\partial \vec{v}_1}{\partial t} = -\vec{\nabla}\Phi_1 - \frac{c_s^2}{\rho_0}\vec{\nabla}\rho_1 \quad (5)$$

$$\frac{\partial \rho_1}{\partial t} = -\rho_0 \vec{\nabla}.\vec{v}_1 \quad (6)$$

Taking spatial derivative of eq. 5 and temporal derivative of eq. 6 we find

$$\vec{\nabla}\left(\frac{\partial \vec{v}_1}{\partial t}\right) = -\nabla^2\Phi_1 - \frac{c_s^2}{\rho_0}\nabla^2\rho_1 \quad (7)$$

and

$$-\frac{1}{\rho_0}\frac{\partial^2 \rho_1}{\partial t^2} = \frac{\partial}{\partial t}(\vec{\nabla}.\vec{v}_1) \quad (8)$$

Recognising that the LHS of eq. 7 and the RHS of eq. 8 are the same, we can write

$$\frac{\partial^2 \rho_1}{\partial t^2} = c_s^2\nabla^2\rho_1 + (4\pi G\rho_0)\rho_1 \quad (9)$$

where we have used Poisson's equation to write

$$\nabla^2\Phi_1 = 4\pi G\rho_1. \quad (10)$$

If we now write a Fourier component of the spatially varying density as

$$\rho_1 = A\exp\{i(\vec{k}.\vec{r} + \omega t)\} \quad (11)$$

We find

$$\omega^2 = c_s^2 k^2 - 4\pi G\rho_0 \equiv c_s^2(k^2 - k_J^2) \quad (12)$$

where

$$k_J^2 = \frac{4\pi G\rho_0}{c_s^2} = \frac{4\pi G\rho_0 m_p \mu}{k_B T} \quad (13)$$

Here m_p is the proton mass, μ is the mean molecular weight, k_B is the Boltzmann constant and T is the temperature of the gas. For $k < k_J$ the value of ω^2 is negative and hence the disturbance grows exponentially. k_J defines a minimum mass scale:

$$M_J = \left(\frac{2\pi}{k_J}\right)^3 \rho_0 = \left[\frac{\pi k_B T}{G\mu m_p}\right]^{3/2} \frac{1}{\rho_0^{1/2}} \tag{14}$$

This is called the Jeans' Mass. Perturbations of size larger than this in a gas cloud would grow, become self-gravitating and collapse.

Implosion and Explosion

Catastrophic gravitational collapse occurs in the cores of massive stars at the end of their evolution. The nuclear burning proceeds until Fe is synthesised at the core, which cannot burn further as the peak of the binding energy has been reached. This is a degenerate white-dwarf-like configuration whose mass continues to grow as ashes are added from the nuclear burning shell around it. Eventually the Chandrasekhar limit is exceeded and collapse occurs. As collapse proceeds, the Fe nuclei are first photodissociated, and then electrons are captured by protons to produce neutron-rich matter. The loss of electrons means the loss of degeneracy pressure, the main support against gravity at this stage. As a result the collapse accelerates and in hydrodynamic time scale of a few seconds produces a very compact configuration, made primarily of neutrons. As the neutrons are squeezed together at densities higher than nuclear density ($\rho_{nuc} = 2.8 \times 10^{14}$ g/cm^3) the mutual repulsion between neutrons will halt the collapse, and the core will bounce back to an equilibrium configuration, which is now a neutron star. The bounce will send a shock wave through the surrounding envelope, making the envelope explode in a type II supernova. The gravitational binding energy released in the collapse of the core is $\sim 10^{53}$ erg, about 1% of which goes into the kinetic energy of the expanding envelope. Neutrinos carry the rest of the energy away.

The expanding ejecta, with the kinetic energy of 10^{51} erg, is heated by the shock, as well as the decay of radioactive elements synthesised and ejected. It therefore shines brightly. The total energy emitted in radiation amounts to $\sim 10^{49}$ erg. Very massive stars would be able to grow cores too massive to be supported as neutron stars, the reason for this being the additional radiation pressure support in the pre-collapse core. Such cores will collapse to black holes. A spinning black hole produced this way will swallow the inner parts of the envelope through a dense accretion disk, and eject a small fraction of matter in a jet along the spin axis. With large amount of energy imparted to this small amount of matter, the material in the jet would move at relativistic speeds. Viewed along the jet, this will be a copious source of high energy radiation. This model is believed to explain the Gamma-Ray Burst sources. The rest of the envelope in this case will get eventually expelled in a supernova-like explosion (often referred to as a "hypernova").

de Laval nozzle: Jets

Let us consider a one-dimensional flow, and assume that gravity can be ignored. This is described by

$$\frac{dv}{dt} = v\frac{dv}{dx} = -\frac{1}{\rho}\frac{dP}{dx} = -\frac{c_s^2}{\rho}\frac{d\rho}{dx}$$

which gives

$$d\ln\rho = -\frac{v^2}{c_s^2}d\ln v$$

If A is the cross sectional area of the flow then $\rho v A$ = constant. Thus

$$d\ln v = -\frac{d\ln A}{1-(v^2/c_s^2)}$$

which shows that if $v^2 < c_s^2$ then decreasing cross sectional area leads to an increase in Mach number, while for a supersonic flow an *increasing* cross sectional area increases the Mach number. So if the flow has a throat, with converging subsonic approach and diverging supersonic exit, highly supersonic jet flows can be produced. This idea has been applied to explain the formation of powerful jets seen in active galaxies - light jet matter forcing its way through interstellar medium and converging, but as the interstellar density drops away from the galactic centre, the flow cross section would expand and supersonic jets may be produced.

Today we know that magnetic fields play a more important role in the production of jets. This involves magnetic fields anchored to an accretion disk, which we discuss below.

Accretion

There are many situations that lead to the accretion of matter from the immediate surroundings onto a compact object. A compact star may capture some of the stellar wind from a binary companion, or exert sufficient tidal force on the companion to strip matter from it and produce a flow dircted towards itself (Roche Lobe Overflow). If the compact object is a stellar mass black hole or a neutron star, they show up as X-ray binaries. Accreting White Dwarfs undergo nova explosions, and are called "Cataclysmic Variables". Supermassive black holes at centres of galaxies can be fed matter from the surrounding interstellar medium or tidally stripped stars. Large accretion rates on such objects lead to the generation of high luminosity at the galactic nucleus, as well as production of powerful jets. These are known as Active Galactic Nuclei (AGNs).

Gravitational capture of matter by a body from a passing flow in which it is immeresed is treated in the classical Bondi-Hoyle picture of accretion. gravitational acceleration by the immersed body bends the trajectory of the flowing matter, causing convergence behind the body (this effect is called gravitational focussing). The trajectories cross behind the object and matter collides at the crossings. In the collision one may assume that the velocity components opposing each other are fully dissipated (and corresponding energy radiated away), while the parallel component remains. Up to a certain distance from the body the remaining parallel component would be less than the escape velocity at that point, and matter will fall in. Matter on trajectories colliding beyond that distance will escape. Tracing these trajectories back to their initial impact parameter one may define a gravitational capture cross section for the body πr_a^2, where the 'accretion radius' r_a is given by

$$r_a = \frac{2GM}{(v_w^2 + c_s^2)}$$

where M is the mass of the accreting body, v_w is the speed of the wind and c_s is the sound velocity in the wind. For wind from hot massive stars usually $v_w \gg c_s$, and the sound speed in the above expression can be ignored.

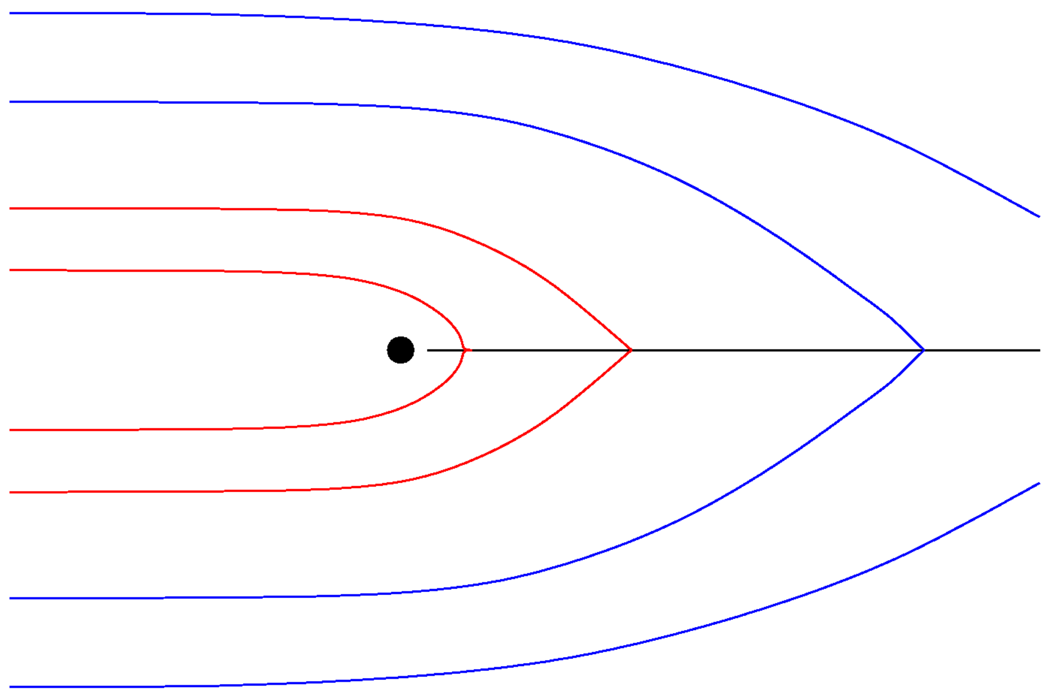


Figure 1: The geometry of Bondi-Hoyle accretion. Wind flows from left to right in the figure, past the accretor (black dot). Trajectories of wind matter are gravitationally focussed and made to collide on a line behind the accretor (horizontal black line). The velocity component perpendicular to this line is dissipated in the collision, while that parallel to this line remains. Matter on outer trajectories (blue) retains sufficient velocity to escape the gravity of the accretor while that on inner ones (red) would be captured.

If the flow past the body is not symmetric, then there is a net angular momentum in the captured matter. This is true also in case of the matter accreted in a Roche Lobe Overflow. The angular momentum will cause the matter to form a ring around the accretor. The ring will intersect the accretion stream and dissipation will ensue. Eventually through viscous dissipation matter will proceed to smaller and smaller orbits, angular momentum being transported outwards in the process. This forms an *accretion disk* around the accretor, which is encountered in a wide variety of accretion situations. At any radius R of the disk the matter rotates around the central mass at the local Keplerian speed $v_\phi = \sqrt{GM/R}$, i.e. the angular speed $\Omega = \sqrt{GM/R^3}$. As matter in inner orbits rotate faster than that in outer orbits, viscosity can make angular momentum flow outwards in the disk, and sustain an inward flow. If $v_r(R)$ is the radial inflow velocity at radius R and $\Sigma(R)$ is the surface mass density at that radius then by continuity of mass $2\pi R\Sigma(R)v_r(R) = \dot{M}$, the mass accretion rate. In a steady state the above product is constant at all radii. Normally this v_r is much smaller than the Keplerian speed v_ϕ at the same radius, and therefore the kinetic energy of matter is dominated by the Keplerian motion. It follows therefore that of the Gravitational potential energy released in the process of matter coming to radius R from far away, nearly half the energy remains in kinetic energy and the rest must have been radiated away.

If ν is the coefficient of kinematic viscosity, then the viscous force per unit length around the circumference at any R is $\nu\Sigma(Rd\Omega/dR)$. So the viscous torque around the whole circumference is $\tau(R) = R(2\pi R)\nu\Sigma(Rd\Omega/dR)$.

Now consider a ring of material between R and $R + dR$. In unit time, material of amount $\dot{M}$ enters $R + dR$ with specific angular momentum $(R + dR)^2\Omega(R + dR)$ and leaves R with specific angular momentum $R^2\Omega(R)$. This loss of angular momentum takes place because of the action of net viscous torque $(d\tau/dR)dR$. Thus

$$\dot{M}\frac{d(R^2\Omega)}{dR} = -\frac{d}{dR}\left[\nu\Sigma 2\pi R^3\frac{d\Omega}{dR}\right]$$

Using Keplerian Ω and integrating, one finds

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

where the boundary condition used is that the shear τ vanishes at an inner radius R_*. This inner radius could be the last stable orbit around a black hole, or the stellar surface in case of a white dwarf or a weakly magnetized neutron star, or approximately the Alfvén radius (distance at which the ram pressure of accreting matter equals the magnetic pressure) around a strongly magnetized accretor.

The viscous dissipation rate per unit area can then be computed:

$$D(R) = \nu\Sigma\left(R\frac{d\Omega}{dR}\right)^2 = \frac{3GM\dot{M}}{4\pi R^3}\left[1-\left(\frac{R_*}{R}\right)^{1/2}\right]$$

This must be radiated away. The disk is nearly optically thick and hence the emitted radiation can be approximated to be a blackbody at the local temperature $T(R)$. Accounting for the two surfaces of the disk, $D(R) = 2\sigma T^4$. Therefore

$$T(R) = \left\{\frac{3GM\dot{M}}{8\pi R^3\sigma}\left[1-\left(\frac{R_*}{R}\right)^{1/2}\right]\right\}^{1/4}$$

Magnetic fields anchored to the accretion disk can play a very important and interesting role. In figure 2 consider a magnetic field line anchored to the disk at P. The field line rotates with the keplerian angular speed at P. The disk being hot, matter will evaporate from the surface and move preferentially along magnetic field lines. One such blob, at R, will now be rotating faster than the local Keplerian speed and feel a net centrifugal acceleration outward. Such an effect can effectvely cause matter to leave the disk. Beyond the Alfvén distance, the field lines will twist and wrap around the rotation axis, resulting in a highly collimated matter outflow along the polar axes. This is now considered the most important mechanism for jet formation in accreting systems.

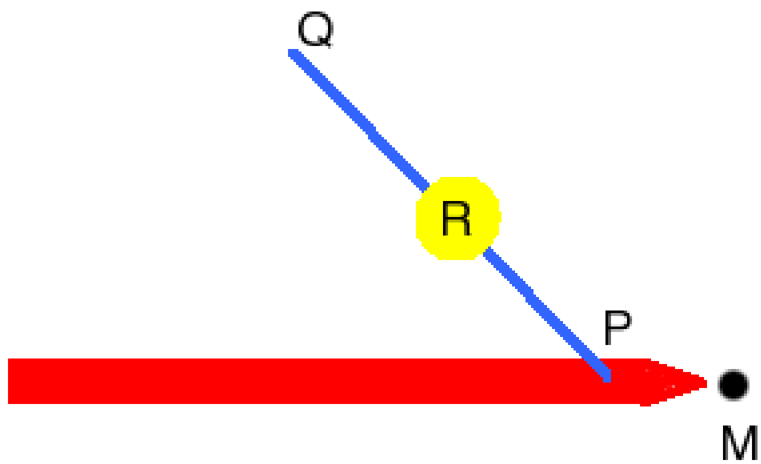


Figure 2: A gas blob R moving on a magnetic field line PQ anchored to an accretion disk (red) around a mass M can be centrifugally ejected from the disk and form part of a jet