Gauge Theory duals of Null and Space-like Singularities

Sumit R. Das

S.R.D, J. Michelson, K. Narayan and S. Trivedi, PRD 74 (2006) 026002

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A. Awad, S.R.D, K. Narayan and S. Trivedi, hep-th/0711.2994

Space-like and Null Singularities

- Space-like or Null singularities are difficult to understand – these are singularities which you cannot "see" and therefore cannot avoid.
- They usually signify a beginning or end of time
- This is hard to think about in the usual context of quantum mechanical time evolution
- In this talk – will summarize one approach to gain insight using dual descriptions of the AdS/CFT type

Usual AdS/CFT

- IIB string theory in asymptotically $AdS_5 \times S^5$ space-times is dual to large-N expansion of $\mathcal{N}$ =4 SYM theory on the boundary with appropriate sources or excitations.
- The usual relationship between the dimensionless parameters on the two sides are

$$g_s = g^2_{YM} \qquad\qquad (R/l_s)^4 = 4\pi \; g^2_{YM} \; N$$

- Where g_s is the string coupling, g^2_{YM} is the square of the Yang-Mills coupling, l_s is the string length and R is the AdS length scale

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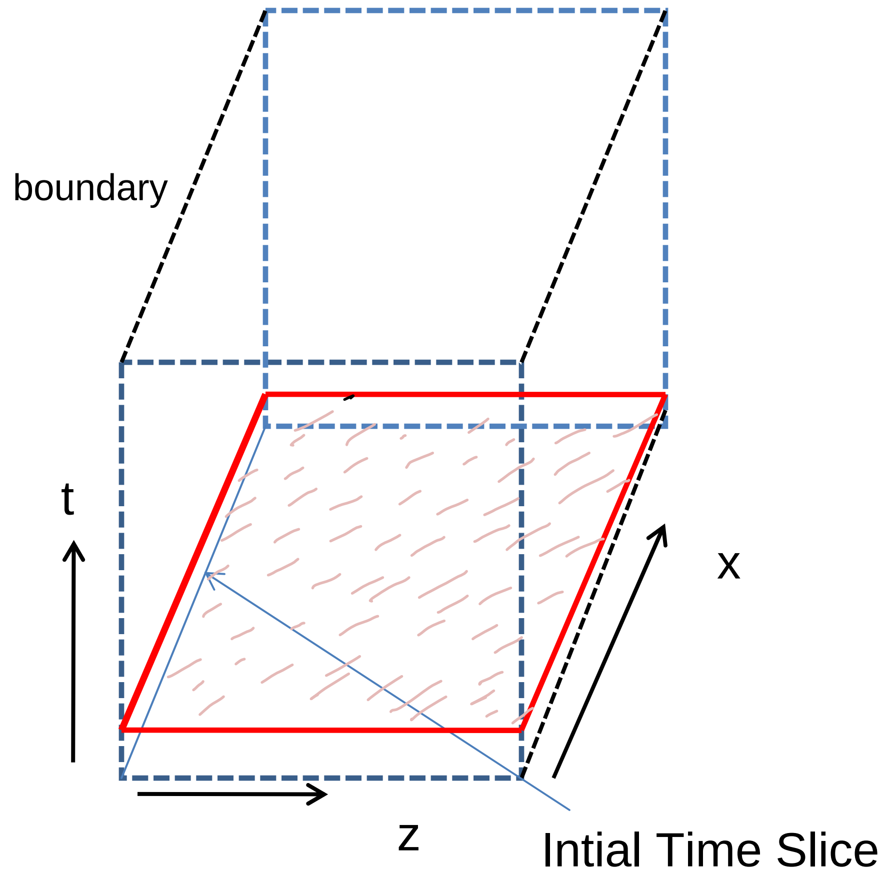
- For generic values of the parameters , the gauge theory hopefully continues to make sense, though there is no interpretation in terms of General Relativity.
- Could this happen near singularities ?

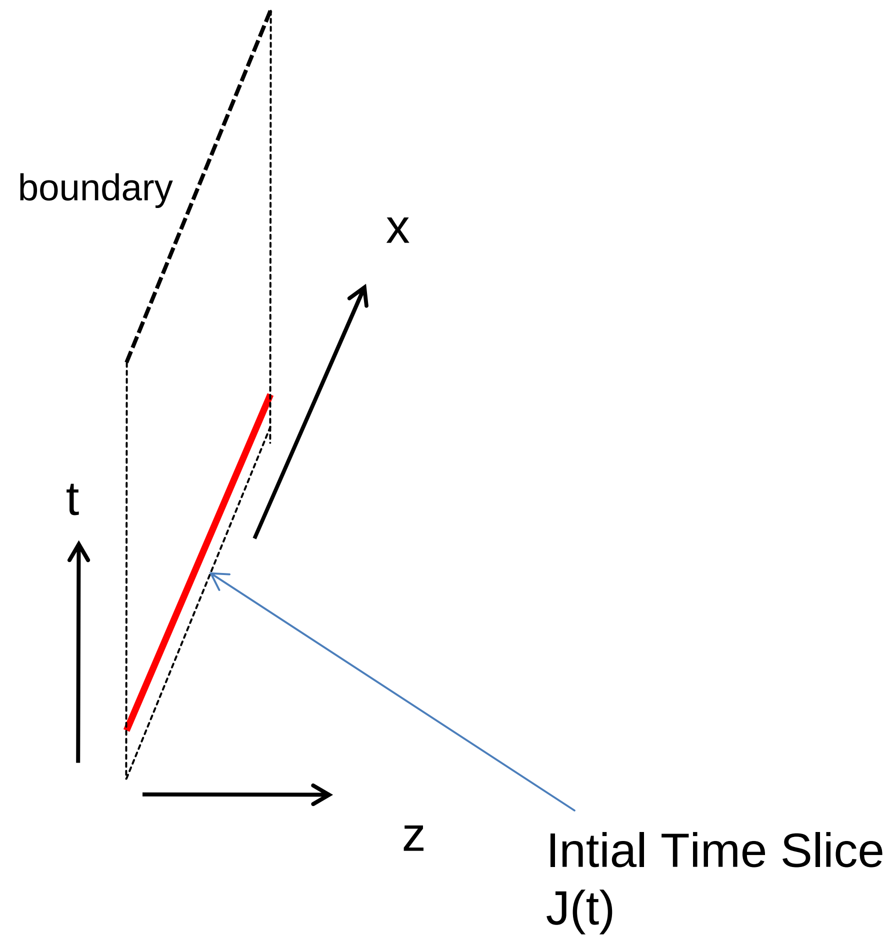
A Scenario

- At early times, start with the ground state of the gauge theory with large ‘t Hooft coupling.
- The physics is now well described by supergravity in usu $AdS_5 \times S^5$



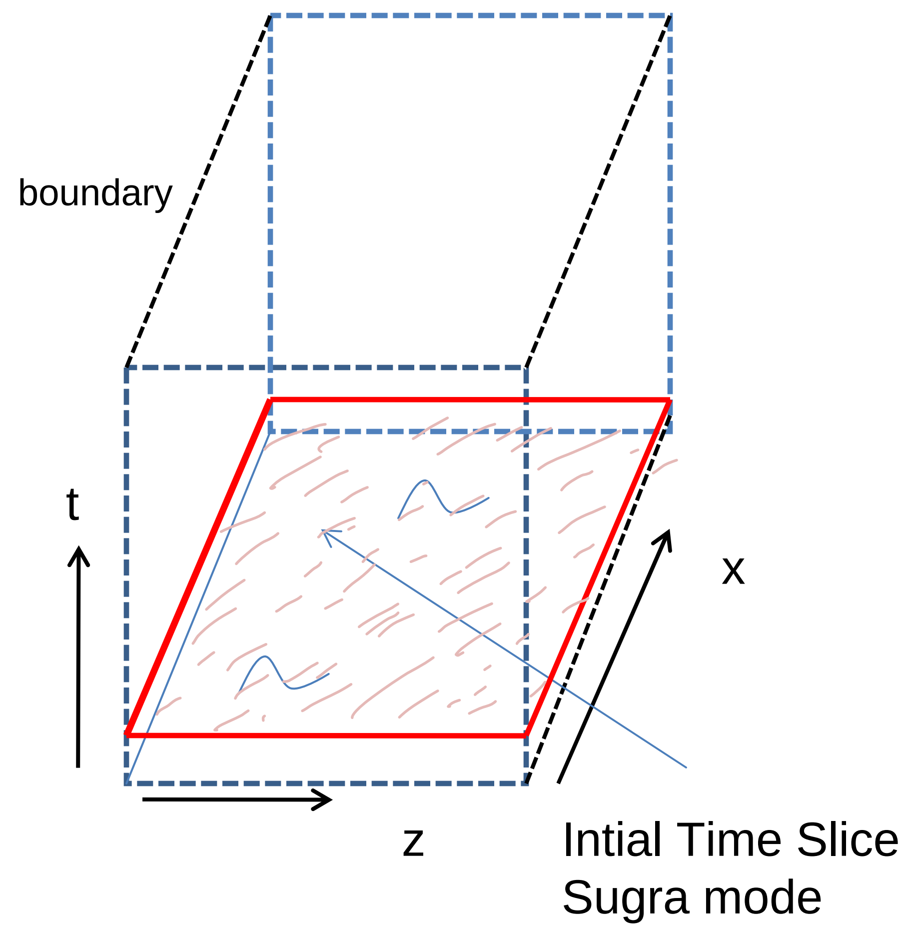
A Scenario

- Now turn on a **time dependent source** in the Yang-Mills theory which deforms the lagrangian.



A Scenario

- Now turn on a time-dependent source in the Yang-Mills theory which deforms the lagrangian.
- This corresponds to turning on a non-normalizable mode of the supergravity in the $AdS_5 \times S^5$ deforming the original



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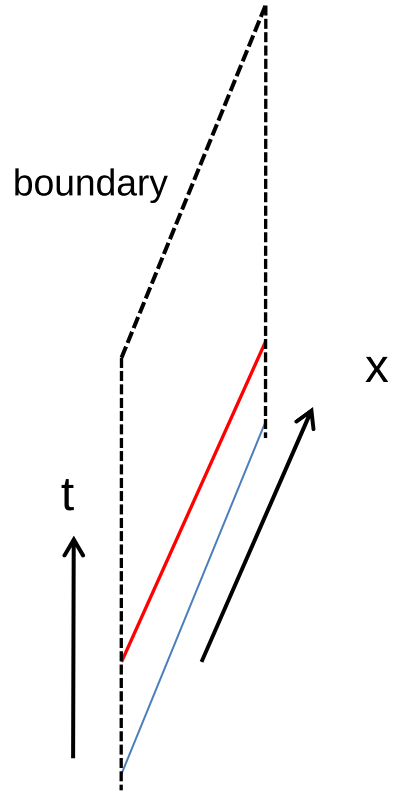
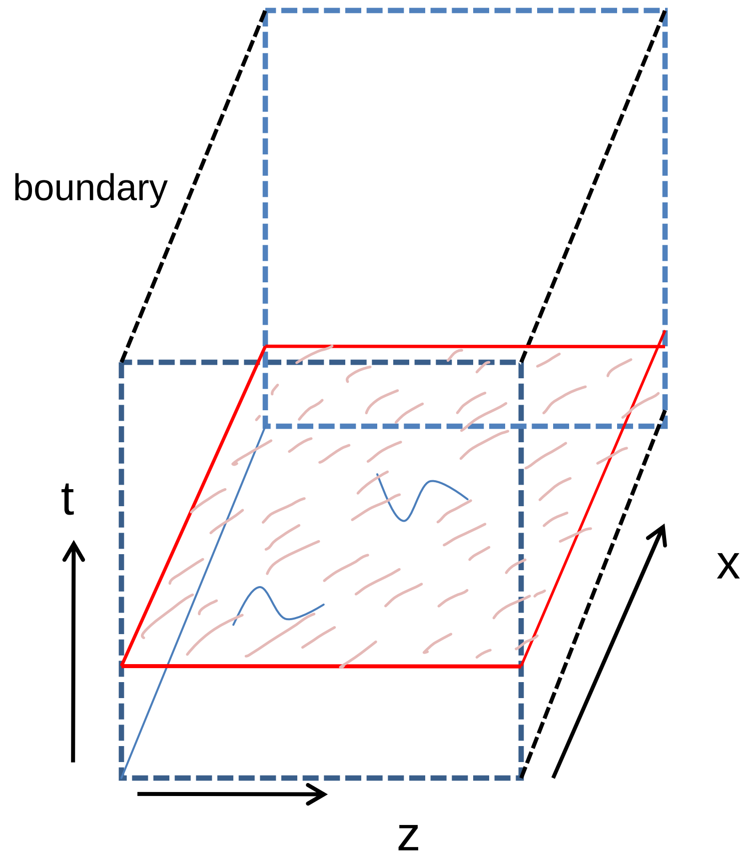
- The gauge theory evolves according to the deformed hamiltonian



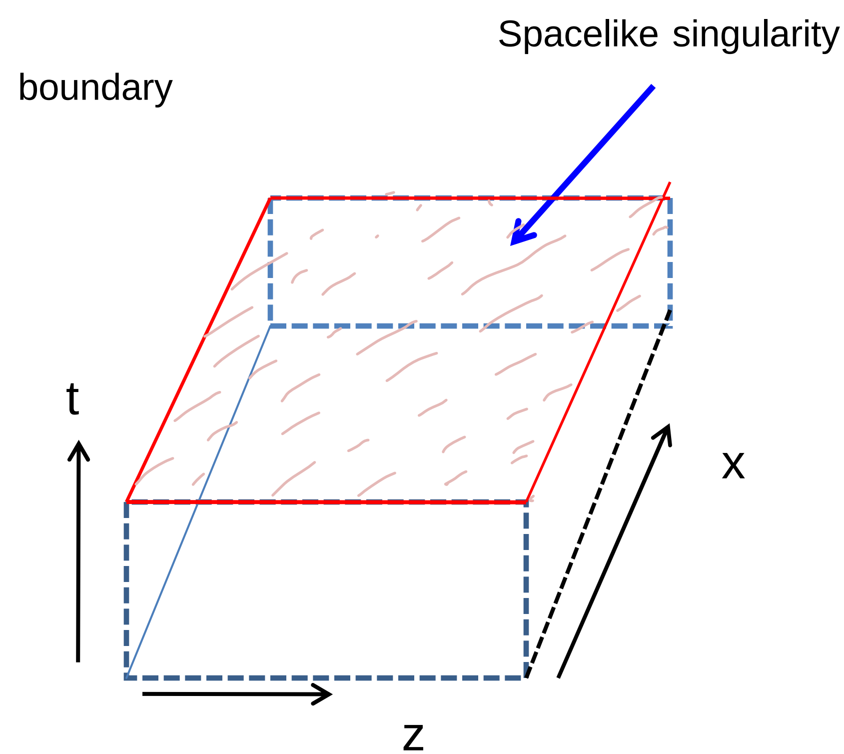
A Scenario

- The gauge theory evolves according to the deformed hamiltonian
- At sufficiently early times the supergravity background evolves according to the classical equations of motion



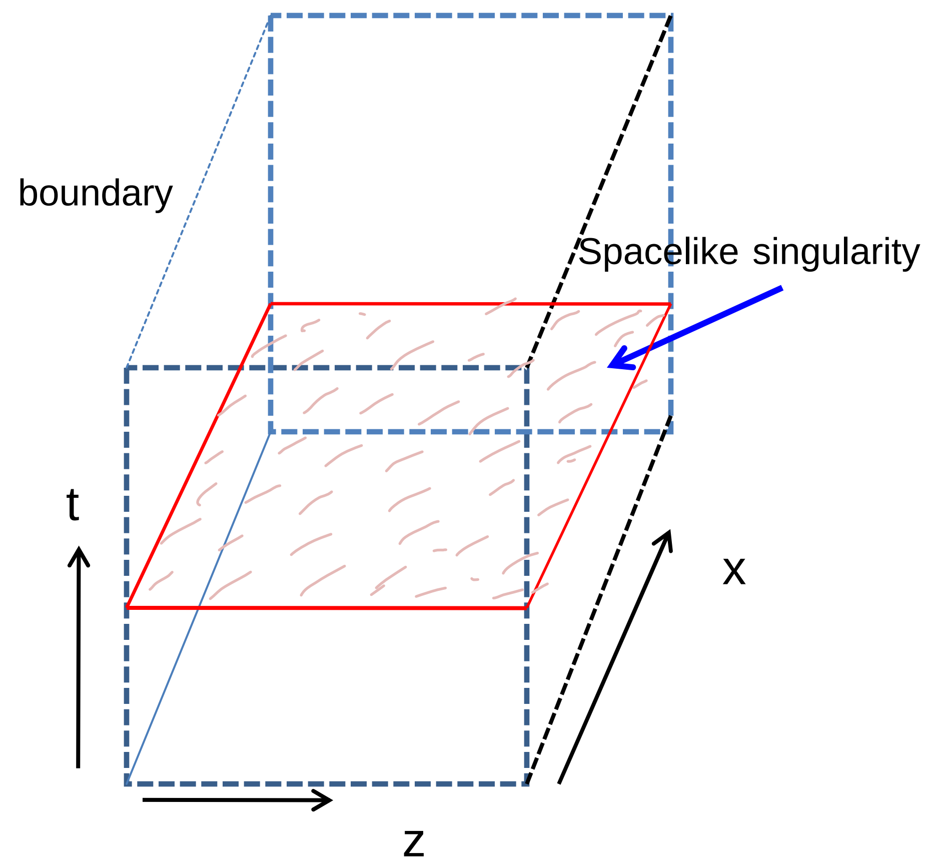
A Scenario

- At later times, the curvatures or other invariants of supergravity start becoming large
- If we nevertheless insist on the supergravity solution we encounter a singularity at some finite time
- Beyond this time, it is meaningless to evolve any further.



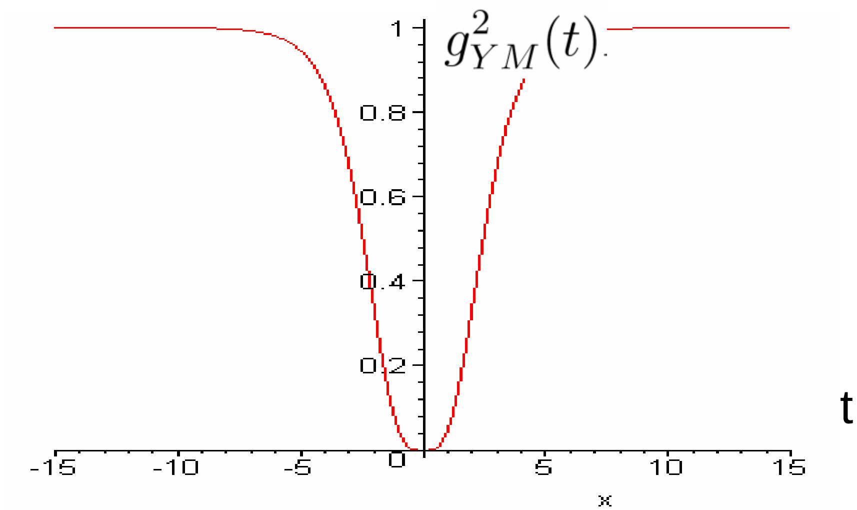
A Scenario

- However, the gauge theory could be still well defined at this time.
- And if we are lucky enough the gauge theory may be evolved beyond this point
- At much later times, the source could weaken again and one may regain a description in terms of supergravity



Models implementing this Scenario

- We will try to implement this scenario by turning on sources in the gauge theory which correspond to time dependen $g^2_{YM}(t)$ ngs
- The gauge theory would still live on flat space-time and there would be no other source.
- We will choose the gauge theory coupling to be bounded everywhere and becoming vanishingly small at some time.



- In supergravity this would correspond to a metric which is constrained to be **FLAT** on the boundary and a dilaton whose boundary value matches the gauge theory coupling.

$$ds^2 = \frac{1}{z^2}\left[dz^2 + g_{\mu\nu}(x,z)dx^\mu dx^\nu\right] + d\Omega_5^2$$

$$F_5 = \omega_5 + \star\omega_5$$

$$\mathrm{Lim}_{z\to 0} \;\; e^{\Phi(x,z)} = g_{YM}^2(t)$$

$$\mathrm{Lim}_{z\to 0} \;\; g_{\mu\nu}(x,z) = \eta_{\mu\nu}$$

At early times this should b $AdS_5 \times S^5$

Null Solutions

- The best controlled solutions of this type are those with null rather than spacelike singularities

$$ds^2 = \frac{1}{w^2}\left[dw^2 - 2dy^+dy^- + d\vec{y}^2 + \frac{1}{4}w^2(\Phi')^2(dy^+)^2\right]$$

- Where $\Phi(y^+)$ is the dilaton which is a function y^+ alone.
- These solutions have been independently obtained and studied by

    Chu and Ho, JHEP 0604 (2006) 013

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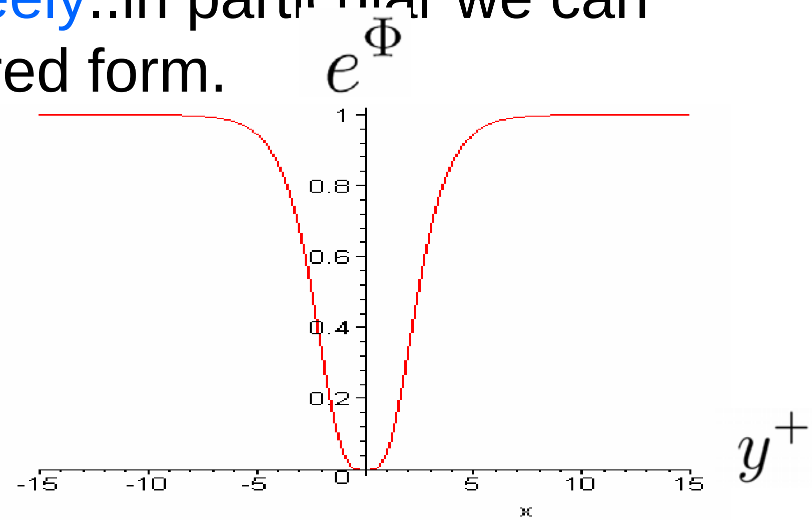
- Where $\Phi(y^+)$ is the dilaton which is a functio y^+ alone.
- This function may be chosen freely..in particular we can choose this function of the desired form.
- For example,

$$e^\Phi = (\tanh\ y^+)^2$$



- There is a singularity at $y^+ = 0$. Null geodesics

$$\rho = z_0 F(y^+)$$

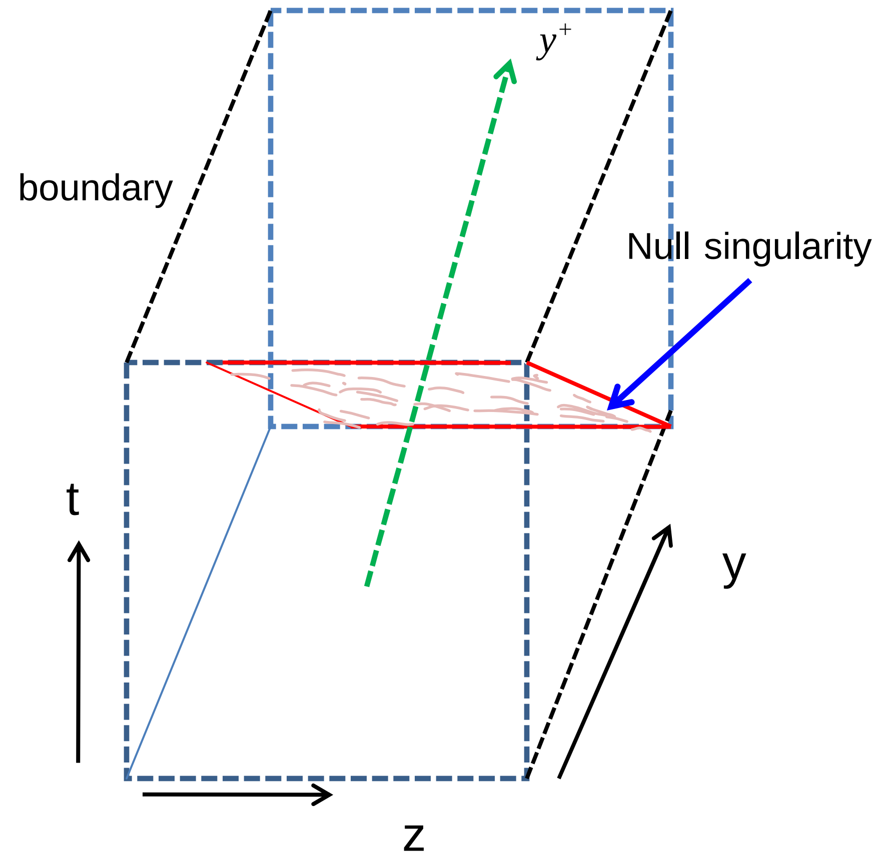
$$y^- = y_0^- - \frac{1}{4} z_0^2 \frac{d}{dy^+}(F(y^+))^2$$

where $\frac{F''}{F} = \frac{1}{4}(\Phi')^2$

reach this at finite affine parameter if

$$\lambda = \int^{y^+} \frac{dx}{(F(x))^2}$$

is finite.



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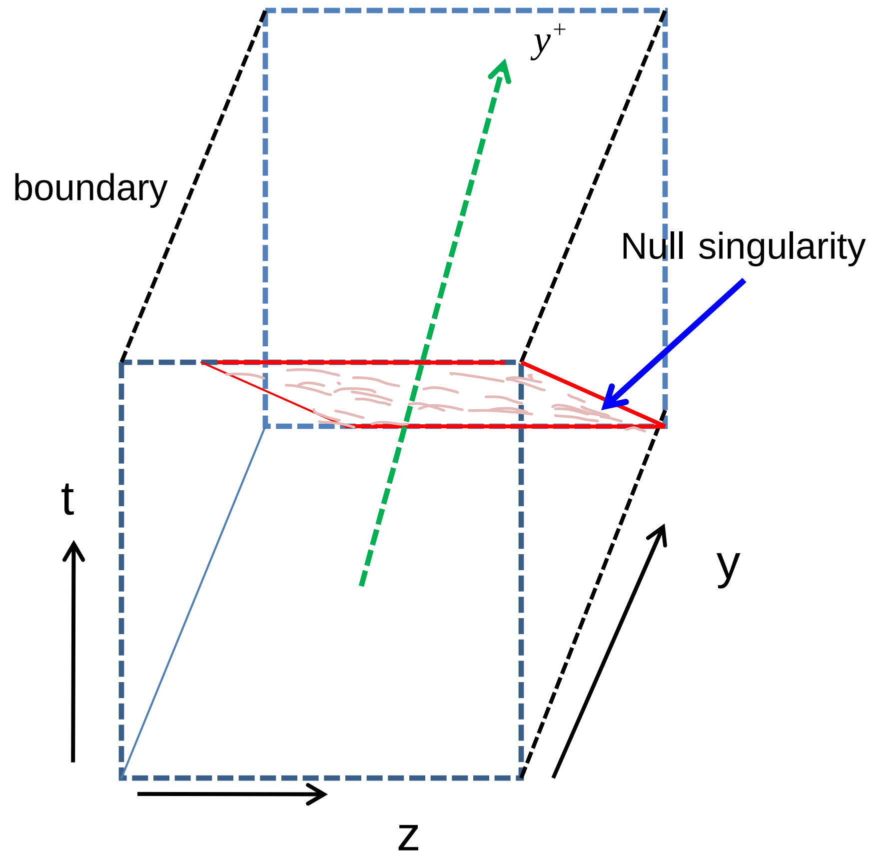
Tidal forces diverge here.



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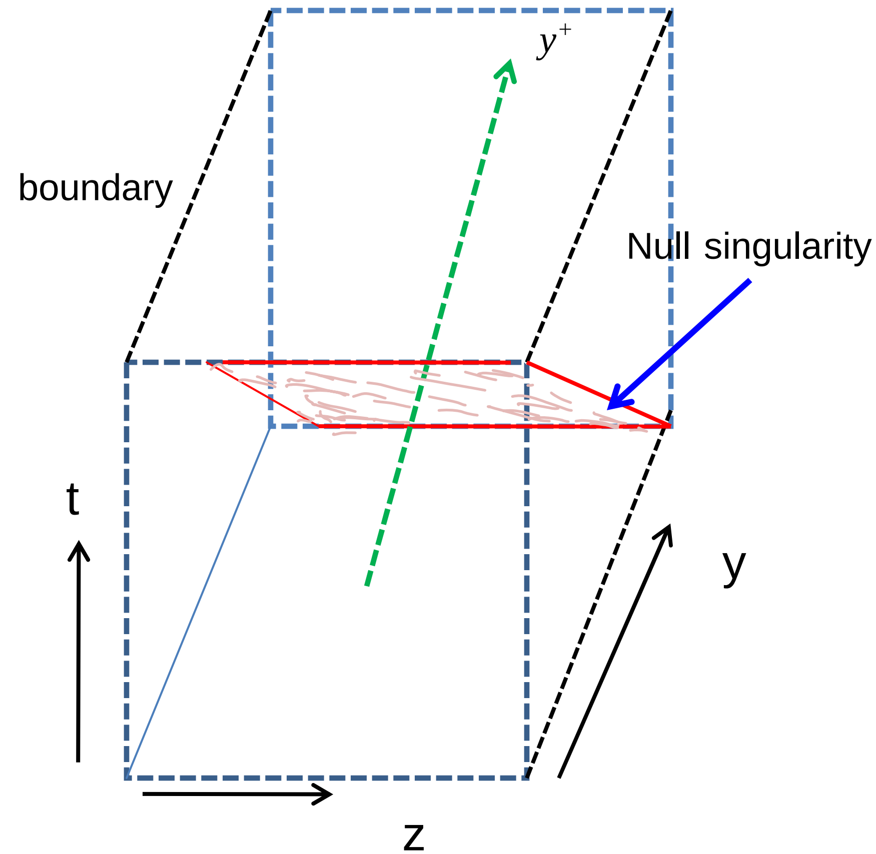
However, the singularity weakens as we approach the boundary



- These solutions are in fact related to

$$ds^2 = \frac{1}{z^2}\left[dz^2 + e^{f(x^+)}(-2dx^+dx^- + d\vec{x}^2)\right]$$

where $F(x^+) = e^{-f(x^+)/2}$ $\qquad \frac{F''}{F} = \frac{1}{4}(\Phi')^2$

by coordinate transformations

$$z = w\, e^{f(y^+)/2} \qquad x^- = y^- - \frac{1}{4}w^2(\partial_+ f)$$

- This is an example of the general fact that a Weyl transformation on the boundary is equivalent to a special class of coordinate transformations in the bulk - the Penrose-Brown-Hanneaux (PBH) transformations.

A more general class

- In fact there is a more general class of solutions of the following form

$$ds^2 = \frac{1}{z^2}\left[dz^2 + \tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu\right] + d\Omega_5^2$$

- The 4d metric $\tilde{g}_{\mu\nu}(x)$ and the dila $\Phi(x)$ are functions of the four x^ν ordinates and the 5-form field strength is standard
- This is a solution if $\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi$ $\nabla^2\Phi = 0$

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- Thus a solution of 3+1 dimensional dilaton gravity may be lifted to be a solution of 10d IIB supergravity with fluxes.

- We will consider solutions of this type where the 4d metric is conformal to flat space

$$\tilde{g}_{\mu\nu} = e^{h(x)} \; \eta_{\mu\nu}$$

 The connection between Weyl transformations on the boundary and and PBH transformations then ensures that there is a different foliation of the AdS space-time in which the boundary is flat – and all we have is a nontrivial dilaton.

- We will always define the dual gauge theory to live on this flat boundary.

Kasner-like Solutions

- The easiest form of time dependent solution is the lift of a usual 4d Kasner universe

$$ds^2 = \frac{1}{z^2}\left[dz^2 - dt^2 + \sum_{i=1}^{3} t^{2p_i} dx^i dx^i\right]$$

$$e^{\Phi(t)} = |t|^{\sqrt{2(1-\sum p_i^2)}} \qquad \sum_{i=1}^{3} p_i = 1$$

- This has a spacelike curvature singularity at t=0.
- The effective string coupling vanishes here – as required.
- However the coupling diverges at infinite past and future.

- Nevertheless it is instructive to see what the dual gauge theory looks like. This can be explicitly worked out for

$$p_1 = p_2 = p_3 = \frac{1}{3}$$

- In this case the 4d metric is conformal to flat space

$$ds^2 \;=\; \frac{1}{z^2}\left[dz^2 + \frac{2t}{3}\left(-dt^2 + (dx^1)^2 + \cdots (dx^3)^2\right)\right]$$

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- The exact PBH transformation may be written down and the metric which has a flat boundary is

$$ds^2 = \frac{1}{w^2}\left[dw^2 - \frac{(16T^2 - 5w^2)^2}{256T^4}dT^2 + \frac{(16T^2 - w^2)^{\frac{4}{3}}(16T^2 + 5w^2)^{\frac{2}{3}}}{256T^4}\left((dx^1)^2 + \cdots (dx^3)^2\right)\right]$$

$$e^{\Phi} = \left(T\ \left[\frac{16T^2 + 5w^2}{16T^2 - w^2}\right]^{2/3}\right)^{\sqrt{3}}$$

FRW-type solutions

- Time dependent solutions with bounded coupling have 3+1 dimensional slices which are conformal to FRW universes with k=-1.

$$ds^2 = \frac{1}{z^2}\left[dz^2 + \sinh(2t)\left(-dt^2 + \frac{dr^2}{1+r^2} + r^2\left(d\theta^2 + \sin\theta^2\, d\phi^2\right)\right)\right]$$

$$e^{\Phi(t)} = g_s\,|\tanh\ t|^{\sqrt{3}}$$

- The 3+1 dimensional slice is in fact conformal to part of Minkowski space. Defining new coordinates

$$r = \frac{R}{\sqrt{\eta^2 - R^2}} \qquad e^{-t} = \sqrt{\eta^2 - R^2}$$

- This solution becomes

$$ds^2 = \frac{1}{z^2}\left[dz^2 + \mid 1 - \frac{1}{(\eta^2 - R^2)^2} \mid [-d\eta^2 + dR^2 + R^2 d\Omega_2^2]\right]$$

$$e^{\Phi} = \mid \frac{\eta^2 - R^2 - 1}{\eta^2 - R^2 + 1} \mid^{\sqrt{3}}$$

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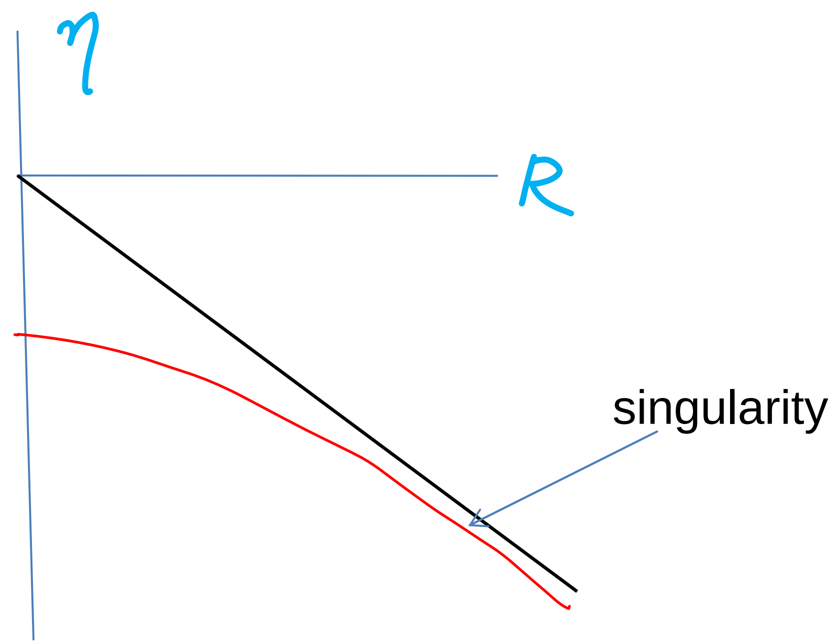
- The original space-time is thus conformal to the past light cone part of Minkowski space
- Singularity is now at

$$\eta^2 - R^2 = 1$$

- This metric can be extended to the whole $\eta - R$ plane



- It is useful to write this solution in slightly different coordinates

$$\tau^2 = \eta^2 - R^2 \qquad\qquad r^2 = \frac{\eta + R}{\eta - R}$$

- In these coordinates the conformal factor depends on a single time variable τ

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}\,(1 - \frac{1}{\tau^4})\,[-d\tau^2 + \frac{\tau^2}{r^2}\,dr^2 + \frac{\tau^2}{4}\,(r - \frac{1}{r})^2\,d{\Omega_2}^2]$$

- The dilaton is now

$$\Phi(\tau) = \sqrt{3}\,\ln\left[\frac{\tau^2 - 1}{\tau^2 + 1}\right]$$

- The spacelike singularity is now $\tau = 1$

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- These transformations are

$$z(\rho,\bar{T}) = \frac{\sqrt{\bar{T}^4-1}}{\bar{T}^2}\,\rho[1+\frac{\rho^2}{\bar{T}^2\,(\bar{T}^4-1)^2}+\frac{\rho^4}{\bar{T}^4\,(\bar{T}^4-1)^4}]+O(\rho^7)$$

$$\tau = \bar{T} + \frac{\rho^2}{(\bar{T}^4-1)\,\bar{T}} + O(\rho^6)$$

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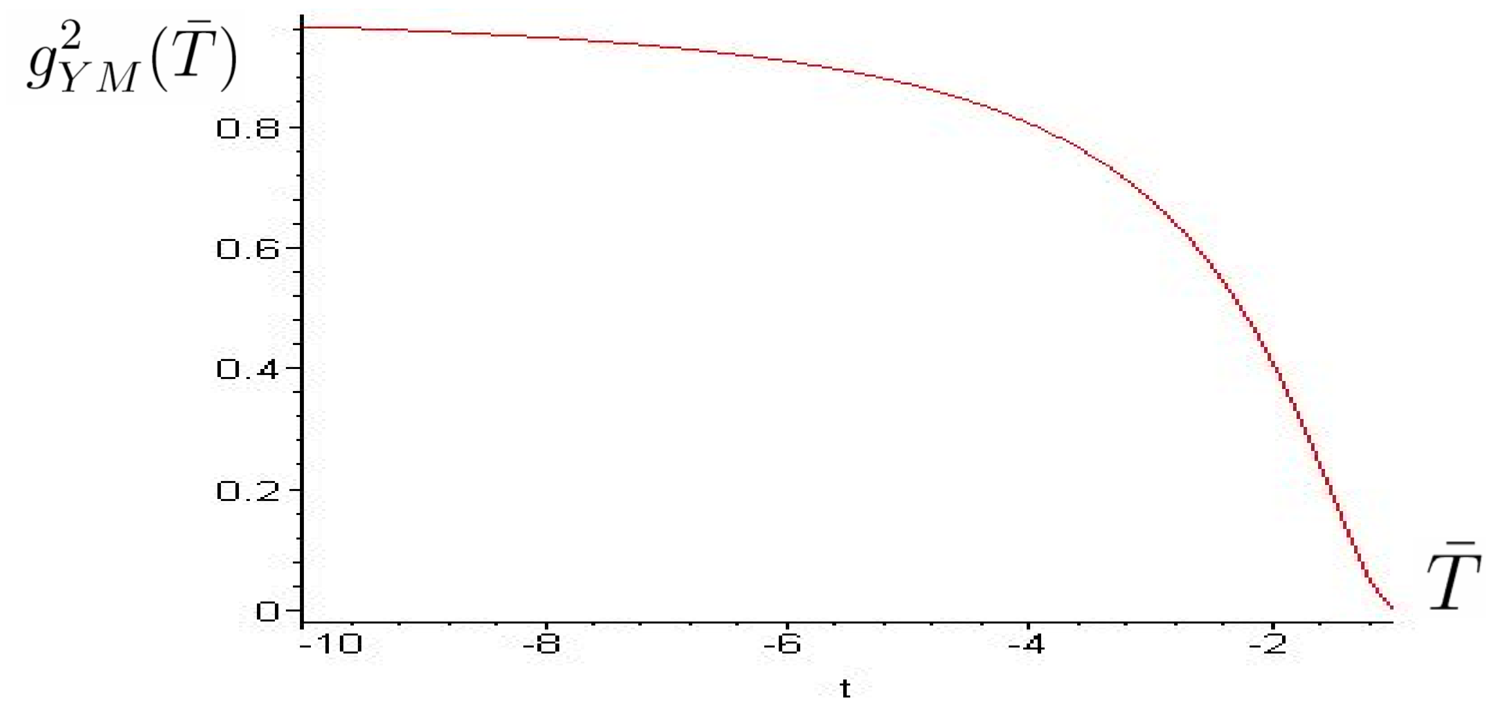
- In this case, we have not been able to determine the exact form of the transformations. However all one needs is the transformation near the boundary. We will, therefore, determine this in an expansion around the boundary.
- The metric near the boundary becomes

$$
\begin{aligned}
ds^2 &= [\frac{1}{\rho^2} + O(\rho^4)]\, d\rho^2 - [\frac{1}{\rho^2} - \frac{10\,\bar{T}^2}{(\bar{T}^4-1)^2} + \frac{25\,\bar{T}^4}{(\bar{T}^4-1)^4}\,\rho^2 + O(\rho^4)]\, d\bar{T}^2 \\
&+ [\frac{1}{\rho^2} + \frac{2\,\bar{T}^2}{(\bar{T}^4-1)^2} + \frac{(\bar{T}^4-8)}{(\bar{T}^4-1)^4}\,\rho^2 + O(\rho^4)] \left[\frac{\bar{T}^2}{r^2}\, dr^2 + \frac{\bar{T}^2}{4}\,(r - \frac{1}{r})^2\, d\Omega_2^2\right]
\end{aligned}
$$

- The boundary metric is now explicitly flat.

- In terms of these new coordinates the spacelike singularity appears a $\bar{T} = -1$ and the asymptotic past $\bar{T} \to -\infty$
- The effective string coupling is bounded, decreasing from a finite value in the past to a zero value at the singularity.
- Therefore, the dual gauge theory lives on flat space and has a time dependent coupling constant which vanishes at some $\bar{T} = -1$ me .



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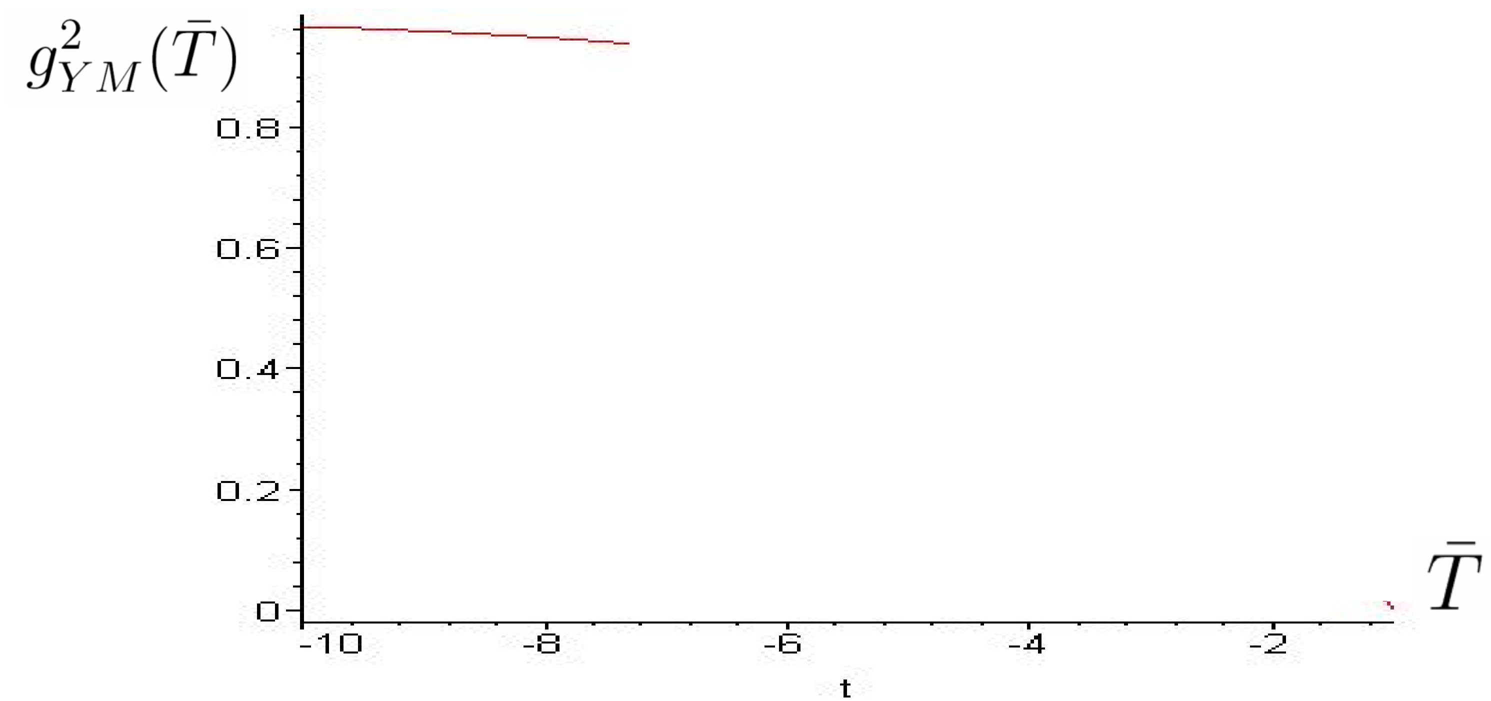
- At early times the 't Hooft coupling is large and supergravity can be trusted.



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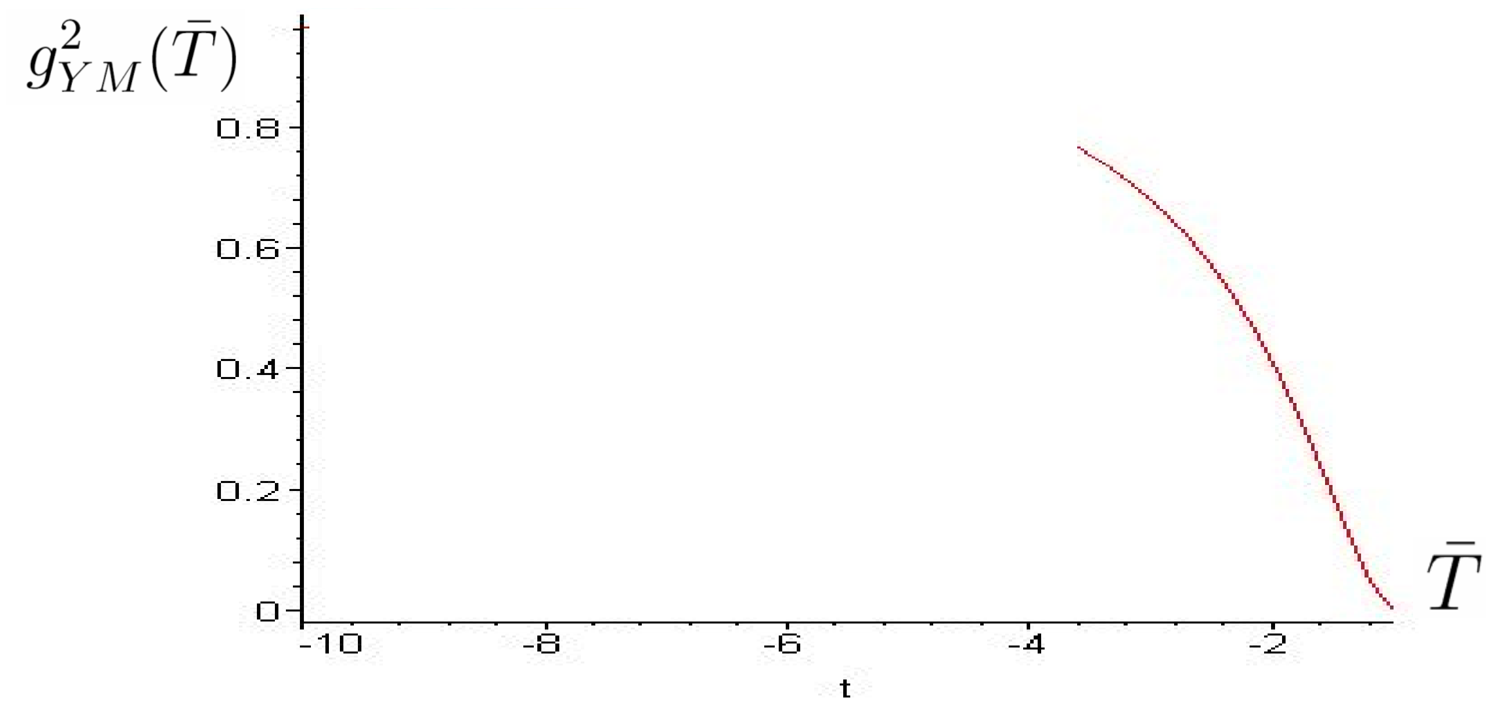
- The effective string coupling is bounded, decreasing from a finite value in the past to a zero value at the singularity.
- At late times the ‘t Hooft coupling becomes small and supergravity is meaningless. This is when the singularity appears in the bulk.



The Energy-Momentum Tensors

- The holographic stress tensor of these backgrounds provide insight into the nature of the quantum state
- In the regime where supergravity is reliable, this evaluates the energy momentum tensor of the dual gauge theory
- For backgrounds which are asymptotic $AdS_5 \times S^5$ at early times we can use this calculation to see whether the initial state is reasonable.
- We will compute this using the method of covariant counterterms

(Henningson and Skenderis ;

Balasubramanian and Kraus ;

Fukuma, Matsura and Sakai;…………….)

- Consider a 5d metric of the form

$$ds^2 = \frac{1}{z^2}\left[dz^2 + g_{\mu\nu}(x,z)dx^\mu dx^\nu\right]$$

- The cutoff boundary is taken to be at $z = z_0$
- With appropriate counterterms the holographic stress tensor is given by

$$T^{\mu\nu} = \frac{1}{8\pi G_5}\left[\Theta^{\mu\nu} - \Theta\, h^{\mu\nu} - 3\, h^{\mu\nu} + \frac{1}{2}G^{\mu\nu} - \frac{1}{4}\nabla^\mu\,\Phi\nabla^\nu\,\Phi + \frac{1}{8}h^{\mu\nu}(\nabla\,\Phi)^2\right]$$

where $\Theta^{\mu\nu}$ is the extrinsic curvature of the boundary, $h_{\mu\nu}(x)$ is the induced metric

$$h_{\mu\nu}(x) = \frac{1}{z_0^2}g_{\mu\nu}(x,z_0)$$

and $G^{\mu\nu}$ is the Einstein tensor computed from the induced metric.

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- However the gauge theory is best defined in a choice of boundary where the boundary metric is flat.
- In this choice of boundary the result depends on the specific background.

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- This ensures that in the gauge theory we are starting off with the vacuum state at early light cone times
- The fact that the em tensor continues to vanish at all later times is probably a reflection of the fact that for backgrounds with a null isometry there is no particle production due to the source.
- In particular, the conformal anomaly vanishes. The general expression for the anomaly agrees with the field theory calculation of *Fradkin and Tseytlin* and *Liu and Tseytlin* – and the holographic calculation of *Nojiri and Odinstov*

EM Tensor : FRW-type solutions

- For the FRW type solutions which have bounded couplings, the metric in flat boundary coordinates is

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ds^2 &= [\frac{1}{\rho^2} + O(\rho^4)]\, d\rho^2 - [\frac{1}{\rho^2} - \frac{10\,\bar{T}^2}{(\bar{T}^4-1)^2} + \frac{25\,\bar{T}^4}{(\bar{T}^4-1)^4}\,\rho^2 + O(\rho^4)]\, d\bar{T}^2 \\
&+ [\frac{1}{\rho^2} + \frac{2\,\bar{T}^2}{(\bar{T}^4-1)^2} + \frac{(\bar{T}^4-8)}{(\bar{T}^4-1)^4}\,\rho^2 + O(\rho^4)] \left[\frac{\bar{T}^2}{r^2}\, dr^2 + \frac{\bar{T}^2}{4}\,(r-\frac{1}{r})^2\, d\Omega_2^2\right]
\end{aligned}
$$

EM Tensor : FRW-type solutions

- For the FRW type solutions which have bounded couplings, the metric in flat boundary coordinates is

$$
\begin{aligned}
ds^2 \;&=\; [\frac{1}{\rho^2} + O(\rho^4)]\, d\rho^2 - [\frac{1}{\rho^2} - \frac{10\,\bar{T}^2}{(\bar{T}^4-1)^2} + \frac{25\,\bar{T}^4}{(\bar{T}^4-1)^4}\,\rho^2 + O(\rho^4)]\, d\bar{T}^2 \\
&+\; [\frac{1}{\rho^2} + \frac{2\,\bar{T}^2}{(\bar{T}^4-1)^2} + \frac{(\bar{T}^4-8)}{(\bar{T}^4-1)^4}\,\rho^2 + O(\rho^4)] \left[\frac{\bar{T}^2}{r^2}\, dr^2 + \frac{\bar{T}^2}{4}\,(r-\frac{1}{r})^2\, d\Omega_2^2\right]
\end{aligned}
$$

- The energy momentum tensor is given by

$$
T_\mu^\nu = \frac{\rho^4}{4\,\pi\, G_5\,(\bar{T}^4-1)^4}\,\mathrm{diag}\left(12 - 3\,\bar{T}^4,\, 4 + 9\,\bar{T}^4,\, 4 + 9\,\bar{T}^4,\, 4 + 9\,\bar{T}^4\right) + \; O(\rho^6)
$$

- The energy momentum of the dual field theo$< T^{\mu\nu} >$ is related to the holographic energy momentum tensor by the relation

$$\sqrt{-g}\, g_{\mu\nu} < T^{\nu\sigma} > = \lim_{\rho\to 0} \sqrt{-h}\, h_{\mu\nu} T^{\nu\sigma}$$

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and the conformal anomaly is

$$< T_\mu^\mu >= \frac{12\, N^2}{\pi^2} \frac{(\bar{T}^4 + 1)}{(\bar{T}^4 - 1)^4}$$

- At early times, $\bar{T} \to -\infty$ the bulk backgrou $AdS_5 \times S^5$ and the $< T_\mu^\nu >$ vanishes

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- However, this is precisely the time when the bulk calculation has no significance in the gauge theory, which is now weakly coupled.

Properties of the gauge theory

- For the null backgrounds, the dual gauge theory is easier to analyze.
- Even though the theory lives on flat space, the dilaton factor is in front of the kinetic term and diverges at the time of bulk singu

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- Normally one would absorb the coupling factor by a field redefinition so that only nonlinear terms involve the coupling.
- However in this case the factor is a function of X^+. Such a redefinition would introduce extra terms in the quadratic terms rendering the propagator non-standard

- Luckily , we can work in a light cone ga $A_- = 0$.
- In this gauge we can perform the following field redef $\tilde{A}_i = e^{-\Phi/2} A_i$ $\tilde{A}_+ = e^{-\Phi/2} A_+$
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$$S_{\rm GF} = -\frac{1}{4} \int d^4x \; [\; {\rm Tr}(\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu)^2 - 2ie^{\Phi/2}\, {\rm Tr}\{(\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu)[\tilde{A}^\mu, \tilde{A}^\nu]\} - e^{\Phi}\, {\rm Tr}([\tilde{A}_\mu, \tilde{A}_\nu])^2 - \partial_{X^-}\{(\partial_+ \Phi)\tilde{A}_i \tilde{A}^i\} \;]$$

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- This factor is bounded and goes to zero in the potentially problematic region.

- There are other gauge choices which are useful. *Chu and Ho* use a gauge of the form

$$\partial_\mu(e^{-\Phi(x^+)}A^\mu) + (\partial_+\Phi)e^{-\Phi(x^+)}A_- = 0$$

- They have used this gauge to calculate the effective action .
- This gauge could be more useful since it may avoid some of vexing issues associated with light cone gauge and light front evolution.

- For time dependent couplings this redefinition of fields does not lead to a standard kinetic term in usual gauges. This typically leads to singular terms, since

$$(\partial_+ \Phi) \sim \frac{1}{\bar{T}^2 - 1}$$

- However, just like the null solutions, this could be a gauge artifact. We have not yet found a suitable gauge to display this.

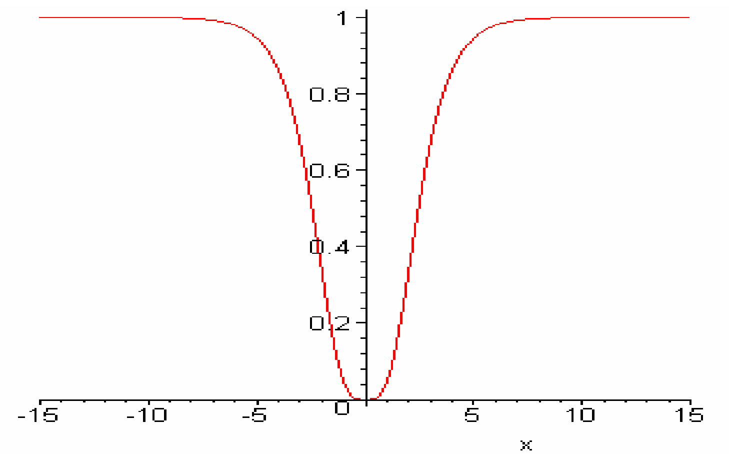
- With space-time dependent couplings loop diagrams can have divergences in addition to usual loop UV divergences. While the latter can be dealt with usual renormalization – the meaning of the former is not clear.
- However, when the coupling

  depends on only one null coordinate

  bounded everywhere

  differs from a constant only in a small region

these additional divergences are absent.



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- Formally, usual UV divergences coming from integrals over loop momenta can be renormalized. In this case

$$g_{\rm bare}(x^+) = Z[g_{\rm R}(x^+)]\ g_{\rm R}(x^+)$$

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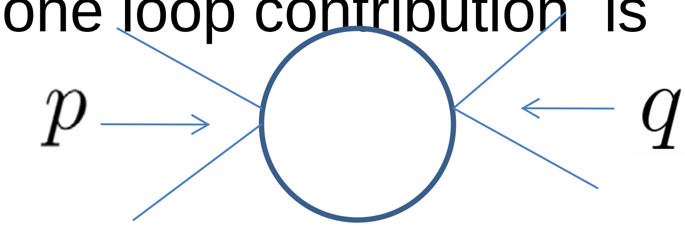
  However, the resulting quantum effective action can have additional divergences even after this renormalization.

- Consider for example ϕ^4 scalar field theory with a space-time dependent coupling $g(x)$ which we take to be general for the moment.
- In terms of the fourier transform of the renormalized coupling $g_R(p)$ the one loop contribution is



$$\int d^4p \; d^4q \; \phi_0^2(p)\phi_0^2(q) \; F(p,q)$$

$$F(p,q) = \int d^4k \; g_R(-p-k) \; g_R(-q+k) \; \log \frac{k^2}{\mu^2}$$

where $\phi_0^2(p)$ is the fourier transform of the background field.

$$F(p,q) = \int d^4k \; g_R(-p-k) \; g_R(-q+k) \; \log \frac{k^2}{\mu^2}$$

- If g_R was a constant (in position space), this would lead to the usual momentum conserving delta function

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- However in our case the integral over k is non-trivial and may lead to a divergent answer.
- This divergence would usually come from large values of k - so that we can ignore the external momenta

$$F(p,q) = \int d^4k \; g_R(-k) \; g_R(k) \; \log\frac{k^2}{\mu^2}$$

- When the coupling depends on a single null coordinate this expression simplifies and the potential for divergence reduces as well. Writing

$$g_R(p) = g_R[\delta^4(p) - h(p)]$$

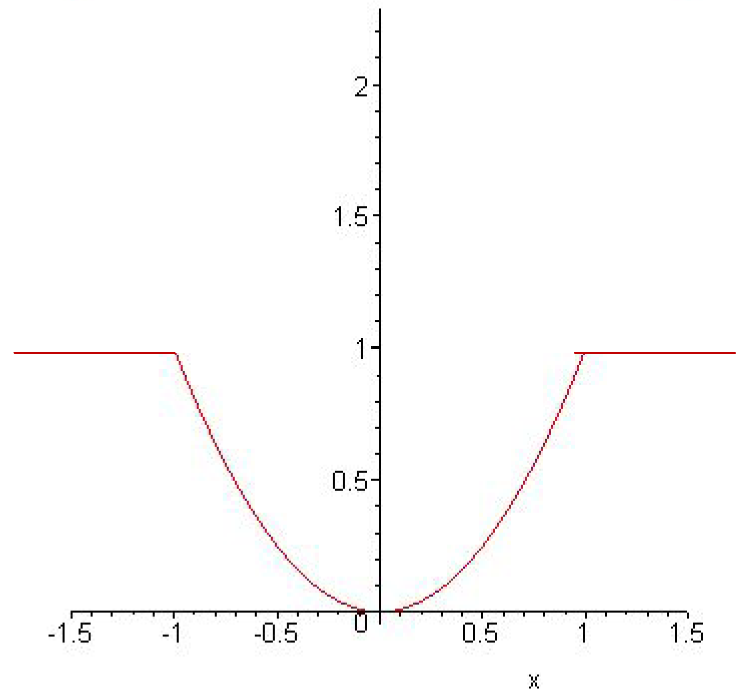
the relevant term becomes

$$F(p,q) = g_R^2[\delta^4(p+q) - \delta^2(\vec{p}+\vec{q})\delta(p_- + q_-)h(-p-q)\ \log\frac{p^2q^2}{\mu^2}$$
$$+\ \delta^2(\vec{p}+\vec{q})\delta(p_- + q_-)\int dk_+\ h(k_+)h(-k_+)\ \log\frac{k_+p_- + \vec{p}^2}{\mu^2}]$$

- For the kind of couplings we have been considering – it is easy to find situations when this last integral is convergent.
- This is because the coupling differs from a constant only in a small region near

- For a coupling which is of the following simple form

$$
\begin{aligned}
g_R(x^+) &= g_R & |x^+| > 1 \\
&= g_R(x^+)^2 & |x^+| \leq 1
\end{aligned}
$$



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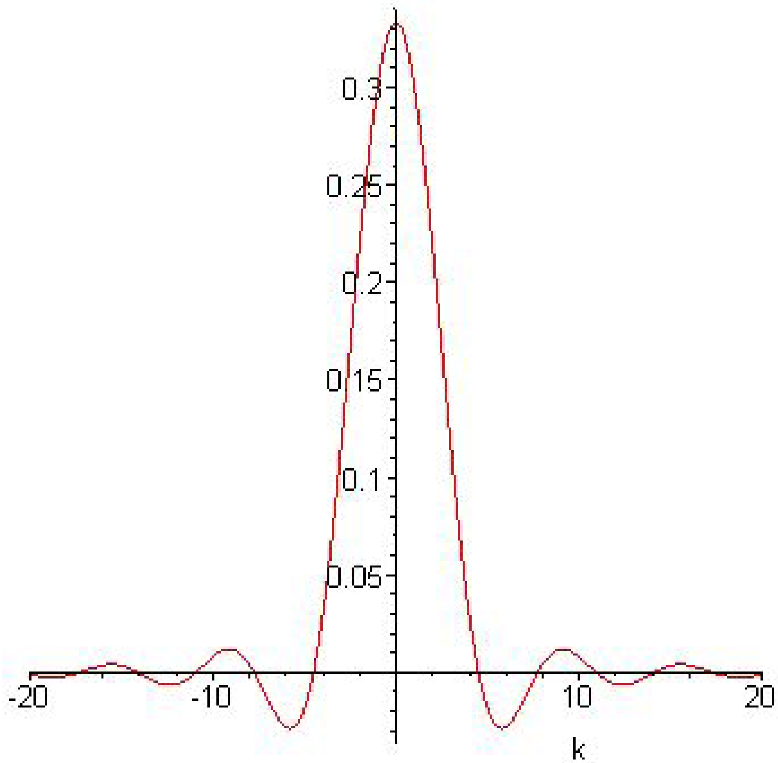
- This guarantees that the integral is convergent.

$h(k)$



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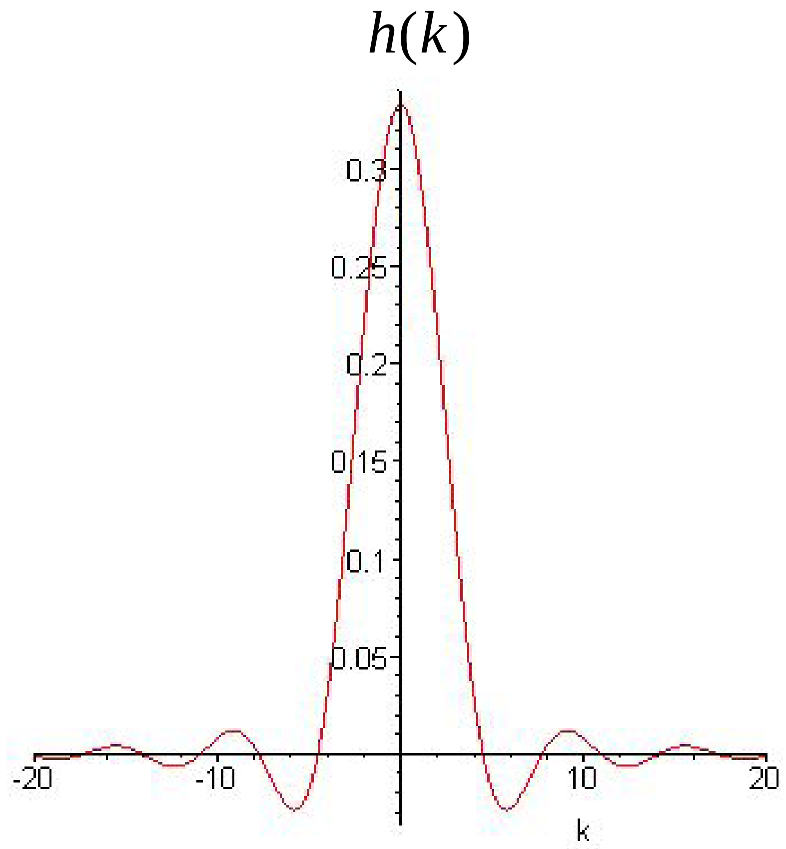
- This guarantees that the integral is convergent.
- For smooth solutions of this general form, the result continues to hold



Features of the Worldsheet

- In these examples, the Yang-Mills coupling becomes vanishingly small at the time when the bulk becomes singular.
- According to the usual wisdom of AdS/CFT one would expect that stringy effects become large.
- Since the string coupling in the bulk also vanishes at this time one might also expect that classical stringy effects could bring about the “resolution” of this singularity.
- The meaning of this is not entirely clear. In fact our point is that a closed string description is useless at this time – we should replace this by a perturbative gauge theory
- Nevertheless it may be useful to look at the worldsheet theory.

- Rewrite the metric in terms of coordinate $\vec{y} = (z, \Omega_5)$

$$ds^2 = \frac{1}{\vec{y}^2}\left[2dx^+dx^- + d\vec{x}^2 + d\vec{y}^2 - \frac{1}{4}\vec{y}^2\ [\partial_+\Phi(x^+)]^2\ (dx^+)^2\right]$$

- The invariant form of the action is, wi $\xi^\mu = (x^\pm, \vec{x}, \vec{y})$

$$S = \frac{1}{2}\int d\sigma d\tau\ \left[\sqrt{-h}h^{ab}\partial_a\xi^\mu\partial_b\xi^\nu\ G_{\mu\nu}(\xi)\right]$$

- The string metric $G_{\mu\nu}$ is related to the Einstein frame metric by

$$G_{\mu\nu} = e^{\Phi/2}g_{\mu\nu}$$

- Now fix a ligl $h_{01} = 0$ $x^+ = \tau$

σ

- And a further chc et coordinate

$$\sqrt{\frac{h_{00}}{h_{11}}} = G_{+-}(y, \tau)$$

- The final form of the worldsheet action is

$$S = \frac{1}{2}\int d\sigma d\tau \left[[(\partial_\tau \vec{x})^2 + (\partial_\tau \vec{y})^2] - \frac{e^{\Phi(\tau)}}{\vec{y}^4}[(\partial_\sigma \vec{x})^2 + (\partial_\sigma \vec{y})^2] - \frac{1}{4}\vec{y}^2(\partial_+\Phi)^2 \right]$$

- At early times this reproduces the known form $AdS_5 \times S^5$
- Since $e^{\Phi} \to 0$ $x^+ = \tau \to 0$, $\vec{x}$ the oscillator states become light.
- However, $(\partial_+\Phi)^2$ diverges $\tau = 0$

$$(\partial_+\Phi)^2 \sim \frac{1}{\tau^2}$$

Thus the $\vec{y}$ oscillator states decouple at this time.

- In a sense we are left with a very floppy tensionless string which can, however, oscillate only in two transverse dimensions.

String and Brane Excitations

- Clearly, higher massive modes of the fundamental string are excited copiously as we approach the singularity.
- It turns out that other branes are excited as well.
- We have studied this by considering the Penrose limit of the null backgrounds and constructing Matrix Theory in the resutling pp-wave
- The Matrix theory shows spherical D-branes (which appear as fuzzy solutions of the model) grow in size as we approach the singularity.

Epilouge

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- In the regions where the bulk solution becomes singular – and therefore cannot be trusted, the gauge theory dual becomes weakly coupled and therefore is not expected to have a gravity dual in any case
- For null singularities, preliminary studies of the gauge theory seem to suggest that the gauge theory evolution may indeed well defined.
- For space-like singularities, things are less clear – but now at least we have examples where the issue can be analyzed.

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- Even if this is not true, we would like to know the precise signatures of cosmological singularities in the gauge theory.

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- Even if this is not true, we would like to know the precise signatures of cosmological singularities in the gauge theory.
- In any case, the emergent nature of space-time in string theory has interesting things to say about singularities.

Thank you