The Magnitude Scale

The Magnitude system is a way to express the intensity of an observed source, used mainly in the optical, near-IR and UV bands. If F_ν is the intensity received from a given source (energy per unit area per unit time per unit bandwidth), then the *apparent* magnitude of the object is defined by

$$m = -2.5 \log_{10} \frac{F_\nu}{F_\nu^0}$$

where F_ν is the flux per unit frequency received from the source, and F_ν^0 is a normalising constant. The normalising constants have been calibrated for standard photometric bands, some of which are listed in table 1 below. Notice that a larger value of the magnitude means that the source is fainter, not brighter. In a clear dark sky the naked eye can detect sources up to about $m = 6$. The brightest star in the night sky, Sirius, has an apparent magnitude $m = -1.5$.

While the apparent magnitude provides a measure of the received flux, if the distance to the source is known, one can estimate its luminosity. A measure of the luminosity in the magnitude scale is provided through the *Absolute Magnitude*, which is the *apparent magnitude* that would have been observed if the source were to be situated at a distance of 10 pc from us. So if m is the apparent magnitude of a source located at a distance of d from the observer, then the absolute magnitude of the source is given by

$$M = m - 5.0 \log_{10} \left(\frac{d}{10 \text{ pc}} \right)$$

The quantity $(m - M)$ is called the *distance modulus*. The relation between the absolute magnitude at a frequency ν and the spectral luminosity L_ν of the source thus works out to be

$$L_\nu = 1.20 \times 10^{17} \left(\frac{F_\nu^0}{1 \text{ Jy}} \right) 10^{-0.4M} \text{ erg/s/Hz}$$

For the Sun, in the visible (V) band, we have $m = -26.78$ and $M = 4.79$.

In general, without reference to a specific photometric band, an effective magnitude called the **AB magnitude** is defined using the following relation:

$$m(\text{AB}) = -2.5 \log_{10}(F_\nu) - 48.60$$

where F_ν is measured in the units of erg/cm^2/s/Hz.

Table 1: Characteristics of some standard photometric bands. The normalising flux F_ν^0 is quoted in the unit Jansky (Jy), which equals 10^{-26} W/m^2/Hz. $\lambda_{\rm eff}$ is the effective wavelength in microns.

System	Band	$\lambda_{\rm eff}$ (μ)	F_ν^0 (Jy)
Johnson	U	0.3652	1920
	B	0.4448	4130
	V	0.5505	3690
	R	0.6930	3170
	I	0.8785	2550
Cousins	$R_{\rm C}$	0.6588	3100
	$I_{\rm C}$	0.8060	2450
Infrared	J	1.22	1610
	H	1.63	1040
	K	2.19	653
	L	3.45	288
	M	4.75	158