Basics of MHD

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Plan

- Magnetic fields in Astrophysics
- MHD Basics
- MHD waves
- Astrophysical batteries

The magnetic Universe

- **Earth (1 Gauss; Irregular reversals over 2×10^5 yr)**
- **Sun (1 - 10^3 gauss; 11 yr Solar cycle)**
- **Neutron Stars: $10^{12} - 10^{15}$ G!**
- **Control of many processes: Accretion and Jets**
- **Galaxies: $B \sim 10\mu G$, ordered on 10 kpc scales + random component**
- **Clusters of Galaxies: few μG strengths on ~ 10 kpc scales**
- **Equally strong B in Young $z \sim 1 - 2$ galaxies (Bernet et al. 2008)**
- **Even in the IGM and voids? ($B \geq 3 \times 10^{-16}$ Gauss; Mpc scales)** (Neronov and Vovk, 2010; ... BUT SEE Broderick et al., 2011)

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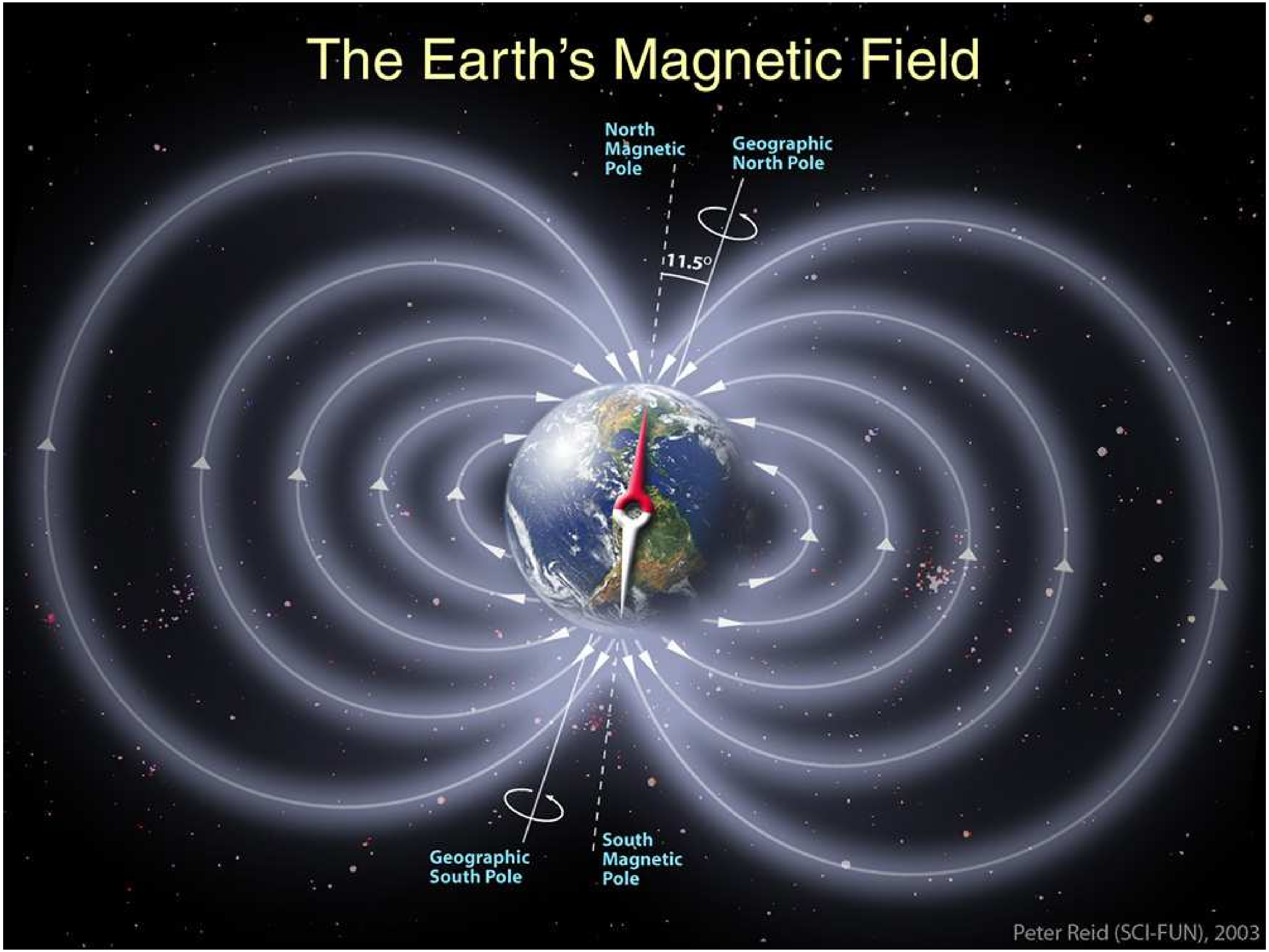
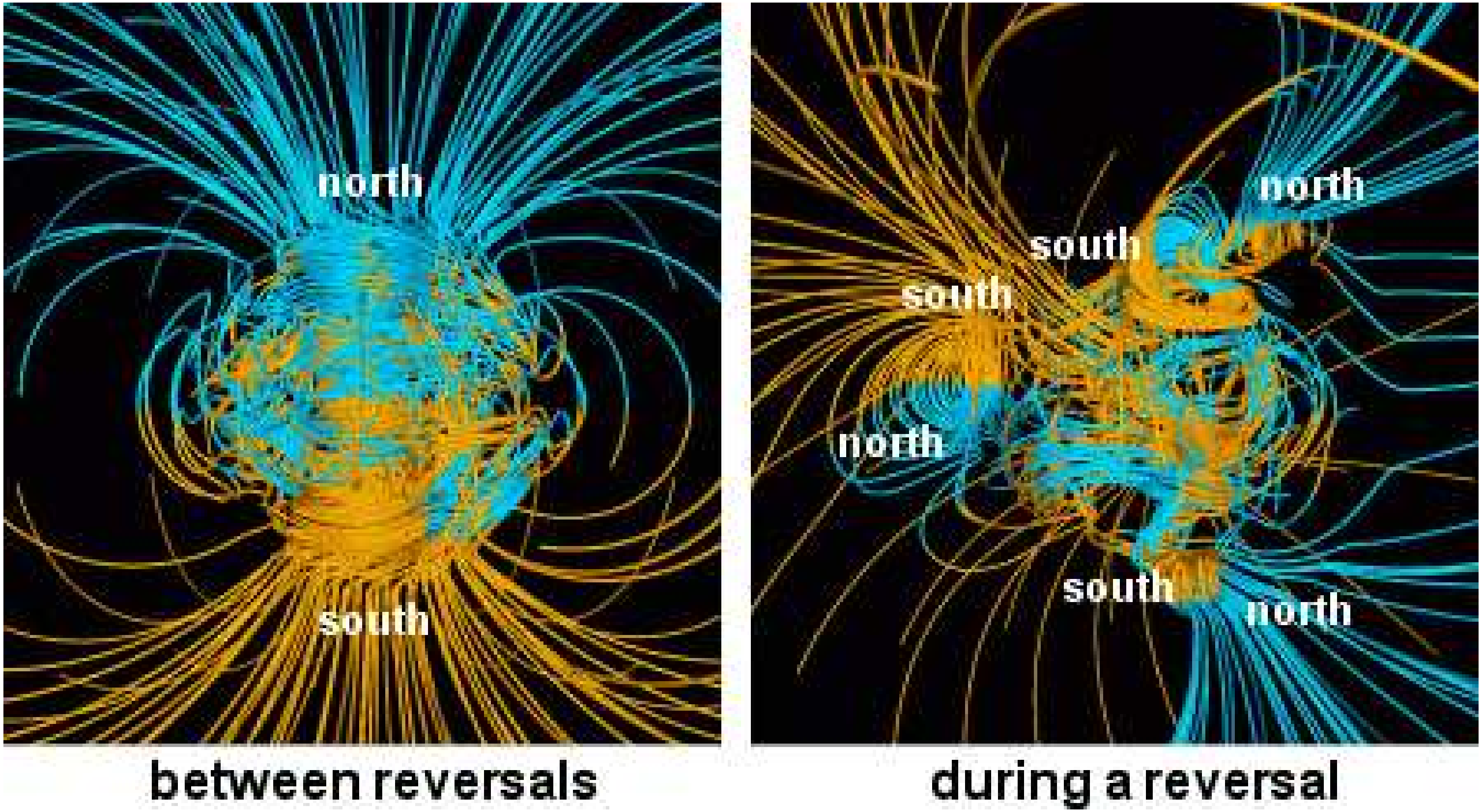
The Earth´s Magnetic Field



The Earth's Magnetic Field



Simulations by Glatzmaier and Roberts, 1995

Why Magnetic Universe?

- Electrically Neutral Universe
  - Both $+$ve and $-$ve Charges Present
- Free Electic charges + No magnetic charges $\Rightarrow$
  - Can short out strong E in plasma rest frame
- Non Relativistic Velocities $\Rightarrow$
  - Simpler to think interms of B than E

Maintaining magnetic fields

- **Magnetic fields decay if not maintained, because of:**
  - Resistance dissipating currents ($\sim 20,000$ yr for earth)
  - Lorentz force Driving motions, which are damped by Viscosity or become turbulent and then decay
- **EM induction by Motions can maintain magnetic fields**

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \frac{\boldsymbol{J}}{\sigma}.$$

- **Motions in a magnetic field induces electric fields**
- **If this electric field has a curl, can re-generate magnetic fields**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\boldsymbol{V} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}.$$

- **Magnetic Field almost frozen to moving plasma.**
  Need initial B field – "Battery".
  Need kinetic to magnetic energy conversion — dynamos

MHD basics: The Induction equation

- **Maxwell's equations**

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \nabla \cdot \mathbf{B} = 0,$$

$$\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c}\mathbf{J}, \quad \nabla \cdot \mathbf{E} = 4\pi\rho_{\rm e},$$

- **Ohm's law**

$$\mathbf{J} = \rho_{\rm e}\mathbf{V} + \sigma\left(\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c}\right)$$

- **Eliminate $\mathbf{J}$ using Ohm+Maxwell, define resitivity** $\eta = c^2/(4\pi\sigma)$

$$\frac{\eta}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \frac{\eta}{c^2}\mathbf{V}(\nabla \cdot \mathbf{E}) + \mathbf{E} = \frac{\eta}{c}(\nabla \times \mathbf{B}) - \frac{\mathbf{V} \times \mathbf{B}}{c}.$$

- **Faraday timescale** $\eta/c^2 \sim 10^{-14} T_4^{-3/2} s$ **for ionized plasma!**

MHD basics: The Induction equation

- Variation timescale (or advective timescale) of $\mathbf{E}$ usually much longer than Faraday time

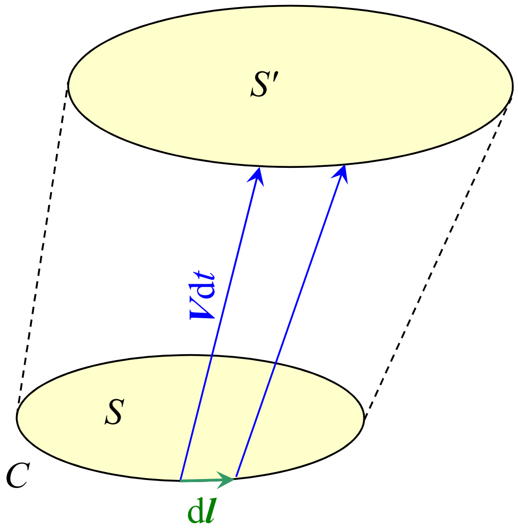
$$\frac{\eta}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \frac{\eta}{c^2}\mathbf{V}(\nabla \cdot \mathbf{E}) + \mathbf{E} = \frac{\eta}{c}(\nabla \times \mathbf{B}) - \frac{\mathbf{V} \times \mathbf{B}}{c}.$$

- So neglect displacement and advective currents terms, take curl, use Faraday $\Rightarrow$ Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \nabla \times \mathbf{B})\,.$$

  - $\mathbf{V} = 0 \Rightarrow$ pure diffusion and decay
  - If $\eta \to 0$, the flux $\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$ is 'frozen' $\to d\Phi/dt \to 0$.
  - Mag. Reynolds number $R_{\rm M} = (VB)/(\eta B/L) = VL/\eta \gg 1$

Flux Freezing



- **Magnetic flux:** $\Phi = \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}$.

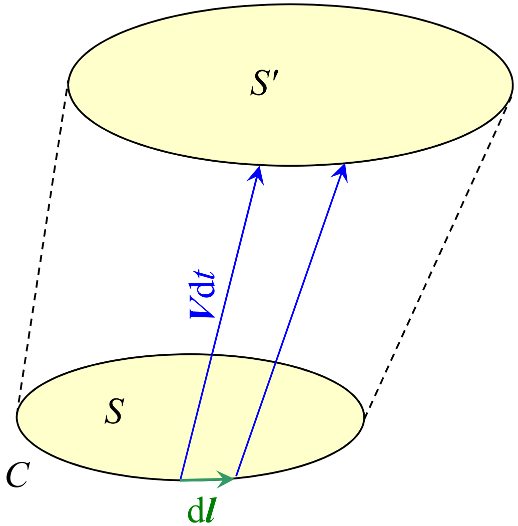
$$\Delta\Phi = \int_{S'} \boldsymbol{B}(t+dt) \cdot \mathrm{d}\mathbf{S} - \int_S \boldsymbol{B}(t) \cdot \mathrm{d}\mathbf{S}.$$

- $\nabla \cdot \boldsymbol{B} = 0$ **at time** $t + dt \Rightarrow$

$$\int_{S'} \boldsymbol{B}(t+dt) \cdot \mathrm{d}\mathbf{S} = \int_S \boldsymbol{B}(t+dt) \cdot \mathrm{d}\mathbf{S} - \oint_C \boldsymbol{B}(t+dt) \cdot (d\mathbf{l} \times \mathbf{V} dt),$$

$$\Delta\Phi = \int_S [\boldsymbol{B}(t+dt) - \boldsymbol{B}(t)] \cdot \mathrm{d}\mathbf{S} - \oint_C \boldsymbol{B}(t+dt) \cdot (d\mathbf{l} \times \mathbf{V}) dt.$$

Flux Freezing



$$\Delta\Phi = \int_S [\boldsymbol{B}(t+dt)-\boldsymbol{B}(t)]\cdot d\mathbf{S} - \oint_C \boldsymbol{B}(t+dt)\cdot(d\mathbf{l}\times\mathbf{V})dt.$$

$$\frac{d\Phi}{dt} = \int_S \frac{\partial \boldsymbol{B}}{\partial t}\cdot d\mathbf{S} - \oint_C (\mathbf{V}\times\boldsymbol{B})\cdot d\mathbf{l}$$

- Using $\oint_C (\mathbf{V}\times\boldsymbol{B})\cdot d\mathbf{l} = \int_S \nabla\times(\mathbf{V}\times\boldsymbol{B})\cdot d\mathbf{S}$

$$\frac{d\Phi}{dt} = \int_S \left[\frac{\partial \boldsymbol{B}}{\partial t} - \nabla\times(\mathbf{V}\times\boldsymbol{B})\right]\cdot d\mathbf{S} = \eta\int_S (\nabla^2\boldsymbol{B})\cdot d\mathbf{S}.$$

- **So as $\eta \to 0$, $d\Phi/dt \to 0$ and so Φ is constant.**

Flux freezing applications

- **Consider a flux tube with field B, length l, density ρ**

$$BA = \text{Constant}, \qquad \rho A l = \text{Constant} \to \frac{B}{\rho l} = \text{Constant}$$

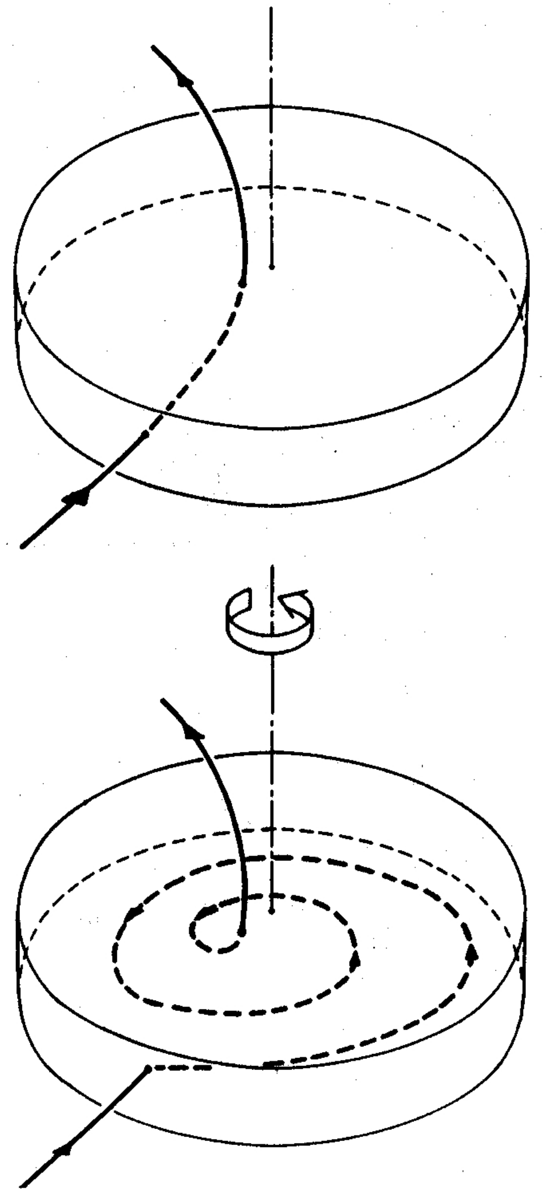
- **Compressing a flux tube of constant lenth l increases B**

- **Incompressible fluid motion with $\rho = \text{Constant}$ gives $B \propto l$ and $A \propto 1/l$: Application to small-scale dynamo**

- **Ingoring diffusion, one can expand induction quation as**

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{V} \times \boldsymbol{B}) = -\underbrace{(\boldsymbol{V} \cdot \boldsymbol{\nabla})\boldsymbol{B}}_{\text{advection}} + \underbrace{(\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{V}}_{\text{stretching}} - \underbrace{\boldsymbol{B}(\boldsymbol{\nabla} \cdot \boldsymbol{V})}_{\text{compression}} ,$$

Galactic Shear



- Stretching due to differential rotation in galaxy
- Let $V = (0, r\Omega(r), 0)$, initially axisymmetric B
- Leads to winding up the field: $B_r = B_r(r,z)$ $B_\phi = B_\phi(r,z,t_0) + r(d\Omega/dr)B_r t$ $B_z = B_z(r,z))$

The Lorentz Force

- **The Lorentz force on a charge:** $\mathbf{F} = q[\boldsymbol{E} + (\boldsymbol{v} \times \boldsymbol{B})/c]$

- **The Lorentz force on fluid $\mathbf{F}$ is** :

$$
\begin{aligned}
\mathbf{F} &= +en_p\left[\mathbf{E} + \frac{\mathbf{v_p} \times \boldsymbol{B}}{c}\right] - en_e\left[\mathbf{E} + \frac{\mathbf{v_e} \times \boldsymbol{B}}{c}\right] \\
&= [en_p - en_e]\mathbf{E} + [en_p\mathbf{v_p} - en_e\mathbf{v_e}] \times \boldsymbol{B}/c \\
&= \rho_e\mathbf{E} + \frac{\mathbf{J} \times \boldsymbol{B}}{c}
\end{aligned}
$$

- **Charge density** $\rho_e = (+en_p - en_e)$

- **Current density:** $\mathbf{J} = (+en_p\mathbf{v_p} - en_e\mathbf{v_e})$

The neglect of the Electric force

- Including the charge density the Lorentz force is:

$$\mathbf{F} = \rho_e \mathbf{E} + \frac{\mathbf{J} \times \boldsymbol{B}}{c}$$

- The ratio of the electric to magnetic parts is:

$$\frac{|\rho_e \mathbf{E}|}{|(\mathbf{J} \times \boldsymbol{B}/c)|} = \frac{|(\nabla \cdot \mathbf{E})\mathbf{E}|}{|(\nabla \times \boldsymbol{B}) \times \boldsymbol{B})|} \sim \frac{V^2}{c^2} \ll 1,$$

- The last part follows from Ohms law applied to a highly conducting medium: $\mathbf{E} + (\mathbf{V}/c) \times \boldsymbol{B} = \mathbf{J}/\sigma \sim 0$.

- So electric part of force can be neglected compared to magnetic part for $V/c \ll 1$.

The Lorentz force

- **Using $\nabla \times \boldsymbol{B} = (4\pi/c)\mathbf{J}$**

$$\mathbf{F} = \frac{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi} = \underbrace{-\nabla\left(\frac{\boldsymbol{B}^2}{8\pi}\right)}_{\text{Pressure}} \underbrace{+\frac{(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}}{4\pi}}_{\text{Tension}}$$

- **Magnetic force broken into a ”Pressure gradient” and Tension force**

MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\rho \left[\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right] = -\nabla p - \rho \nabla \phi + \mathbf{F}_{\rm viscous} + \frac{\mathbf{J} \times \boldsymbol{B}}{c}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \,.$$

$$p = K \rho^{\gamma}; \quad \nabla^2 \phi = 4\pi G \rho$$

Magnetic energy evolution

- **Start from Faraday's Law and dot with B**

$$\frac{1}{2}\frac{\partial B^2}{\partial t} = \boldsymbol{B} \cdot \frac{\partial \boldsymbol{B}}{\partial t} = -c\boldsymbol{B} \cdot (\boldsymbol{\nabla} \times \boldsymbol{E}) = -c\boldsymbol{\nabla} \cdot (\boldsymbol{E} \times \boldsymbol{B}) - c\boldsymbol{E} \cdot (\boldsymbol{\nabla} \times \boldsymbol{B})$$

- **Use Ohm's Law**: $\boldsymbol{E} = -(\boldsymbol{V} \times \boldsymbol{B})/c + (4\pi/c^2)\eta \boldsymbol{J}$

$$\frac{1}{2}\frac{\partial B^2}{\partial t} = -c\boldsymbol{\nabla} \cdot (\boldsymbol{E} \times \boldsymbol{B}) - (4\pi/c)\eta \boldsymbol{J} \cdot (\boldsymbol{\nabla} \times \boldsymbol{B}) + (\boldsymbol{V} \times \boldsymbol{B}) \cdot (\boldsymbol{\nabla} \times \boldsymbol{B})$$

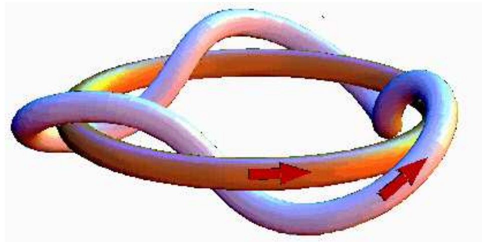
- **Shuffle the triple product, devide by 4π**

$$\frac{1}{8\pi}\frac{\partial B^2}{\partial t} + \underbrace{\boldsymbol{\nabla} \cdot \left[\frac{c}{4\pi}\boldsymbol{E} \times \boldsymbol{B}\right]}_{\text{Poynting flux}} = \underbrace{\frac{4\pi}{c^2}\eta \mathbf{J}^2}_{\text{Dissipation}} \underbrace{- \mathbf{V} \cdot \frac{(\mathbf{J} \times \mathbf{B})}{c}}_{\text{LF Work}}$$

Magnetic Helicity

- **Magnetic Helicity** $H = \int_V \mathbf{A} \cdot \mathbf{B} \, dV$, $\quad \nabla \times \boldsymbol{A} = \boldsymbol{B}$
  $\mathbf{A}$ is vector potential, V is **closed** volume

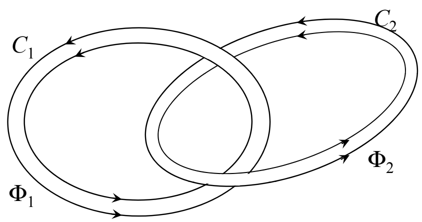
- **Measures links and twists in** B



- H **invariant under gauge transformation for closed fields**

$$H' = \int_V \boldsymbol{A}' \cdot \boldsymbol{B}' \, \mathrm{d}V = H + \int_V \nabla\Lambda \cdot \boldsymbol{B} \, \mathrm{d}V = H + \oint_{\partial V} \Lambda \boldsymbol{B} \cdot \hat{\boldsymbol{n}} \mathrm{d}S = H,$$

- $H = \Phi_1 \oint_{C_1} \boldsymbol{A} \cdot \mathrm{d}\mathbf{l} + \Phi_2 \oint_{C_2} \boldsymbol{A} \cdot \mathrm{d}\mathbf{l}, = 2\Phi_1\Phi_2$



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IUCAA Linking number is

$$(\mathbf{4} \times \mathbf{1}) + (\mathbf{4} \times -\mathbf{1}) = \mathbf{0}!!$$

Magnetic helicity evolution

- **Using Faraday's law and** $(\partial \boldsymbol{A}/\partial t) = -c(\boldsymbol{E} + \nabla\phi)$

$$\frac{1}{c}\frac{\partial}{\partial t}(\boldsymbol{A}\cdot\boldsymbol{B}) = (-\boldsymbol{E} - \nabla\phi)\cdot\boldsymbol{B} + \boldsymbol{A}\cdot(-\nabla\times\boldsymbol{E})$$
$$= -2\boldsymbol{E}\cdot\boldsymbol{B} + \nabla\cdot(\boldsymbol{A}\times\boldsymbol{E} - \phi\boldsymbol{B}).$$

- **Use Ohm's Law**: $\boldsymbol{E} = -(\boldsymbol{V}\times\boldsymbol{B})/c + (4\pi/c^2)\eta\boldsymbol{J}$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -2\int_V \boldsymbol{E}\cdot\boldsymbol{B}\mathrm{d}V + \oint_{\partial V}(\boldsymbol{A}\times\boldsymbol{E} - \phi\boldsymbol{B})\cdot\hat{\boldsymbol{n}}\mathrm{d}S$$
$$= -2\eta\int_V(\frac{4\pi}{c})\boldsymbol{J}\cdot\boldsymbol{B}\,\mathrm{d}V$$

Helicity Conservation

- **Helicity evolution**
$$\frac{dH}{dt} = -2\eta \int_V dV \, \frac{4\pi}{c} \mathbf{J} \cdot \mathbf{B}.$$
- **For energy**
$$\frac{dE_B}{dt} = -\eta \int_V dV \, \frac{4\pi}{c^2} \mathbf{J}^2 - \int_V dV \, \mathbf{V} \cdot \frac{(\mathbf{J} \times \mathbf{B})}{c}$$
- **As $\eta \to 0$, $dE_B/dt \to$ constant with $|\mathbf{J}| \propto \eta^{-1/2}$, $B \propto \eta^0$.**
- **BUT $dH/dt \to 0$!**
- **Helicity is nearly conserved even when energy dissipated**

How does helicity arise in astrophysical systems?

Writhe and Twist Helicities

Anvar and Natasha Shukurov 2009

WRITHE AND TWIST Helicities

MHD Waves

- **Magnetostatic equilibrium, with ρ_0, p_0, $\boldsymbol{B}_0$ uniform and $\boldsymbol{V}_0 = 0$**
- **Consider small perturbations ρ_1, p_1, $\boldsymbol{b} = \boldsymbol{B} - \boldsymbol{B}_0$, $\boldsymbol{v}$ and linearize MHD eqns in the perturbations**

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot (\mathbf{v}) = 0$$

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla \left[p_1 + \frac{\boldsymbol{B}_0 \cdot \boldsymbol{b}}{4\pi} \right] + \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\boldsymbol{b}}{4\pi}$$

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}_0) = (\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\boldsymbol{v} - \boldsymbol{B}_0(\boldsymbol{\nabla} \cdot \boldsymbol{v}).$$

$$p_1 = (dp/d\rho)_0 \rho_1 = (\gamma p_0/\rho_0) = c_s^2 \rho_1$$

- **Solve by taking Fourier transform: modes with $\exp i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)$**

MHD waves

- For Fourier modes we have

$$-i\omega\hat{\rho}_1 + \rho_0(i\boldsymbol{k}\cdot\hat{\boldsymbol{v}}) = 0; \quad -i\omega\hat{\boldsymbol{b}} = (i\boldsymbol{k}\cdot\boldsymbol{B}_0)\hat{\boldsymbol{v}} - \boldsymbol{B}_0(i\boldsymbol{k}\cdot\boldsymbol{v})$$

$$-i\omega\hat{\boldsymbol{v}} = -i\boldsymbol{k}\left[\frac{\hat{p}_1}{\rho_0} + \frac{\boldsymbol{B}_0\cdot\hat{\boldsymbol{b}}}{4\pi}\right] + \frac{(i\boldsymbol{k}\cdot\boldsymbol{B}_0)\hat{\boldsymbol{b}}}{4\pi}; \quad \hat{p}_1 = c_s^2\hat{\rho}_1$$

- Multiply by $i\omega$ eliminate b and ρ_1; Define Alfvén velocity: $\boldsymbol{V}_A = \boldsymbol{B}_0/\sqrt{4\pi\rho_0}$.

$$\omega^2\hat{\boldsymbol{v}} = (c_s^2+V_A^2)(\boldsymbol{k}\cdot\hat{\boldsymbol{v}})\boldsymbol{k}+(\boldsymbol{k}\cdot\boldsymbol{V}_A)\left[(\boldsymbol{k}\cdot\boldsymbol{V}_A)\hat{\boldsymbol{v}} - (\boldsymbol{V}_A\cdot\hat{\boldsymbol{v}})\boldsymbol{k} - (\boldsymbol{k}\cdot\hat{\boldsymbol{v}})\boldsymbol{V}_A\right].$$

- Have homogengenous matrix equation for $\hat{v}_i$ of form $M_{ij}\hat{v}_j = 0$
- Dispersion relation from $Det(M) = 0$, 3-modes of Oscillation, for magnetic case unlike fluid case

MHD Waves

- **Alfvén Waves: For $\hat{\boldsymbol{v}} \perp \boldsymbol{k}$ and $\hat{\boldsymbol{v}} \perp \boldsymbol{B}_0$, so $\boldsymbol{k} \cdot \hat{\boldsymbol{v}} = 0$ and $\hat{\boldsymbol{v}} \cdot \boldsymbol{V}_A = 0$**

$$\omega^2 = (\boldsymbol{k} \cdot \boldsymbol{V}_A)^2 = (k V_A \cos\theta)^2; \quad \omega/k = \pm V_A \cos\theta; \qquad (\cos\theta = \hat{\boldsymbol{B}}_0 \cdot \hat{\boldsymbol{k}})$$

- **Group velocity $\nabla_{\boldsymbol{k}} \omega = \pm \boldsymbol{V}_A$, waves proagate along the field, incompressible**
- **From induction equation: $\hat{v} \parallel \hat{b}$, so $\hat{b} \cdot B_0 = 0$, and driven by field tension**
- **Other two modes taking dot product with $\boldsymbol{k}$ and $\boldsymbol{V}_A$**

$$\begin{pmatrix} \omega^2 - k^2(c_s^2 + V_A^2) & k^2(\boldsymbol{V}_A \cdot \boldsymbol{k}) \\ -(\boldsymbol{V}_A \cdot \boldsymbol{k}) c_s^2 & \omega^2 \end{pmatrix} \begin{pmatrix} \boldsymbol{k} \cdot \hat{\boldsymbol{v}} \\ \boldsymbol{V}_A \cdot \hat{\boldsymbol{v}} \end{pmatrix} = 0$$

- **Gives dispersion relation for fast/slow magnetosonic waves**

$$\left(\frac{\omega}{k}\right)^2 = \frac{1}{2}(V_A^2 + c_s^2) \pm \left[\frac{1}{4}(V_A^2 + c_s^2)^2 - V_A^2 c_s^2 \cos^2\theta\right]^{1/2}.$$

The first "seed" fields in the universe

- **Primordial fields from Early Universe? Uncertain Physics**
  Constrained by observations of CMB, FRM

- **Astrophysical Batteries using postive/negative charge asymmetry**

- **Biermann Batteries**: $\mathbf{E}_{Bier} = -\nabla p_e/en_e + ...$

$$(\partial \mathbf{B}/\partial t) = -c\nabla \times \mathbf{E}_{Bier} = -(ck/en_e)\nabla n_e \times \nabla T_e$$

  - **Re-Ionization fronts**: $B < 10^{-19}$ G (Subramanian, Narasimha, Chitre, MN, 1994; Gnedin, Ferrara and Zweibel, ApJ, 2000)
  - **Structure formation Shocks** (Kulsrud et al, 1997)

- **During recombination:** $\gamma - e/p$ **scattering asymmetry**
  $B_0 \sim 10^{-30}$ G at MPc (Gopal & Sethi, 2005; Mattarrese et al, 2005);
  $B_0 \sim 10^{-21}$ G at pc (Ichiki et al 2007)

- **Seed fields from first supernovae and AGN outflows**

Need Dynamos to explain observed fields and maintain against decay

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Biermann Battery

- **Write 2-fluid equations for electrons/protons and subtract**

$$\frac{D_e \boldsymbol{v}_e}{Dt} = -\frac{\boldsymbol{\nabla} p_e}{n_e m_e} - \frac{e}{m_e}\left(\boldsymbol{E} + \boldsymbol{v}_e \times \boldsymbol{B}\right) - \boldsymbol{\nabla}\phi_g - \frac{(\boldsymbol{v}_e - \boldsymbol{v}_p)}{\tau_{ei}},$$

$$\frac{D_i \boldsymbol{v}_i}{Dt} = -\frac{\boldsymbol{\nabla} p_i}{n_i m_i} + \frac{e}{m_i}\left(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B}\right) - \boldsymbol{\nabla}\phi_g + \frac{m_e n_e}{m_i n_i}\frac{(\boldsymbol{v}_e - \boldsymbol{v}_i)}{\tau_{ei}}.$$

- **Assume $m_e \ll m_i$, neglect nonlinear: Generalized Ohm's law**

$$\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B} = -\frac{\boldsymbol{\nabla} p_e}{e n_e} + \frac{\boldsymbol{J}}{\sigma} + \frac{1}{e n_e}\boldsymbol{J} \times \boldsymbol{B} + \frac{m_e}{e^2}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{J}}{n_e}\right)$$

- **Take curl, use Ampere, neglect Hall, inertial terms**

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left[\boldsymbol{V} \times \boldsymbol{B} - \eta(\boldsymbol{\nabla} \times \boldsymbol{B})\right] - \frac{c k_{\rm B}}{e}\frac{\boldsymbol{\nabla} n_e}{n_e} \times \boldsymbol{\nabla} T.$$

- **Fields generated from zero if $\boldsymbol{\nabla} T$ not parallel to $\boldsymbol{\nabla} n_e$**

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