BLACK HOLES IN

LOOP QUANTUM GRAVITY

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BLACK HOLES AND QUANTUM GRAVITY ?

Black Holes are, as Chandrasekhar used to say:
"... the most perfect objects there are in The Universe: the only elements in their construction are our concepts of space and time. Since GR predicts a single family of solutions, they are the simplest as well." **They are the crown of classical physics in terms of their simplicity and beauty.**

But, Bekenstein and Hawking told us that :

i) Black Holes satisfy some 'thermodynamic-like laws'.

$$\delta M = \frac{\kappa}{8\pi G}\,\delta A \;\;\Rightarrow\;\; M \leftrightarrow E, \quad \kappa \leftrightarrow T\,, \quad A \leftrightarrow S$$

ii) When one invokes quantum mechanics ($\hbar$) then something weird happens:

$$E = M$$

$$T = \frac{\kappa\, \hbar}{2\pi} \ ,$$

and

$$S = \frac{A}{4\, G\hbar}$$

The black holes appear to have thermodynamic properties!

But, what are the underlying degrees of freedom responsible for entropy?

The standard wisdom is that only with a full marriage of the Quantum and Gravity will we be able to understand this.

Different approaches:

- **String Theory**
- **Causal Sets**
- **Entanglement Entropy**
- **Loop Quantum Gravity**

MOTIVATION

- **How do we characterize black holes in equilibrium?**
- **What are quantum horizon states?**
- **Which states should we count?**
- **How does the entropy behave?**
- **Large BH: Bekenstein-Hawking entropy**
- **What happens when we look at small BH's?**

PLAN OF THE TALK

1. **Some History**

2. **Classical Preliminaries**

3. **Quantum Preliminaries**

4. **Quantum Horizon Geometry**

5. **Counting and Entropy**

6. **New Results: Counting by Numbers**

Work of many people, including A. Ashtekar, J. Baez, AC, M. Domagala, J. Lewandowski, K. Meissner, J. Engle, E. Fernandez-Borja, J. Diaz-Polo, K. Krasnov, R. Kaul, A. Ghosh, P. Majumdar, P. Mitra, C. Rovelli, H. Sahlmann and more ...

1. SOME HISTORY

- **94' The area operator is defined (Smolin & Rovelli)**
- **96' Krasnov and Rovelli consider punctures as horizon degrees of freedom.**
- **97' Isolated Horizon boundary conditions understood.**
- **99' Quantum Horizon Geometry fully understood (ABK).**
- **00' Logarithmic corrections computed**
- **02' Possible relation to QNM proposed (SO(3) *vs* SU(2))**
- **04' Error in original ABK computation found. A new counting proposed (DLM)**
- **05'- Several new countings proposed (GM, Dreyer et al, ...)**
- **06' Direct counting of small BH states. New structures found.**

The Beginning

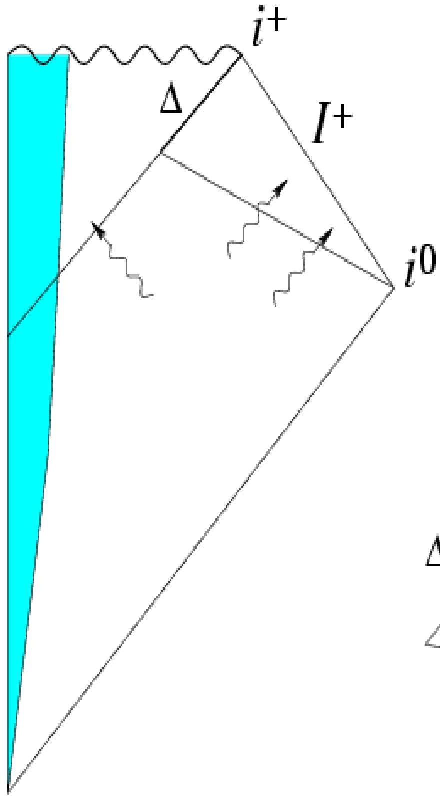
Physically, one is interested in describing black holes in equilibrium. That is, equilibrium of the horizon, not the exterior. Can one capture that notion via boundary conditions?

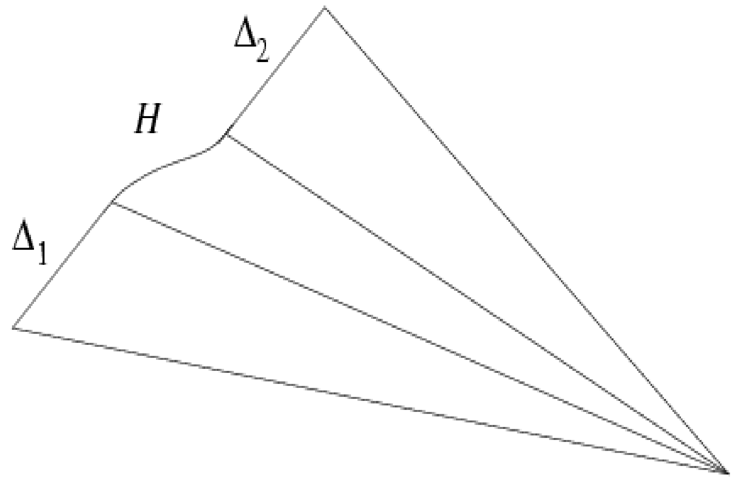
Yes! Answer: Isolated Horizons

Isolated horizon boundary conditions are imposed on an inner boundary of the region under consideration.

The interior of the horizon is cut out. In this a physical boundary?
No! but one can ask whether one can make sense of it:

What is then the physical interpretation of the boundary?





• The boundary Δ, the 3-D isolated horizon, provides an effective description of the degrees of freedom of the inside region, that is cut out in the formalism.

- **The boundary conditions are such that they capture the intuitive description of a horizon in equilibrium and allow for a consistent variational principle.**

- **The quantum geometry of the horizon has independent degrees of freedom that flctuate 'in tandem' with the bulk quantum geometry.**

- **The quantum boundary degrees of freedom are then responsible for the entropy.**

- **The entropy thus found can be interpreted as the entropy assigned by an 'outside observer' to the (2-dim) horizon** $S = \Sigma \cap \Delta$**.**

- **Interpretational issues: is this to be regarded as the entropy contained by the horizon? Is there some 'holographic principle' in action? Can the result be associated to entanglement entropy between the interior and the exterior?, etc.**

ISOLATED HORIZONS

An isolated horizon is a null, non-expanding horizon Δ with some notion of translational symmetry along its generators. There are two main consequences of the boundary conditions:

• The gravitational degrees of freedom induced on the horizon are captured in a $U(1)$ connection,
$W_a = -\frac{1}{2}\,\Gamma^i_a\, r_i$

• The total symplectic structure of the theory (and this is true even when matter is present) gets split as, $\Omega_{\rm tot} = \Omega_{\rm bulk} + \Omega_{\rm hor}$ with

$$\Omega_{\rm hor} = \frac{a_0}{8\pi\, G} \oint_S {\rm d}W \wedge {\rm d}W'$$

• The 'connection part' and the 'triad part' at the horizon must satisfy the condition, $F_{ab} = -\frac{2\pi\,\gamma}{a_0}\, E^i_{ab}\, r_i$, the 'horizon constraint'.

CONSTRAINTS

The formalism tells us what is gauge and what not. In particular, with regard to the constraints we know that:

- **The relation between curvature and triad, the horizon constraint, is equivalent to Gauss' law.**

- **Diffeomorphims that leave S invariant are gauge (vector field are tangent to S).**

- **The scalar constraint must have $N|_{\rm hor} = 0$. Thus, the scalar constraint leaves the horizon untouched; any gauge and diff-invariant observable is a Dirac observable.**

In the quantum theory of the horizon we have to implement these facts.

QUANTUM THEORY: THE BULK

A canonical description:

$$A^i_a \quad SU(2)\,\text{connection} \quad ; \quad E^a_i \quad \text{triad}$$

with $A^i_a = \Gamma^i_a - \gamma\, K^i_a$**. Loop Quantum gravity on a manifold without boundary is based on two fundamental observables of the fundamental variables :**

Holonomies, $h_e(A) := \mathcal{P}\exp(\int_e A)$

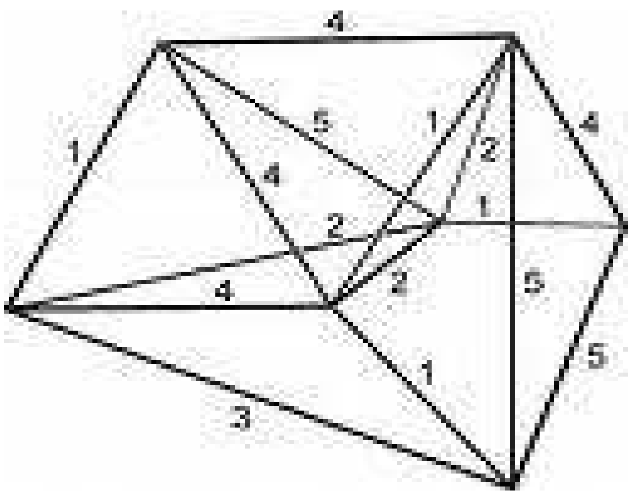
and

Electric Fluxes, $E(f,S) := \int_S \mathrm{d}S^{ab} E^i_{ab}\, f^i$

The main assumption of Loop Quantum Gravity is that these quantities become well defined operators. LOST Theorem: There is a unique represntation on a Hilbert space of these observabes that is *diffeomorphism invariant***.**

Hilbert space:

$$\mathcal{H}_{\mathrm{AL}} = \oplus_{\mathrm{graphs}} \mathcal{H}_{\Upsilon} = \text{Span of all Spin Networks } |\Upsilon, \vec{j}, \vec{m}\rangle \qquad (1)$$

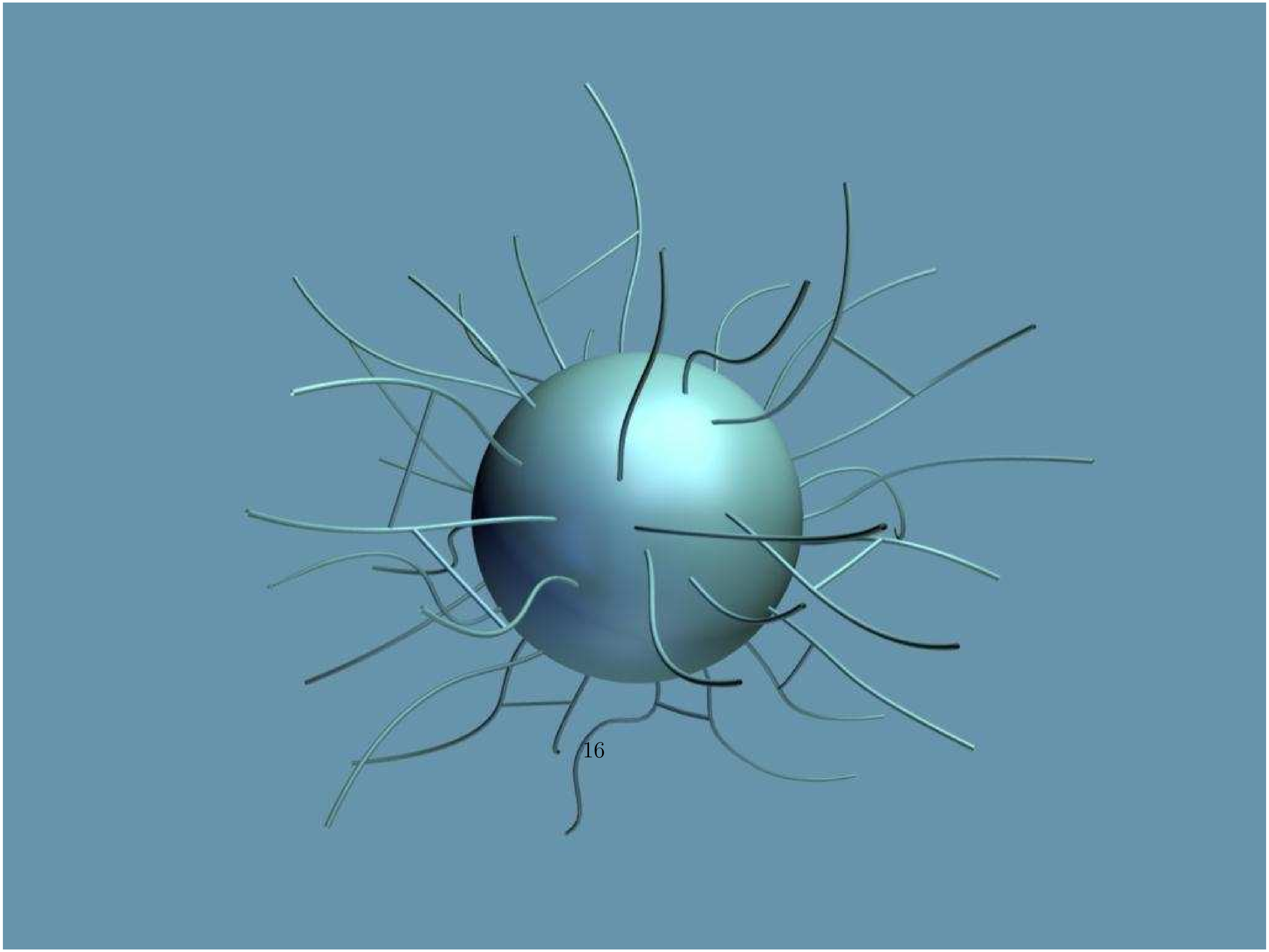


A Spin Network $|\Upsilon, \vec{j}, \vec{m}\rangle$ is a state labelled by a graph Υ, and some colourings $(\vec{j}, \vec{m})$ associated to edges and vertices.

The spin networks have a very nice interpretacion. They are the eigenstates of the quantizad geometry, such as the area operator,

$$\hat{A}[S] \cdot |\Upsilon, \vec{j}, \vec{m}\rangle = 8\pi\ell_{\rm Pl}^2 \gamma \sum_{\rm edges} \sqrt{j_i(j_i+2)}\, |\Upsilon, \vec{j}, \vec{m}\rangle \tag{2}$$

One sees that the edges of the graph, excite the quantum geometry of the surface S at the intersection points between S and Υ.



HORIZON QUANTUM THEORY

Total Hilbert Space is of the form:

$$\mathcal{H} = \mathcal{H}_{\mathrm{V}} \otimes \mathcal{H}_{\mathrm{S}}$$

where $\mathcal{H}_{\mathrm{S}}$, the surface Hilbert Space, can be built from Chern Simons Hilbert spaces for a sphere with punctures.

The conditions on $\mathcal{H}$ that we need to impose are: Invariance under diffeomorphisms of S and the quantum condition on Ψ:

$$\left(\mathrm{Id} \otimes \hat{F}_{ab} + \frac{2\pi\,\gamma}{a_0}\,\hat{E}^i_{ab}\,r_i \otimes \mathrm{Id}\right) \cdot \Psi = 0$$

Then, the theory we are considering is a quantum gravity theory, with an isolated horizon of fixed area a_0 (and multiple-moments). Physical state would be such that, in the bulk satisfy the ordinary constraints and, at the horizon, the quantum horizon condition.

ENTROPY

We are given a black hole of area a_0. What entropy can we assign to it? Let us take the microcanonical viewpoint. We shall count the number of states $\mathcal{N}$ such that they satisfy:

- **The area eigenvalue $\langle \hat{A} \rangle \in [a_0 - \delta, a_0 + \delta]$**
- **The quantum horizon condition.**

The entropy $\mathcal{S}$ will be then given by

$$\mathcal{S} = \ln \mathcal{N}.$$

The challenge now is to identify those states that satisfy the two conditions, and count them.

CHARACTERIZATION OF THE STATES

There is a convenient way of characterizing the states by means of the spin network basis. If an edge of a spin network with label j_i ends at the horizon S, it creates a puncture, with label j_i. The area of the horizon will be the area that the operator on the bulk assigns to it: $A = 8\pi\gamma\ell_{\rm Pl}^2 \sum_i \sqrt{j_i(j_i+1)}$.

Is there any other quantum number associated to the punctures p_i? Yes! the eigenstates of $\hat{E}_{ab}$ that are also half integers m_i, such that $-|j_i| \leq m_i \leq |j_i|$. The quantum horizon condition relates these eigenstates to those of the Chern-Simons theory. The requirement that the horizon is a sphere (topological) then imposes a 'total projection condition' on $m's$:

$$\sum_i m_i = 0$$

A state of the quantum horizon is then characterized by a set of punctures p_i and to each one a pair of half integer (j_i, m_i).

If we are given N punctures and two assignments of labels (j_i, m_i) and (j'_i, m'_i). Are they physically distinguishable? or a there some 'permutations' of the labels that give indistinguishable states?

That is, what is the statistics of the punctures?

As usual, we should let the theory tell us. One does not postulate any statistics. If one treats in a careful way the action of the diffeomorphisms on the punctures one learns that when one has a pair of punctures with the same labels j and m, then the punctures are indistinguishable and one should not count them twice. In all other cases the states are distinguishable.

THE COUNTING

We start with an isolated horizon, with an area a_0 and ask how many states are there compatible with the two conditions, and taking into account the distinguishability of the states.
First Approach: Count just the different configurations and forget about $\sum_i m_i = 0$. Thus, given $(n_{1/2}, n_1, n_{3/2}, \ldots, n_{k/2})$,we count the number of states:

$$N = \frac{N!}{\Pi_j \, (n_j!)} \Pi_j \, (2j+1)^{n_j} \tag{3}$$

Taking the *large area approximation* $A >> \ell_{\rm Pl}$**, and using the Stirling approximation. One gets:**

$$S = \frac{A}{4\ell_{\rm Pl}^2} \, \frac{\gamma_0}{\gamma} \tag{4}$$

with γ_0 the solution to $\sum_j (2j+1) e^{2\pi\,\gamma_0 \sqrt{j_i(j_i+1)}} = 1$.
(and $\sum_j 2\, e^{2\pi\,\gamma_M \sqrt{j_i(j_i+1)}} = 1$ for DLM).

The introduction of the projection constraint introduces a first correction to the entropy area relation as

$$S = \frac{A}{4\ell_{\rm Pl}^2}\,\frac{\gamma_0}{\gamma} - \frac{1}{2}\ln(A) + \ldots$$

- **If we want to make contact with the Bekenstein-Hawking we have to chose $\gamma = \gamma_0$.**
- **The coefficient of the logarithmic correction seems to be universal.**
- **The formalism can be generalized to more general situations, and the result is *the same*:**
  - **Maxwell, Dilatonic and Yang Mills Couplings**
  - **Cosmological, Distortion and Rotation**
  - **Non-minimal Couplings**

COUNTING BY NUMBERS

We tell a computer how to count for a range of area a_0 at the Planck scale.

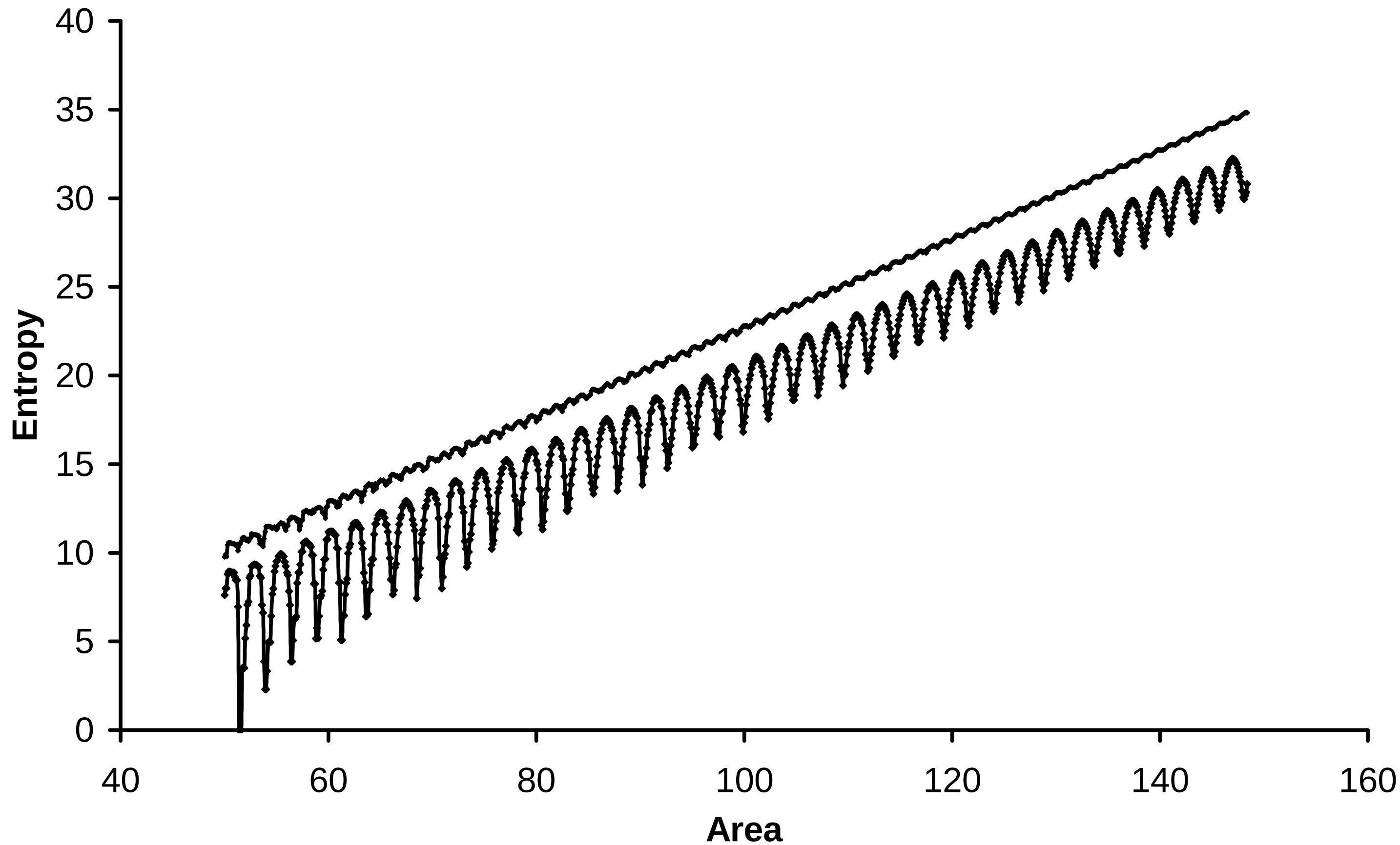
• **How does the incorporation or not of the projection constraint $\sum_i m_i = 0$ affect the number of states?**

• **Can we see for such small black holes that the entropy tends to be a linear function of the area?**

• **Can we say anything about the Barbero-Immirzi parameter?**

• **Is the entropy area relation for such small black holes like anything we had imagined?**

• **Are there new structures found?**


Entropy
Area
0
5
10
15
20
25
30
35
40
40
60
80
100
120
140
160

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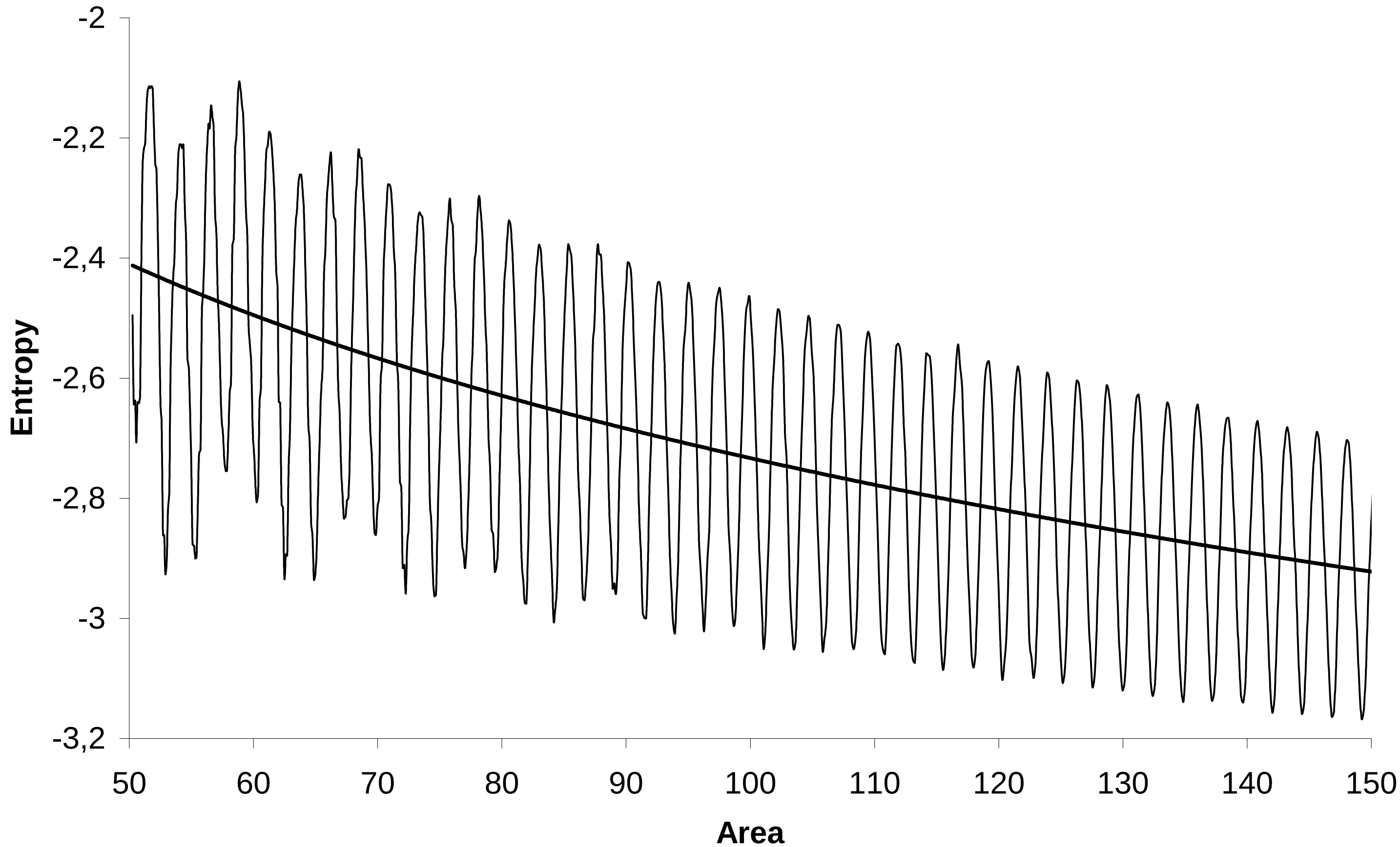
In the analytical computations, the introduction of the projection constraint introduces a first correction to the entropy area relation as

$$S = \alpha A - \frac{1}{2} \ln(A) + \ldots$$

We then subtracted the two plots and found:


Entropy
-2
-2,2
-2,4
-2,6
-2,8
-3
-3,2
50
60
70
80
90
100
110
120
130
140
150
Area

What we see in that, on average, the entropy tends to the analytical relations.

What is the nature of the oscillations? What we see is that the frequency of the oscillations is independent of the size δ of the interval used in the counting.

What about the Immirzi parameter?

We found the Barbero-Immirzi parameter from our counting (by interpolating the curve) is very close to the analytical value:
γ_0 and γ_M (depending on the counting).

For Planck scale horizons, Barbero-Immirzi parameter and the logarithmic correction are recovered.

But what about the oscillations ?

ENTROPY QUANTIZATION

Both the oscillations found with a large value of δ as well as these structures in the 'spectrum' posses the same periodicity

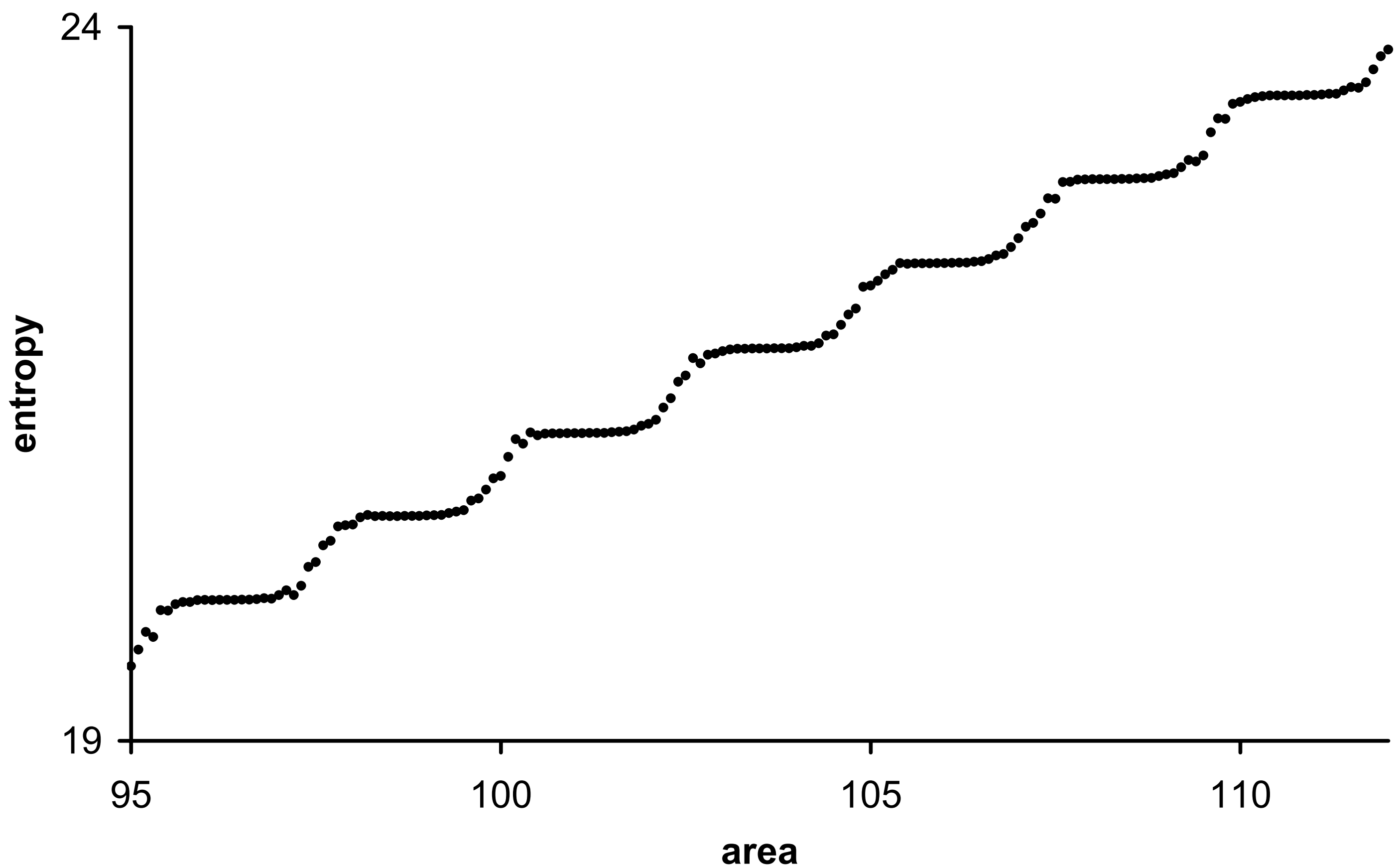
$$\delta A_0 \approx 2.41\ \ell_p^2$$

Is there any physical significance to this periodicity?

we chose the interval:

$$2\,\delta = \Delta A_0$$

With this choice, the plot of the entropy *vs* area becomes:


24
19
entropy
95
100
105
110
area

WHAT DOES THIS MEAN?

Instead of oscillations, Entropy seems to increase in discrete steps.

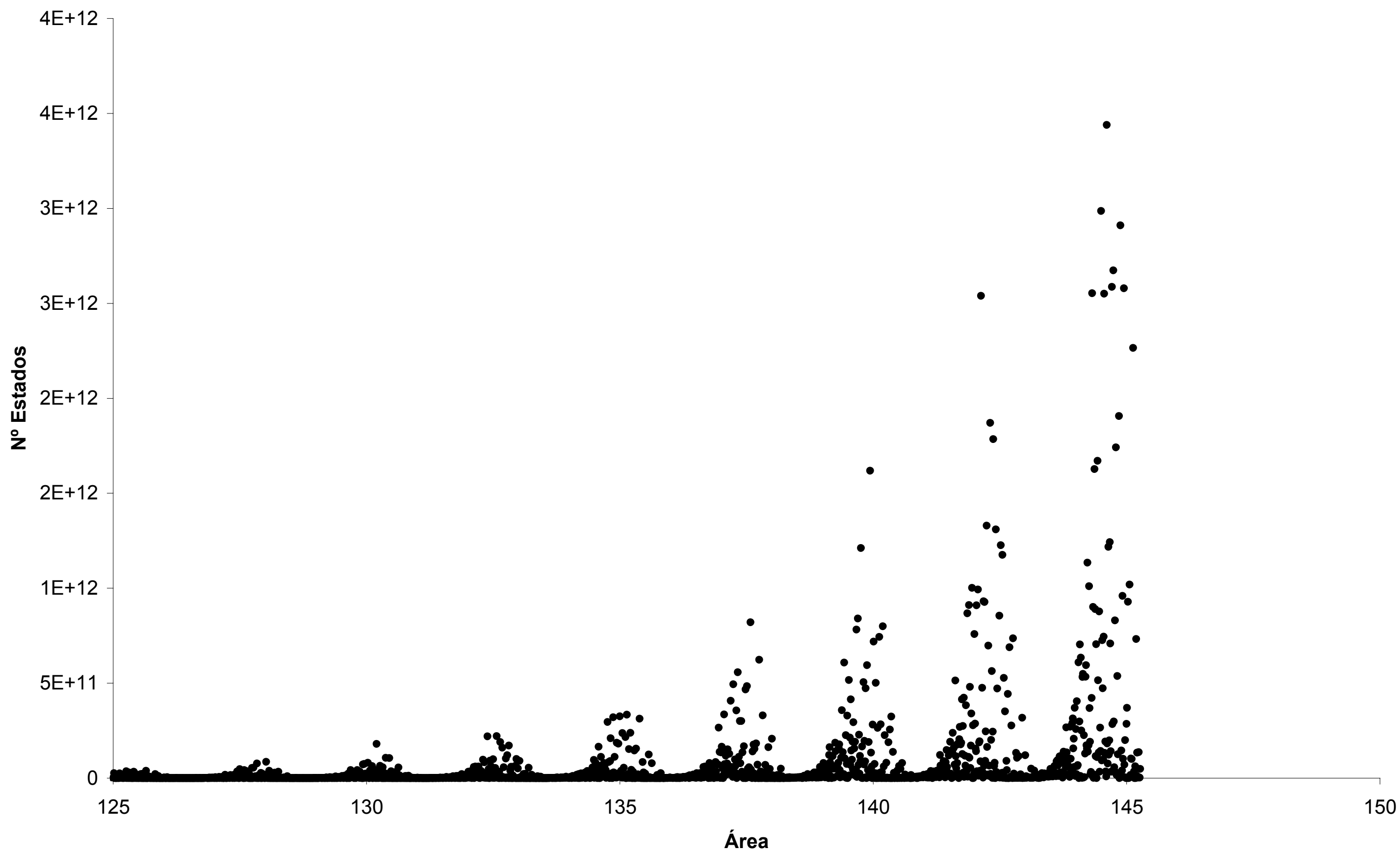
Furthermore, the height of the steps seems to approach a constant value as the area of the horizon grows, thus implementing in a rather subtle way the conjecture by Bekenstein that entropy should be equidistant for large black holes.

This result is robust: Independent of the counting!

Is there any way of understanding this? Maybe

While the constant number in which the entropy of large black holes 'jumps' is:

$$\Delta S \mapsto 2\,\gamma_0\,\ln{(3)}$$


4E+12
4E+12
3E+12
3E+12
2E+12
2E+12
1E+12
5E+11
0
Nº Estados
125
130
135
140
145
150
Área

Some recent analytic understanding (Sahlmann, Fernandez-Borja, Diaz-Polo)

- **One can think of the states organized in** *bands*.
- **One uses the analytic n_j distribution that maximazes degeneracy.**
- **One can find the 'average area', for each band associated with this maximum degeneracy configuration:**

$$\Delta A = \frac{8\pi\gamma \sum_s \sqrt{s(s+2)}(s+1)e^{-2\pi\gamma_0\sqrt{s(s+2)}}}{3(\sum_s s(s+1)e^{-2\pi\gamma_0\sqrt{s(s+2)}}) + 2}$$

with

$$\frac{\Delta A - \gamma\, 8\ln(3)}{\Delta A} \approx 0.003\%$$

Very exciting possibility!

STAY TUNED!

CONCLUSIONS

- **Isolated Horizons provide a consistent framework to incorporate black holes.**
- **One can consistently quantize the theory**
- **Entropy is** *finite* **and dominant term linear in Area.**
- **Any black hole of interest is included**
- **Unexpected features appear by considering Planck size horizons.**
- **Contact with Bekenstein's heuristic model, and Mukhanov-Bekenstein in a subtle manner**
- **Is there more?**

OUTLOOK

- **We have not dealt with the singularity**
- **Ashtekar-Bojowald 'paradigm' for and extended quantum space-time**
- **Based on expectations about singularity resolution coming from LQC**
- **Hawking radiation?**
- **Lost Information Puzzle**
- **Full Theory: How to specify quantum black holes from the full theory?**