# Post Main-Sequence Evolution of Stars

At the end of the Main Sequence phase Hydrogen burning is over in the core. The core now consists of Helium, generates no energy, and slowly contracts in its thermal time scale. The luminosity crossing the surface of the core is negligible, and hence the temperature gradient through the core is also very small. The core can thus be approximated as an isothermal configuration. Just above the core lies a hydrogen-burning shell, which defines the base of the envelope. The helium produced by the burning shell is added to the core and the core mass increases in nuclear timescale. Clearly, the structure of the star changes with time, but we can try to model it as a series of near-equilibrium configurations.

The structure of any one of these configurations can be obtained by matching an isothermal core solution with an envelope solution at the boundary of the core. The main features of such stars can be inferred from an approximate discussion as follows.

For the core, let the temperature be $T_0$, the surface pressure $P_0$, mass $M_\mathrm{c}$ and radius $R_\mathrm{c}$. From Virial Theorem, we have for the core

$$2E_\mathrm{i} + E_\mathrm{g} = 4\pi R_\mathrm{c}^3 P_0$$

or

$$P_0 = \frac{2E_\mathrm{i}}{4\pi R_\mathrm{c}^3} + \frac{E_\mathrm{g}}{4\pi R_\mathrm{c}^3} = \frac{C_V M_\mathrm{c} T_0}{2\pi R_\mathrm{c}^3} - \Theta \frac{G M_\mathrm{c}^2}{4\pi R_\mathrm{c}^4}$$

where $\Theta$ is a numerical factor of order unity that depends on the actual distribution of density in the core. $C_V$ is the specific heat per unit mass in the core. We write the above result as

$$P_0 = C_1 \frac{M_\mathrm{c} T_0}{R_\mathrm{c}^3} - C_2 \frac{M_\mathrm{c}^2}{R_\mathrm{c}^4}$$

To obtain a solution this has to be matched to the envelope:

$$T_0 = T_\mathrm{e}, \quad P_0 = P_\mathrm{e}$$

where $T_\mathrm{e}$ and $P_\mathrm{e}$ are the temperature and pressure at the base of the envelope. Hydrogen burning keeps $T_\mathrm{e}$ at nearly a fixed value. For a given $M_\mathrm{c}$ this matching has to be obtained by adjusting $R_\mathrm{c}$ to an appropriate value.

We now find that the core surface pressure $P_0$ goes through a maximum as a function of $R_c$. The maximum is reached at

$$R_{c,\max} = C_3 \frac{M_c}{T_0}$$

and the value of the maximum pressure is

$$P_{0,\max} = C_4 \frac{T_0^4}{M_c^2}$$

Here $C_3$ and $C_4$ are positive constants.

For the envelope we may write

$$P_e \propto \frac{M^2}{R^4}, \quad T_e \propto \frac{M}{R}$$

where $M$, $R$ are the total mass and radius of the star respectively. We then have

$$P_e = C_5 \frac{T_0^4}{M^2}$$

A balance with the core is only possible if

$$\frac{P_e}{P_{0,\max}} \le 1, \quad \textit{i.e. } q_0 \equiv \frac{M_c}{M} \le q_{sc}$$

i.e. the core mass fraction $q_0$ cannot exceed $q_{sc} = \sqrt{C_4/C_5}$, which is called the Schönberg-Chandrasekhar limit. The value of the limit turns out to be

$$q_{sc} = 0.37 \left(\frac{\mu_{env}}{\mu_{core}}\right)^2$$

For an envelope containing 75% H and 25% He $\mu_{env} \approx 0.7$ and a pure Helium core has $\mu_{core} = 4/3$. Substituting these values, $q_{sc} \approx 0.1$. This suggests that as the core grows beyond about 10% of the total mass it can no longer be pressure supported at any radius and must collapse. At $q < q_{sc}$ there are two possible solutions, one stable (at larger $R_c$) and one unstable (at smaller $R_c$) (fig. 1).

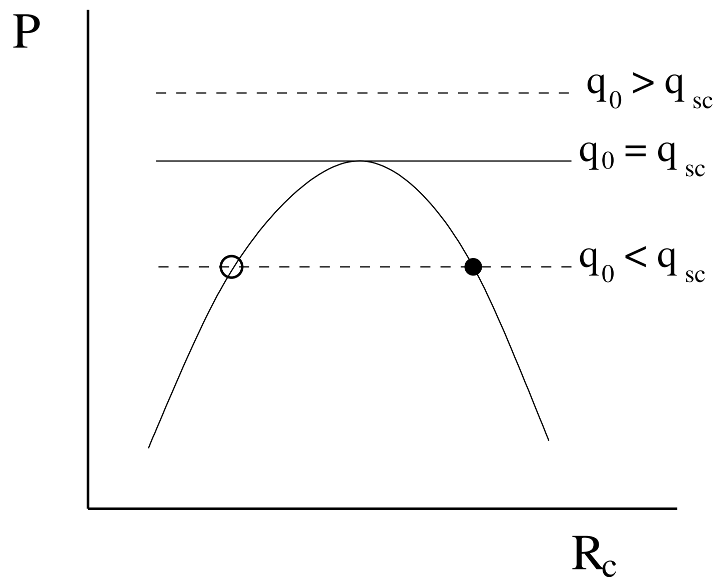


Figure 1: Surface pressure vs. core radius for an inert isothermal core of a given mass (curved line) showing the existence of a maximum. Pressure at the base of the envelope for different values of the total mass of the star (and hence different $q_0$) are indicated by the horizontal lines. At $q_0 = q_{sc}$, the Schönberg-Chandrasekhar limit, the envelope solution is tangent to the core surface pressure. At $q_0 < q_{sc}$ there are two possible solutions, the one on the left, indicated by the open circle, is unstable. As $q_0$ increases the stable solution moves to smaller core radii. For $q_0 > q_{sc}$ there is no solution possible.

The situation illustrated in fig. 1 however includes only thermal pressure. Inclusion of degeneracy pressure introduces a third branch at small values of $R_c$, on which a stable solution can be obtained (fig. 2). Cores with $q_0 > q_{sc}$ can find support only on the degenerate branch. At $q_0 \leq q_{sc}$ two stable solutions are possible, evolutionary history decides which of them is actually realised. As will be clear from the following discussion, the solution with larger $R_c$ is the one chosen in practice.

Let us now try to understand the evolution of the core with the help of fig. 2, which is a modified version of fig. 1, taking into account degeneracy pressure at the core and more realistic solutions for the envelope. Pressure at the base of the envelope is displayed for two different masses. We see that $P_e$ is not exactly independent of $R_c$ but has a weak dependence on it. The main factor determining $P_e$, however, remains the total mass $M$ as seen from the figure. The pressure at the surface of the core $P_0$ has a dependence on $R_c$ similar to that in fig. 1 but at small values of $R_c$ the additional branch due to degeneracy pressure dominates. The local peak of the curve between the ideal gas and degenarate branches moves up and to the left for smaller values of core mass $M_c$ and becomes less pronounced, disappearing altogether for very small core masses.

It is clear from fig. 2 that for a given core mass, the core is more compact in less massive stars. This results in helium cores becoming strongly degenerate before ignition in stars of mass 1 $M_\odot$ or less. If the total mass of the star is 0.45 $M_\odot$ or less then the degenerate helium core can grow to the full mass of the star before ignition, and these objects would end their lives as helium white dwarfs. The core of a several solar mass star, on the other hand, starts as a much less dense, non-degenerate configuration. As shell burning proceeds, the core mass increases, the distance between the degenerate and non-degenerate mass increases, the peak moves down and at some point goes below the $P_e$ curve. The core then makes a rapid transition from thermal pressure supported configuration to a degeneracy pressure supported one, the core radius decreasing suddenly by a large factor. The overlying shell sources become luminous, and the whole star becomes convective. This feature is generic in the evolution of all stars more massive than 1.4 $M_\odot$. In stars of mass below this the core avoids the discontinuous jump and makes a slow, gradual transition from non-

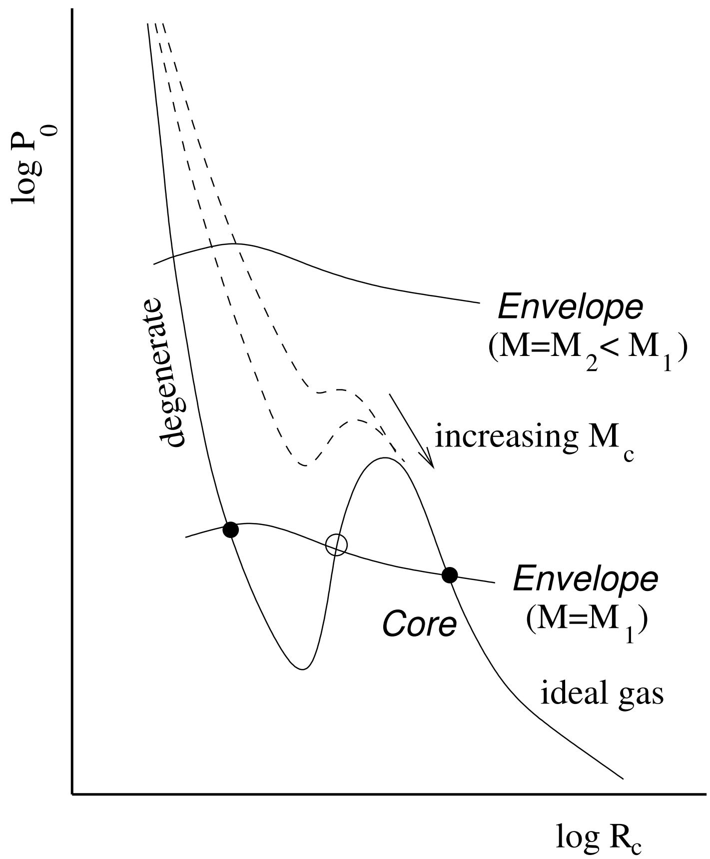


Figure 2: Surface pressure of the isothermal core and base pressure of the envelope shown as a function of the core radius $R_c$. Intersections of the core and the envelope curves represent possible solutions for the structure. Up to three solutions may exist for a given core mass $M_c$ and total mass $M$, with the middle solution (shown with an open circle) being unstable. For stars of relatively low mass only one solution is possible.

degenerate to degenerate state.

One consequence of the core being very compact in low-mass stars is that the pressure gradient

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

becomes very large in magnitude at $R_c$. As a result, the pressure falls to such small values within the layers immediately above the core that the core evolves practically independently, irrespective of the rest of the envelope. The structure of the star is then largely determined by the core mass alone. The luminosity and the radius of the star, after core hydrogen burning and before core helium ignition, are functions of $M_c$ alone, and as $M_c$ grows due to shell burning, these quantities also evolve accordingly. When $M_c$ reaches $0.45M_\odot$ helium ignites in the core, and after an initial helium flash the luminosity settles down to about $100L_\odot$. During the flash, however, some matter may be lost from the surface, and the amount of remaining mass determines the effective temperature of these helium burning stars. They occupy a horizontal strip around $L = 100L_\odot$ in the HR diagram and are called "Horizontal Branch" stars or"Giants". Because Helium burning is a relatively stable phase, a significant number of these stars are actually observed.

When shell burning is generating a large amount of luminosity around an inert core, a star normally needs to become convective to transport this energy out. As discussed before, convection ensures that in the interior $\nabla$ becomes very close to $\nabla_{ad}$, and the star can be well described by a $n = 3/2$ polytrope (for $\nabla_{ad}$ =0.4). The constant $K$ appearing the in Equation of State, while being constant through any given such configuration, however does not have the same value for all such stars. From the polytropic solutions it is easy to show that in this case the pressure in the interior would have the dependences

$$P = CR^{-3/2}M^{-1/2}T^{5/2}$$

The interior model would apply up to the base of the photosphere, located at optical depth $\tau = 2/3$ as measured from outside. The temperature at this point is $T_{eff}$ and the pressure is equal to $P_{ph}$ at the base of the overlying atmosphere. From hydrostatic equilibrium one obtains

$$P_{\rm ph} = \frac{2}{3}\frac{GM}{R^2}\frac{1}{\kappa_{\rm ph}}$$

wherw $M$ is the mass of the star and R is the photospheric radius. If the photospheric opacity $\kappa_{\rm ph}$ is given by

$$\kappa_{\rm ph} = \kappa_0 P^a T^b$$

then

$$P_{\rm ph} = \text{const.}\left(\frac{M}{R^2}T_{\rm eff}^{-b}\right)^{1/(a+1)}$$

The balance between the atmospheric and interior pressure can be carried out in the $\log T$–$\log P$ plane as shown in fig. 3. The solutions depend on the radius $R$ and mass $M$. Writing

$$L = 4\pi R^2 \sigma T^4$$

these solutions yield

$$\log T_{\rm eff} = A \log L + B \log M + \text{const.}$$

where

$$A = \frac{0.75a - 0.25}{b + 5.5a + 1.5} \quad \text{and} \quad B = \frac{0.5a + 1.5}{b + 5.5a + 1.5}$$

For cool stars represented by the fully convective sequences the atmospheric opacity is dominated by $H^-$ ions, for which $a \approx 1$ and $b \approx 3$. This gives

$$A \approx 0.05 \quad \text{and} \quad B \approx 0.2$$

The fully convective solutions therefore represent near-vertical lines in the HR diagram, the lines shifting slightly to the right with decreasing mass of the star. These lines are known as the “Hayashi lines” or “Hayashi tracks”, and separate the accessible part of the HR diagram from inaccessible one. No stellar model in equilibrium can occupy a position to the right of the corresponding Hayashi line.

The fully convective shell burning stars stay on the Hayashi line, changes in luminosity causing them to move up and down the line. When convection

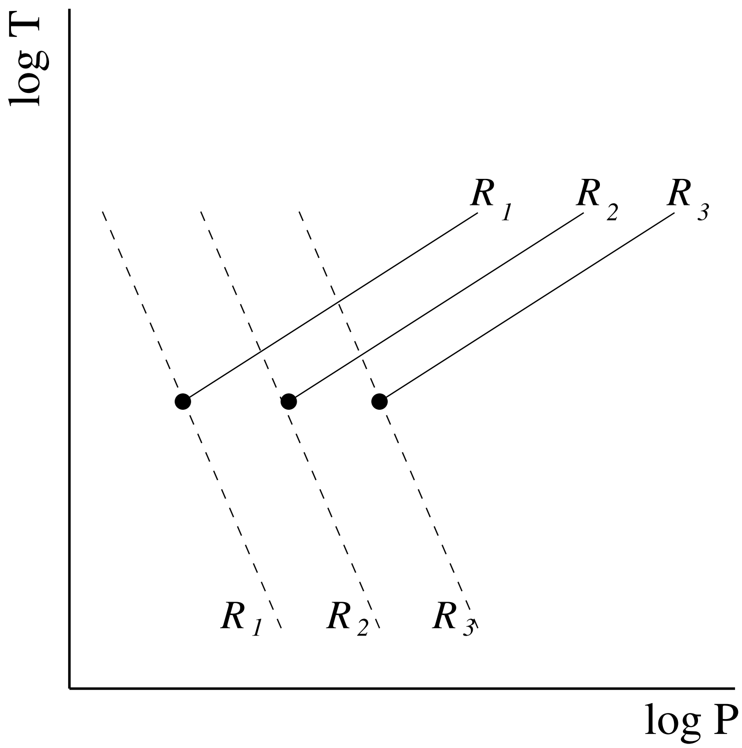


Figure 3: Matching of the interior and atmospheric solutions in fully convective stars of a given mass $M$. Dashed lines represent atmospheres and solid lines the interior solutions for different assumed values of $R$, with $R_1 > R_2 > R_3$ in the illustration above

subsides, for example during a core burning phase, the model moves to the left of the Hayashi line.

Stars with mass less than about $1.4M_{\odot}$, after finishing hydrogen burning, slowly make their way across the HR diagram, gradually as the core contracts, up to the Hayashi line. The star in this phase of evolution is called a subgiant. Upon reaching the Hayashi line the star climbs upwards along the line as the shell sources become more and more luminous. In this phase the star is a "red giant". Once helium ignites in the core the star climbs down the Hayashi line to about $100L_{\odot}$, but may move to the left depending on the amount of envelope material left, and occupy its position on the horizontal branch ("Giants").

More massive stars make a very fast transition from a point slightly to the right of the main sequence to the Hayashi line, as the core goes through non-equilibrium transition from non-degenerate to degenerate state. Because of this quick transition such stars are not usually seen in this phase, giving rise to the so-called "Hertzsprung gap" in the HR diagram. Once the core settles on the degenerate branch the star is already on the Hayashi line. After a while helium ignites in the core and the star moves back to the left from the Hayashi line. At the end of core Helium burning the star returns to the Hayashi line. Each subsequent burning stage causes the star to execute such a "blue loop".

When the final white dwarf configuration is reached in the stellar core the envelope has normally become very distended, in this phase the star is called an "asymptotic giant". The matter in the asymptotic giant envelope is lost in strong winds, leaving the cooling white dwarf at the centre of a slowly expanding shell of envelope matter, which shines as a "planetary nebula", excited by the ultraviolet radiation from the hot white dwarf at the centre.

Massive stars ($> 10M_{\odot}$) form Fe cores, which collapse upon exceeding the Chandrasekhar limit. As collapse proceeds, the Fe nuclei are first photodissociated, and then electrons are captured by protons to produce neutron-rich matter. The loss of electrons means the loss of degeneracy pressure, the main support against gravity at this stage. As a result the

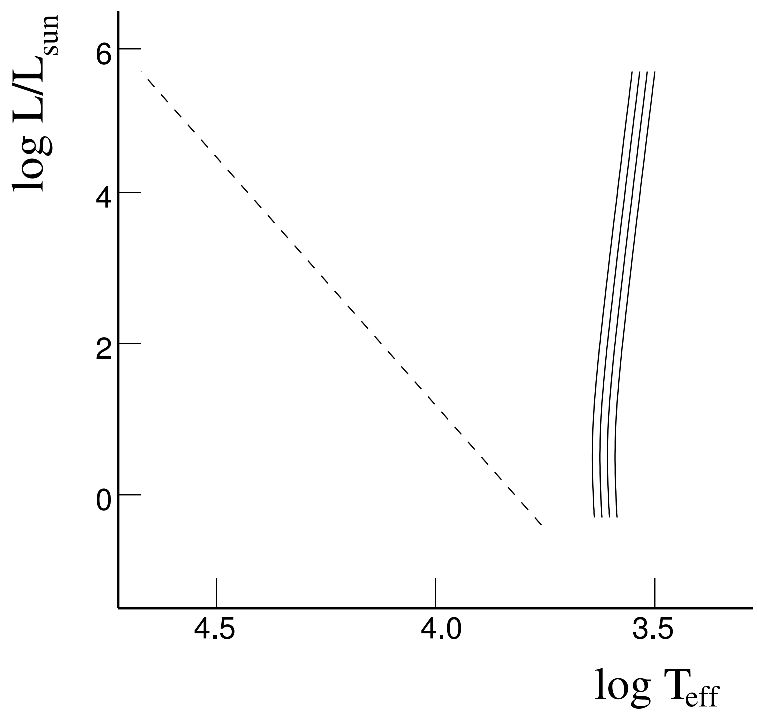


Figure 4: Hayashi lines in the Hertzsprung-Russell diagram, shown for a range of masses. The dashed line shows the approximate location of the main sequence.

collapse accelerates and in hydrodynamic time scale of a few seconds produces a very compact configuration, made primarily of neutrons. As the neutrons are squeezed together at densities higher than nuclear density ($\rho_{\rm nuc} = 2.8 \times 10^{14}$ g/cm$^3$) the mutual repulsion between neutrons will halt the collapse, and the core will bounce back to an equilibrium configuration, which is now a neutron star. The bounce will send a shock wave through the surrounding envelope, making the envelope explode in a type II supernova. The gravitational binding energy released in the collapse of the core is $\sim 10^{53}$ erg, about 1% of which goes into the kinetic energy of the expanding envelope. Neutrinos carry the rest of the energy away, of which we now have observational confirmation through the detection of neutrinos from the supernova SN1987A which occurred in our neighbouring galaxy, the Large Magellanic Cloud (LMC). Twelve neutrino events were detected in the Kamiokande set-up in Japan, and the energy implied by these neutrinos is indeed $10^{53}$ erg for the supernova. The expanding ejecta, with the kinetic energy of $10^{51}$ erg, is heated by the shock, as well as the decay of radioactive elements synthesised and ejected. It therefore shines brightly. The total energy emitted in radiation amounts to $\sim 10^{49}$ erg. Very massive stars would be able to grow cores too massive to be supported as neutron stars, the reason for this being the additional radiation pressure support in the pre-collapse core. Such cores will collapse to black holes. A spinning black hole produced this way will swallow the inner parts of the envelope through a dense accretion disk, and eject a small fraction of matter in a jet along the spin axis. With large amount of energy imparted to this small amount of matter, the material in the jet would move at relativistic speeds. Viewed along the jet, this will be a copious source of high energy radiation. This model is believed to explain the Gamma-Ray Burst sources. The rest of the envelope in this case will get eventually expelled in a supernova-like explosion (often referred to as a "hypernova").