Binary Stars

A large fraction of stars are born in binary systems. In close binaries with initial orbital periods less than a few years, the tidal forces due to the companion can significantly influence the course of evolution of a star.

Let us consider two stars of mass M_1 and M_2 in a binary system with a circular orbit of separation a. In a frame corotating with the binary the force at any point P can be derived from an effective potential of the form

$$V = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{\omega^2 r_3^2}{2} \tag{1}$$

where r_1 and r_2 are the distances of P from M_1 and M_2 respectively, and r_3 is its distance from the rotation axis, which passes through the centre of mass. $\Omega = \sqrt{GM/a^3}$ is the orbital frequency of the binary, the orbital period being $P_{\rm orb} = 2\pi/\Omega$. The distances r_1, r_2 and r_3 can be written in terms of the coordinate $\vec{r}$ of the point P to obtain V as a function of $\vec{r}$. This is called the "Roche potential". If the point P is located in the orbital plane then r_3 is simply the distance from the centre of mass (see fig. 1).

Equipotentials of the Roche potential function in the orbital plane reveal an interesting structure: there are three saddle points of the potential and two maxima, as shown in figure 2. These are called the Lagrangian points and are designated L_1 to L_5, as shown in the figure. The lobes through the L_1 point that encircle the two stars are called the "critical potential lobes" or "Roche lobes". In the course of evolution if one of the stars expands and fills its Roche Lobe then matter would flow from it to the companion. Usually one expresses the size of the Roche lobe in terms of the critical radius R_L, defined as the radius of the sphere which has the same volume as the Roche Lobe. An expression (given by Eggleton) for R_L (around M_1) accurate to within one percent for all mass ratios is as follows:

$$\frac{R_L}{a} = \frac{0.49}{0.6 + q^{2/3}\ln(1 + q^{-1/3})}$$

where $q = M_2/M_1$.

If both components of the binary system are smaller than their respective

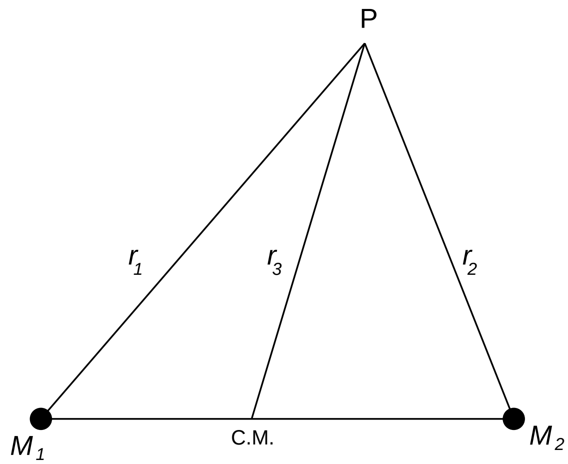

Figure 1: The three distances r_1, r_2 and r_3, in the orbital plane of a binary system of masses M_1 and M_2, which enter the expression for the Roche Potential in a reference frame co-rotating with the binary. C.M. indicates the centre of mass of the binary.

Roche Lobes, then the binary is called "detached". If one component fills its Roche Lobe then the binary is called "semi-detached", and when both components grow to fill their corresponding Roche Lobes together the binary is said to be a "contact binary". In this last situation an envelope may form covering both the components. This is known as the "common envelope" phase of binary evolution. Usually a large amount of matter is lost from the binary through the L_2 and L_3 points during a common envelope phase.

Transfer of mass from one star to another would change the orbital parameters of the binary system. The orbital angular momentum of the binary can be written as

$$J = M_1 M_2 \sqrt{\frac{Ga}{M}} \tag{2}$$

where $M = M_1 + M_2$ is the total mass of the system. From this, one may write

$$\frac{\dot{a}}{a} = 2\frac{\dot{J}}{J} - 2\frac{\dot{M}_1}{M_1} - 2\frac{\dot{M}_2}{M_2} + \frac{\dot{M}}{M} \tag{3}$$

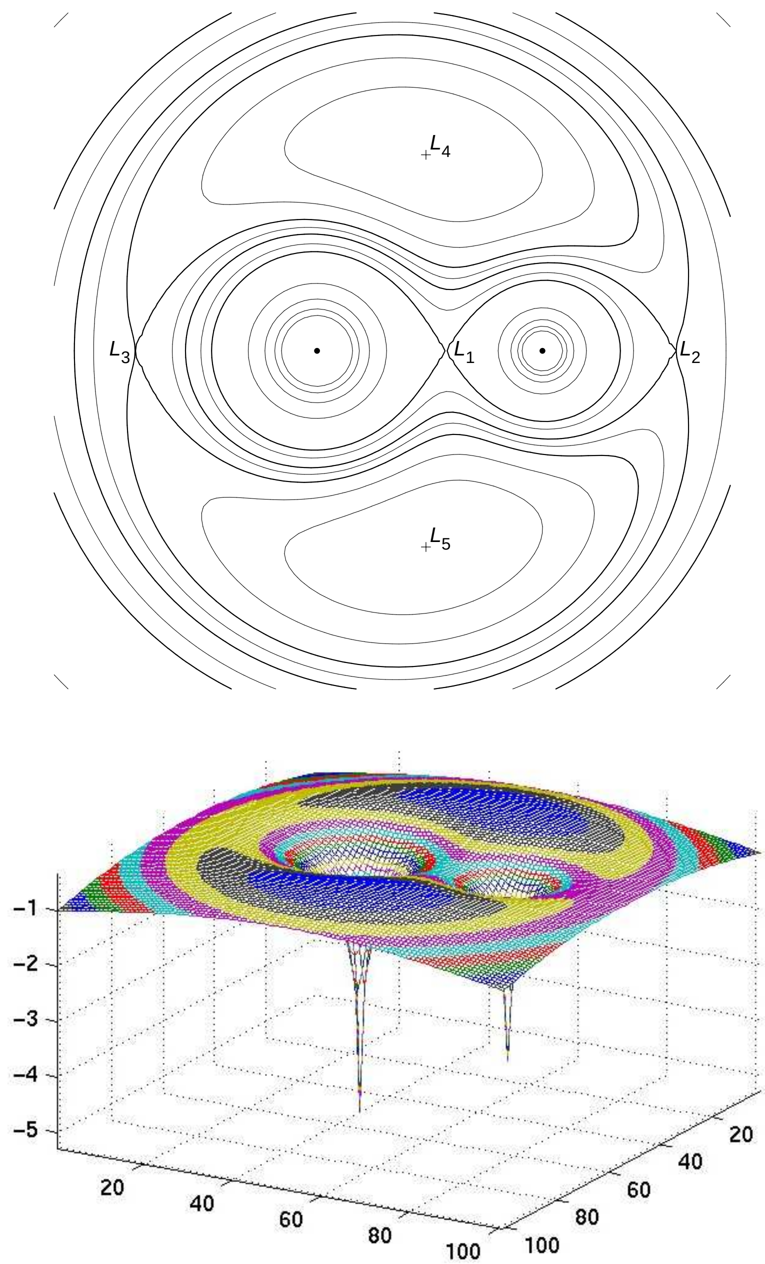

Figure 2: Roche potential in the equatorial plane of a binary system plotted as equipotential contours (above) and in a 3-d surface plot (below). The five Lagrangian points L_1 to L_5 are marked in the upper panel. The mass ratio of the stars is 2:1.

If the evolution is "conservative", namely that no matter or angular momentum leaves the binary system, then both $\dot{M}$ and $\dot{J}$ vanish. Eq. 3 then yields

$$a \propto \frac{1}{(M_1 M_2)^2}$$

or

$$\frac{\dot{a}}{a} = -2\frac{\dot{M}_1}{M_1}\left(1 - \frac{M_1}{M_2}\right)$$

where we have used $\dot{M}_2 = -\dot{M}_1$ due to mass conservation. If M_1 is the mass losing star, $\dot{M}_1$ is negative. We therefore see that if the donor is more massive than the accretor, then the orbit will shrink, and if the donor is less massive than the accretor then the orbit will widen upon conservative mass transfer.

Since the size of the Roche lobe around the components are proportional to the orbital separation a, shrinkage of the orbit also implies shrinking of the Roche Lobe. This can make the Roche-Lobe-Overflow (RLOF) mass transfer highly unstable when matter goes from the more massive to the less massive star. As seen in fig. 3, starting from first contact at the right-most point in the diagram, the size of the orbit, and hence the Roche lobe begins to decrease rapidly while the equilibrium radius of the star does not decrease so quickly. As a result the RLOF mass transfer rate becomes very large and would finally stop only when the mass ratio is much past reversal, i.e. the donor star has become much less massive than the accretor star. At this point normally just the Helium core of the donor star is left, the rest of the matter having been transferred onto the accretor. This is the resolution of the classical *Algol paradox*, involving the binary star *Algol* in which the less massive component was found to be more evolved, contrary to the expectation from single star evolution.

When the mass donating star is less massive than the accretor, as can happen when the accretor is a compact star: white dwarf, neutron star or black hole, the Roche Lobe would expand upon mass transfer and mass transfer would tend to stop. In such a situation sustained mass transfer can take place via one of two mechanisms: nuclear evolution and angular momentum loss.

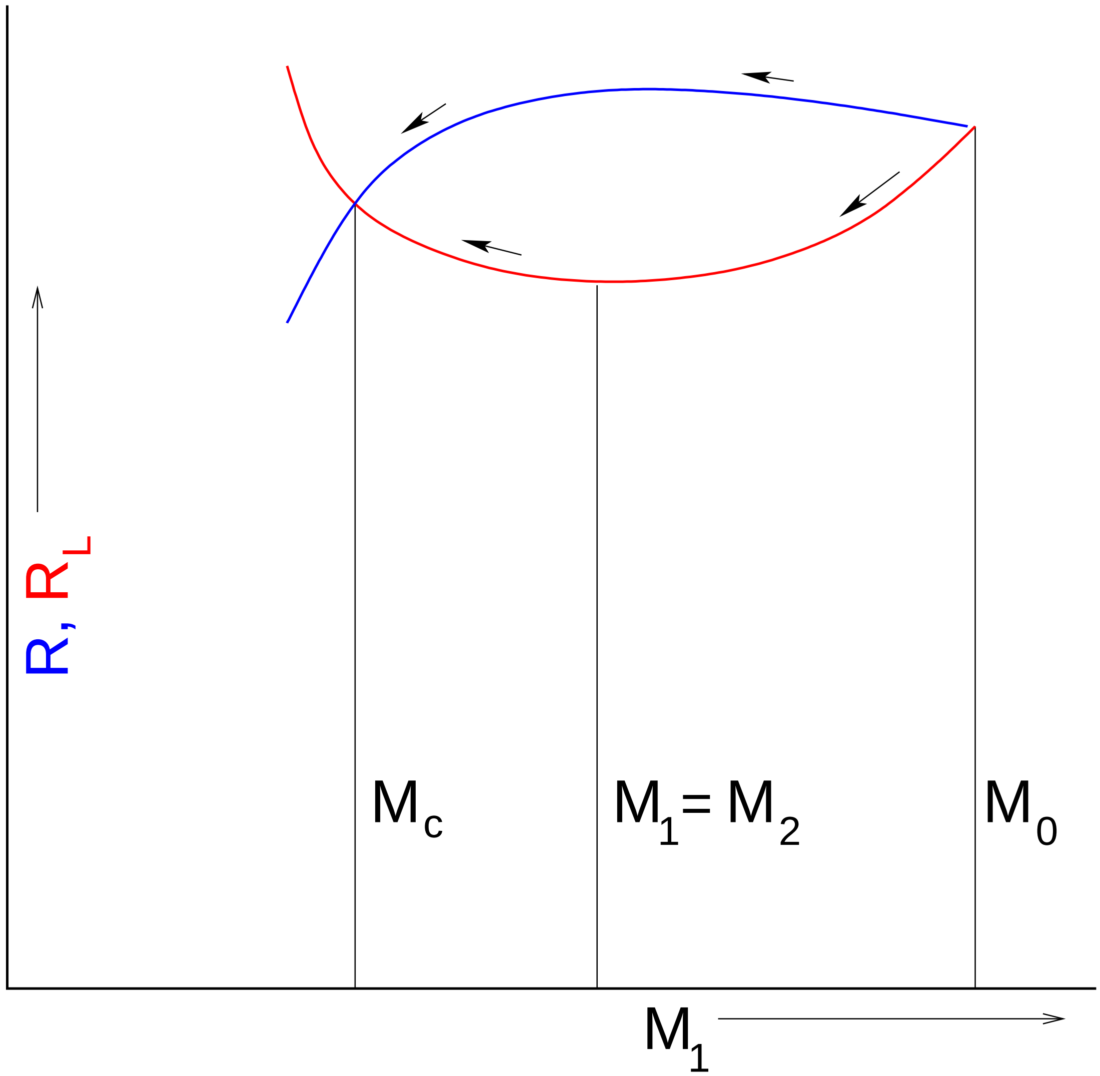

Figure 3: Evolution of the Roche Lobe (red) and the radius (blue) of the mass-donating star of mass M_1 during conservative mass transfer in a binary. The accretor star (of mass M_2) is less massive than the donor star at the start of the mass transfer (rightmost point in the figure). Mass transfer causes M_1 to decrease and M_2 to increase, and the orbital separation and hence the Roche Lobe size passes through a minimum when the mass ratio becomes unity. The evolution of the stellar radius ensures continuous, heavy mass transfer until only the Helium core (of mass M_c) of the donor is left.

If the initial orbit is sufficiently wide (orbital period of a few days or more), the donor star comes into first contact in the post main-sequence phase of evolution. upon mass transfer the orbit, and hence the Roche Lobe expands, but the radius of the star also expands due to nuclear evolution. Mass transfer is therefore sustained via nuclear evolution, and a controlled, long period of substantial transfer rate can be obtained. Wide low-mass X-ray binaries are examples of this kind. They are among the brightest, long-lived X-ray sources in the Galaxy where neutron stars and stellar mass black holes are accreting from less massive companions via nuclear evolution.

If the initial orbit is smaller, then the donor star would normally fill its Roche lobe while still on the main sequence, due to the loss of angular momentum from the binary system via gravitational radiation and magnetic braking. Angular momentum loss ($\dot{J} < 0$) causes the orbit to shrink even without transfer of mass (eq. 3).

Of the Angular Momentum Loss (AML) meshanisms, gravitational radiation has been discussed in the notes on gravity. The mechanism of magnetic braking proceeds as follows. Low-mass stars, such as the sun, have convective envelopes and consequent magnetic activity on the surface. The escaping wind from the star, consisting of highly conducting plasma, interacts with the star via this magnetic field and exerts a spin-down torque. In a close binary, tidal interaction attempts to keep the star in corotation with the orbit (as for example has happened to the moon in the earth-moon system). As the star loses angular momentum via magnetic braking, the tidal forces pump in orbital angular momentum into the star's spin to keep it in corotation. As a result, magnetic braking serves to drain the orbital angular momentum of the binary system.

The evolution of the orbit during the AML-driven mass transfer follows the evolution of the stellar radius. In the main sequence the radius of the star shrinks on mass transfer and so does the orbit. As more and more matter is taken away from the donor, degeneracy pressure begins to become more important in supporting the star. Once the remaining configuration is almost fully degenerate the mass-radius relation reverses (star expands upon loss of mass) and the orbit begins to expand again. Such systems would pass through a minimum in orbital period (see fig. 4). For initial $\sim 1M_\odot$ components and solar composition, this minimum orbital period works out to be $\sim$ 80 minutes which is indeed observed among binary systems such as Cataclysmic Variables, consisting of White Dwarf accretors and low-mass donors.

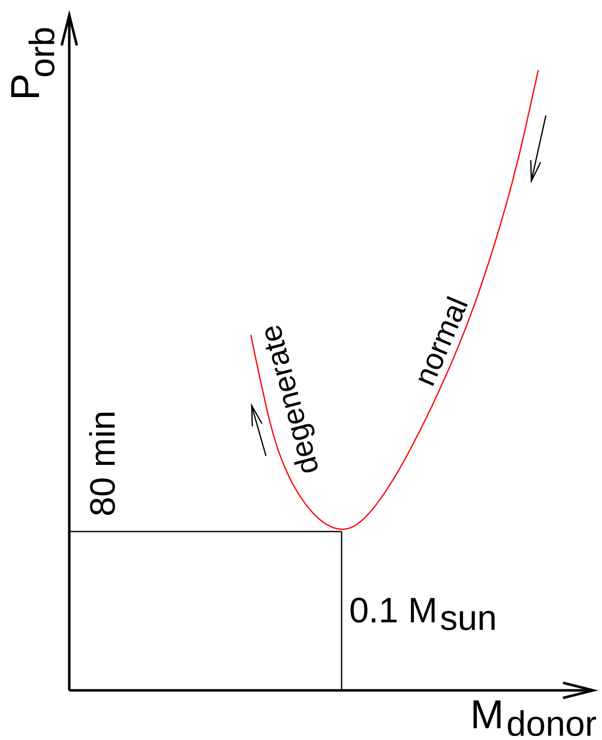

Figure 4: Schematic evolution of the orbital period of a binary containing a low-mass donor and a compact star accretor. The mass transfer is sustained by angular momentum loss. The orbital period passes through a minimum as the donor equation of state, originally thermal, turns degenerate causing a reversal of the mass-radius relation.

In binaries containing White Dwarf accretors the transferred mass is usually not retained on the White Dwarf because of the following reason. After a period of accretion, at the base of the accreted layer the temperature and density become favourable to initiate nuclear fusion. The nuclear burning takes place in degenerate condition and consequently the temperature of the layer rises so rapidly that the entire accreted mass becomes involved in nuclear burning. The nuclear binding energy generated in Hydrogen burning is much larger than the gravitational binding energy of the material on the White dwarf surface, and hence the whole accreted layer, as well as some of the original mass from the White Dwarf is explosively ejected in a *nova explosion*. In the long term, nova explosions end up in the erosion of the mass of a white dwarf. In some exceptional cases, however, the White Dwarf mass may grow with time. This requires the white dwarf to be heavy and compact ($M > 1M_{\odot}$), and the accreted mass to be either transferred very rapidly ($\dot{M} > 10^{-7}M_{\odot}$/yr), or to be devoid of hydrogen and helium (which reduces the nuclear energy generation efficiency). In fact a heavy CO white dwarf accreting matter from another CO white dwarf is the ideal progenitor for a Type Ia supernova. The accretor grows in mass and on approaching Chandrasekhar limit undergoes quick adiabatic compression which raises the interior temperature and ignites Carbon in degenerate condition. The entire white dwarf explodes as a result, giving rise to a Type Ia supernova. About 10^{51} erg of nuclear energy is released in the process, going into the kinetic energy of the ejecta. Since the final condition leading to the Type Ia supernova is very similar in all such cases, this type of supernova has very standard properties (such as peak luminosity, decay rate etc), and is therefore being used as a cosmological distance indicator.

On the surface of a neutron star the gravitational binding energy of matter is much larger than the possible energy generation by nuclear burning, and hence the accreted matter cannot be fully ejected due to nuclear burning. Nevertheless if the conditions are right then thermonuclear explosions on the neutron star surface are able to eject a small fraction of accreted matter in events known as *X-ray bursts*.

Among binaries containing compact star accretors, systems with massive donors (mass much larger than the accretor) are usually not observed in the Roche-lobe overflow phase since due to the heavy transfer rate the system lasts for a very short time. The accretion rate that the compact star can accept on its surface is usually limited to the Eddingto rate (see below), and during RLOF the transfer rate can often exceed this. As a result a common envelope would form and a large fraction of mass would leave the system. X-ray emitting neutron star binaries with massive companions *are* observed, however. In these objects the donor is not yet in a Roche-lobe filling stage, but the accretion on the compact star takes place from the stellar wind of the massive star.

Gravitational capture of matter by a body from a passing flow in which it is immeresed is treated in the classical Bondi-Hoyle picture of accretion. gravitational acceleration by the immersed body bends the trajectory of the flowing matter, causing convergence behind the body (this effect is called gravitational focussing). The trajectories cross behind the object and matter collides at the crossings. In the collision one may assume that the velocity components opposing each other are fully dissipated (and corresponding energy radiated away), while the parallel component remains. Up to a certain distance from the body the remaining parallel component would be less than the escape velocity at that point, and matter will fall in. Matter on trajectories colliding beyond that distance will escape. Tracing these trajectories back to their initial impact parameter one may define a gravitational capture cross section for the body $\pi r_{\rm a}^2$, where the 'accretion radius' $r_{\rm a}$ is given by

$$r_{\rm a} = \frac{2GM}{(v_{\rm w}^2 + c_{\rm s}^2)}$$

where M is the mass of the accreting body, $v_{\rm w}$ is the speed of the wind and $c_{\rm s}$ is the sound velocity in the wind. For wind from hot massive stars usually $v_{\rm w} \gg c_{\rm s}$, and the sound speed in the above expression can be ignored.

If the flow past the body is not symmetric, then there is a net angular momentum in the captured matter. This is true also in case of the matter injected through the L_1 point in Roche Lobe Overflow. The angular momentum will cause the matter to form a ring around the accretor. The ring will intersect the accretion stream and dissipation will ensue. Eventually through viscous dissipation matter will proceed to smaller and smaller orbits, angular momentum being transported outwards in the process. This forms an *accretion disk* around the accretor, which is encountered in a wide variety of accretion situations. At any radius r of the disk the matter rotates around the central mass at the local Keplerian speed $v_\phi = \sqrt{GM/r}$, i.e. the

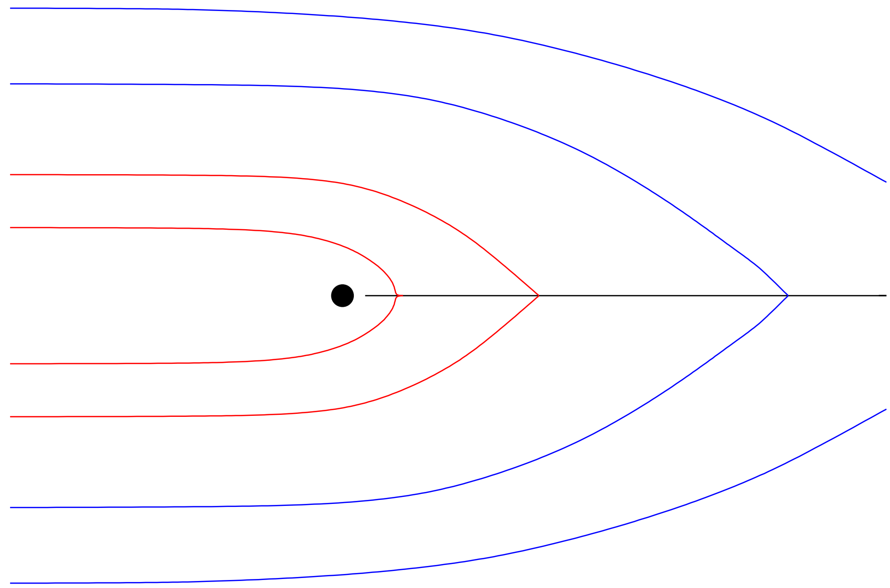

Figure 5: The geometry of Bondi-Hoyle accretion. Wind flows from left to right in the figure, past the accretor (black dot). Trajectories of wind matter are gravitationally focussed and made to collide on a line behind the accretor (horizontal black line). The velocity component perpendicular to this line is dissipated in the collision, while that parallel to this line remains. Matter on outer trajectories (blue) retains sufficient velocity to escape the gravity of the accretor while that on inner ones (red) would be captured.

angular speed $\omega = \sqrt{GM/r^3}$. As matter in inner orbits rotate faster than that in outer orbits, viscosity can make angular momentum flow outwards in the disk, and sustain an inward flow. If $v_r(r)$ is the radial inflow velocity at radius r and $\Sigma(r)$ is the surface mass density at that radius then by continuity of mass $2\pi r\Sigma(r)v_r(r) = \dot{M}$, the mass accretion rate. In a steady state the above product is constant at all radii. Normally this v_r is much smaller than the Keplerian speed v_ϕ at the same radius, and therefore the kinetic energy of matter is dominated by the Keplerian motion. It follows therefore that of the Gravitational potential energy released in the process of matter coming to radius r from far away, nearly half the energy remains in kinetic energy and the rest must have been radiated away.

This process of dissipation heats up the accretion disk, and in a steady state the energy dissipated locally is radiated away locally by radiation from the surface of the disk. The disk is nearly optically thick and hence the emitted radiation can be approximated to be a blackbody at the local temperature $T(r)$. The gravitational potential energy released between r and $r - dr$ is

$$\frac{dE_g}{dt} = GM\dot{M}\left[\frac{1}{r-dr} - \frac{1}{r}\right] = \frac{GM\dot{M}}{r^2}dr$$

half of which must be radiated away. The surface area for radiation is $4\pi r dr$ considering both the uppear and the lower surface of the disk. Hence an estimate of the local temperature $T(r)$ can be obtained from

$$4\pi r dr \sigma T^4(r) = \frac{1}{2}\frac{GM\dot{M}}{r^2}dr$$

yielding

$$T(r) = \left(\frac{G}{8\pi\sigma}\right)^{1/4} M^{1/4}\dot{M}^{1/4}r^{-3/4}$$

A more accurate estimate taking into account finite optical depth in the vertical direction and the variation of $\Sigma(r)$ with radius yields a very similar result:

$$T(r) \propto \dot{M}^{3/10}M^{1/4}r^{-3/4}$$

For typical accretion rates, setting the inner edge of the accretion disk at the surface of a neutron star yields a temperature of $\sim 10^7$ K, which explains why these objects are seen to emit X-rays. For white dwarfs the disk temperature could go up to 10^5 K, emitting Ultraviolet and optical radiation. We see that the disk radiates away half the gravitational energy released in accretion. The remaining energy is released in a thin boundary layer upon impact on the surface of the accretor, producing radiation with somewhat harder spectrum. In case of a black hole the inner edge of the disk would be located at $r = 3r_g$, the smallest stable circular orbit. The disk will be visible in X-rays. However since there is no surface there would be no boundary layer, and the remaining energy may be accreted by the hole.

We have mentioned above the limit to the accretion rate on a gravitating body. This comes from the fact that the gravitational energy released by the accreting matter generates a luminosity, and there is a maximum limit to the luminosity of a given object, called the Eddington luminosity $L_{\rm Edd}$. In a spherically symmetric situation, let us consider the luminosity escaping the surface of an object of mass M and radius R. The corresponding intensity at the surface is $I = L/4\pi R^2$. This exerts a force on electrons present there. Since Coulomb forces prevent large scale charge separation, this force is also transmitted to the associated protons. The maximum luminosity is obtained when this effective outward radiation force equals the gravitational force on the proton, i.e.

$$\frac{L_{\rm Edd}}{4\pi R^2 c}\sigma_{\rm T} = \frac{GMm_p}{R^2}$$

and hence

$$L_{\rm Edd} = \frac{4\pi GMm_p c}{\sigma_{\rm T}}$$

is the Eddington Limit. Here m_p is the proton mass and $\sigma_{\rm T}$ is the Thompson scattering cross section. The maximum luminosity of an object therefore depends only on its mass. If this luminosity is generated by accretion, $L = GM\dot{M}/R$, then the above limit implies a maximum limit on the accretion rate, $\dot{M}_{\rm Edd}$.

Situations where the mass transfer rate from the donor somewhat exceeds the Eddington rate, disk accretion may still be stable, but some matter must escape the system. In this situation, due to the predominance of radiation pressure the disk puffs up in the inner regions and creates a funnel-like structure around the rotation axis of the central object. Radiation from the inner part of the disk and the boundary layer impacts on the walls of the funnel, strips off matter and ejects it along the rotation axis in jets. Such jets arising out of heavy accretion are also seen universally, starting from stellar-mass binaries in the Galaxy to the supermassive black holes in active galactic nuclei (AGNs). Often the jets are seen to acquire great speed and move relativistically. The observational proof of relativistic motion comes from the apparent superluminal motion of blobs in the jets of quasars (which arise in AGNs) as well as some galactic binaries (called microquasars).

Superluminal motion arises when a relativistically moving blob in the jet is viewed at a small angle with respect to the jet. Consider the geometry shown in fig. 6. The blob moves at a speed $v \approx c$, and in time $(t_B - t_A)$ has moved from position A to B. The arrival times at the observer of photons emitted by the blob while at A is $t_1 = t_A + d_A/c$, and similarly for photons emitted from position B it is $t_2 = t_B + d_B/c$. The observer therefore sees the blob executing a transverse motion of amount $v(t_B - t_A)\sin\theta$ in an observed time

$$t_2 - t_1 = (t_B - t_A) - \frac{(d_A - d_B)}{c} = (t_B - t_A)\left[1 - \frac{v\cos\theta}{c}\right]$$

The apparent transverse speed is therefore

$$v_{\rm app} = c\left(\frac{\beta\sin\theta}{1 - \beta\cos\theta}\right)$$

where $\beta = v/c$. The quantity in the parentheses has a maximum at $\cos\theta = \beta$, with the maximum value being equal to $\Gamma\beta$ where $\Gamma = (1 - \beta^2)^{-1/2}$ is the Lorentz factor of motion. For $\beta \approx 1$ this quantity could be much larger than unity and the blob will be seen to move at an apparent faster-than-light (superluminal) speed. This is indeed observed in the jets of Quasars as well as the galactic microquasars (e.g. GRS 1919+105), confirming relativistic outflow.

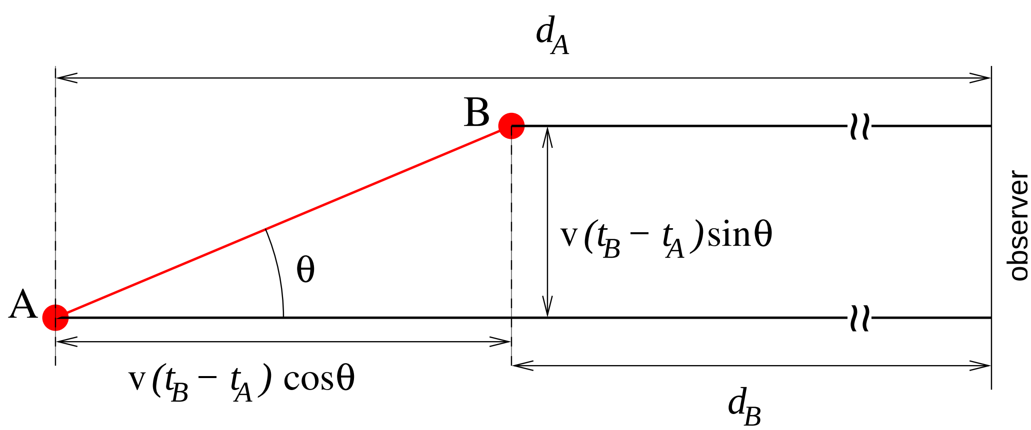

Figure 6: The geometry of apparent superluminal motion (see text). The emitting blob is at A at time t_A and moves to B at time t_B; t_A and t_B are measured by clocks at rest with respect to the observer, but located at A and B respectively.