Interfacing analytical and numerical relativity in modeling binary black hole coalescences

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- **Understanding gravity in the weak and strong regime**
  e.g., comparing with post-Newtonian theory; grasping the transition inspiral to merger to ringdown

- **Detecting gravitational waves and extracting unique information**
  e.g., building analytic templates

- **Making astrophysical predictions**
  e.g., recoil velocity of merging black holes; how supermassive black holes formed

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Modeling the long inspiral phase using PN theory

[Blanchet, Damour, Iyer, Faye, Deruelle; Wagoner, Will, Wiseman, Kidder, ...]

- **In general relativity radiation-reaction effects appear at order $\sim v^5/c^5$ beyond the Newtonian force law**

$$m\,\frac{d\mathbf{v}}{dt} = \mathbf{F}_{\rm Newt} + \cdots\ + \left(\frac{v}{c}\right)^5 \mathbf{F}_{\rm RR}$$

- **Throughout the inspiral $T_{\rm RR} \gg T_{\rm orb} \Rightarrow$ natural** *adiabatic parameter*

$$\frac{\dot{\omega}}{\omega^2} = \mathcal{O}\left[\left(\frac{v}{c}\right)^5\right]$$

- **PN expansion: formal expansion in $1/c$ when $c \to +\infty$**

- **For compact bodies, such as neutron stars and black holes,**

$$\frac{v^2}{c^2} \sim \frac{Gm}{c^2 r} \sim \frac{R_S}{r} \ll 1$$

Waveforms in the adiabatic approximation

- **Inspiral as an adiabatic sequence of circular orbits:**

$$h(t) \propto \ddot{Q} \propto \frac{v^2}{c^2} \cos 2\varphi \propto \left(\frac{GM\,\omega}{c^3}\right)^{2/3} \cos 2\varphi$$

- **Energy-balance equation:** $\frac{dE(v)}{dt} = -F(v)$

$E(v) \to$ center-of-mass energy $\quad F(v) \to$ gravitational-wave energy flux

$E(v)$ **and** $F(v)$ **known as a PN expansion in** $v/c = (GM\omega/c^3)^{1/3}$

$$\Rightarrow \dot{\omega} = -\frac{F(\omega)}{[dE(\omega)/d\omega]} \quad \Rightarrow \quad \varphi_{\rm GW}(t) = 2\varphi(t) = 1/\pi \int \omega\, dt$$

Effective-one-body and Padé resummation

[AB & Damour 99]

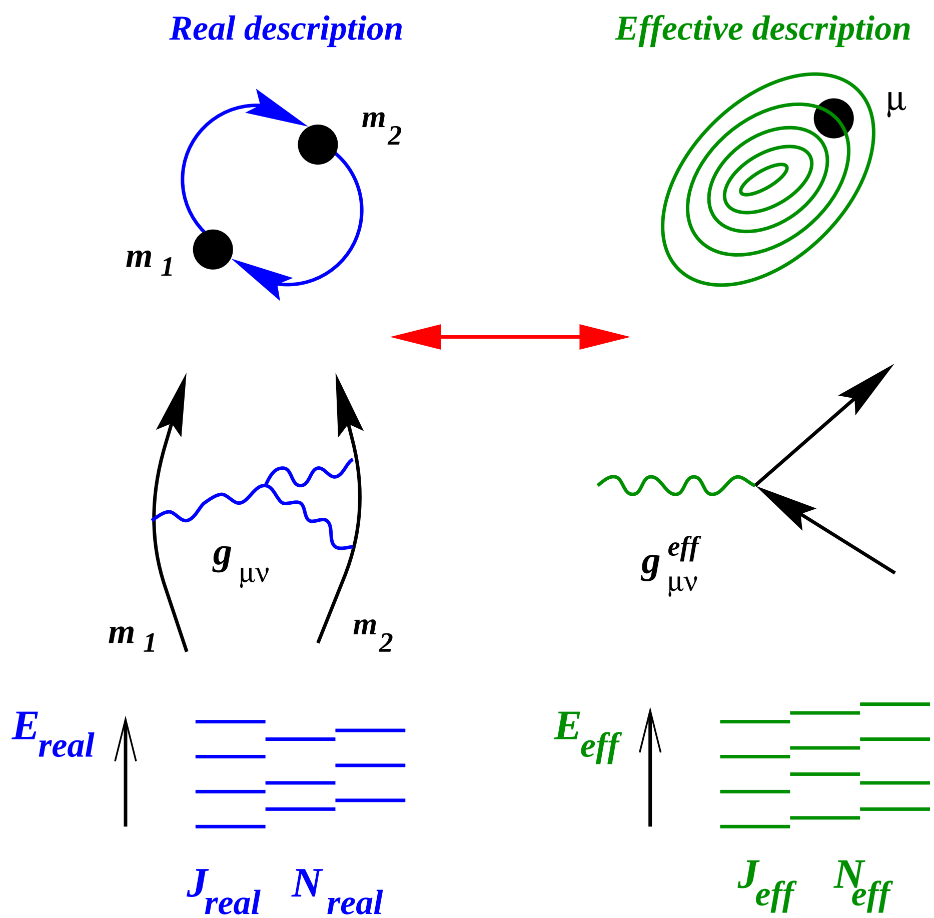
- **Resum so that *known* test mass limit results are recovered**
- **Resum the PN expansion assuming that the equal-mass limit is a η-deformation of the test-mass limit**

$\eta = m_1\, m_2/M^2$

$0 \leq \eta \leq 1/4$

- **Padé resummation of the energy flux F**

  [Damour, Iyer & Sathyaprakash 97]

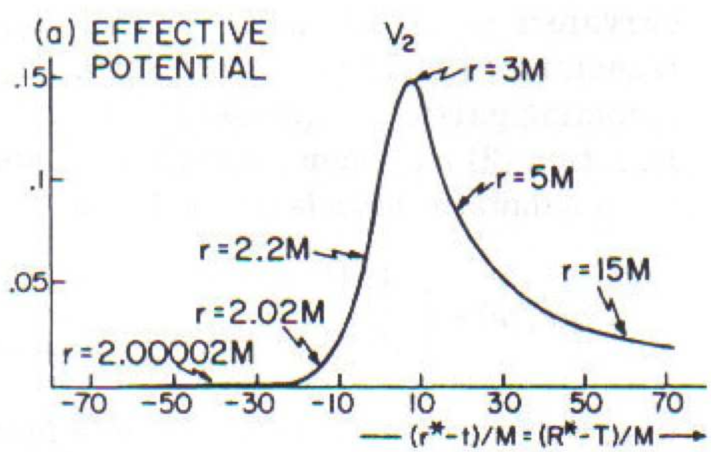


$E_{\rm real} = f(E_{\rm eff})$

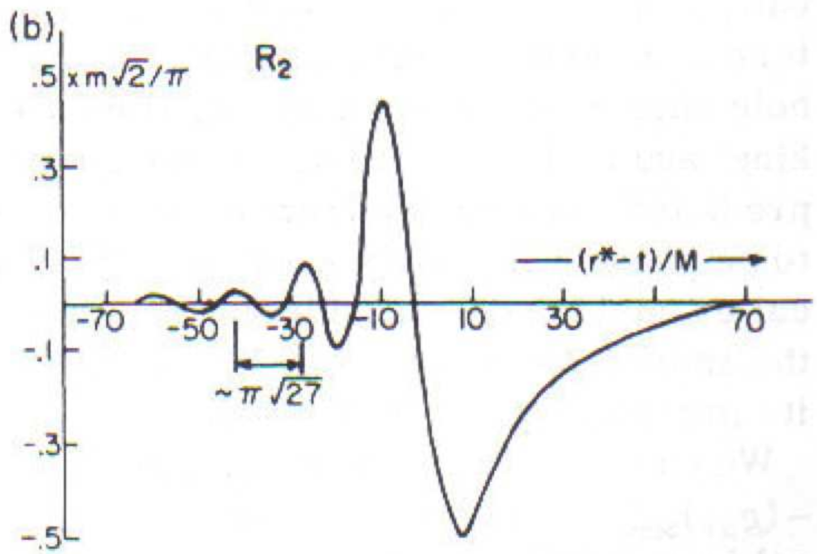
Features of the GW signal emitted by a test-particle falling radially in a Schwarzschild black hole

$$\frac{d^2}{dr_*^2} Z_l + (V_l - \omega^2)\, Z_l = \mathcal{S}_l$$

$Z_l \rightarrow$ perturbation $\quad \mathcal{S}_l \rightarrow$ source



Outgoing field

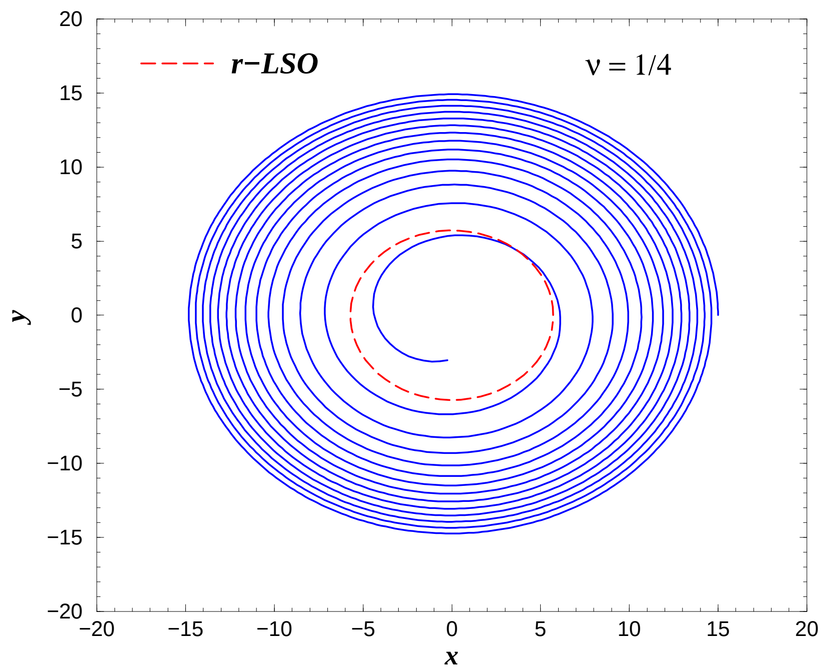


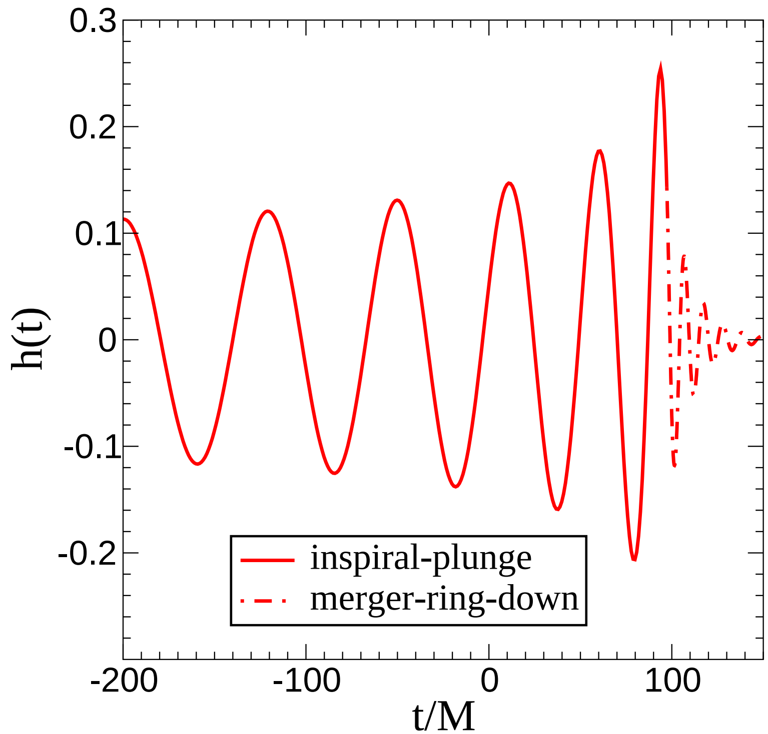
... part of the energy produced in the strong-burst region is stored in the resonant cavity of the geometry, and then slowly released in ringdown modes.

[Press 71; Davis, Ruffini, Press & Price 71; Davis, Ruffini & Tionmo 72]

Full waveform as predicted by the EOB-Padé model

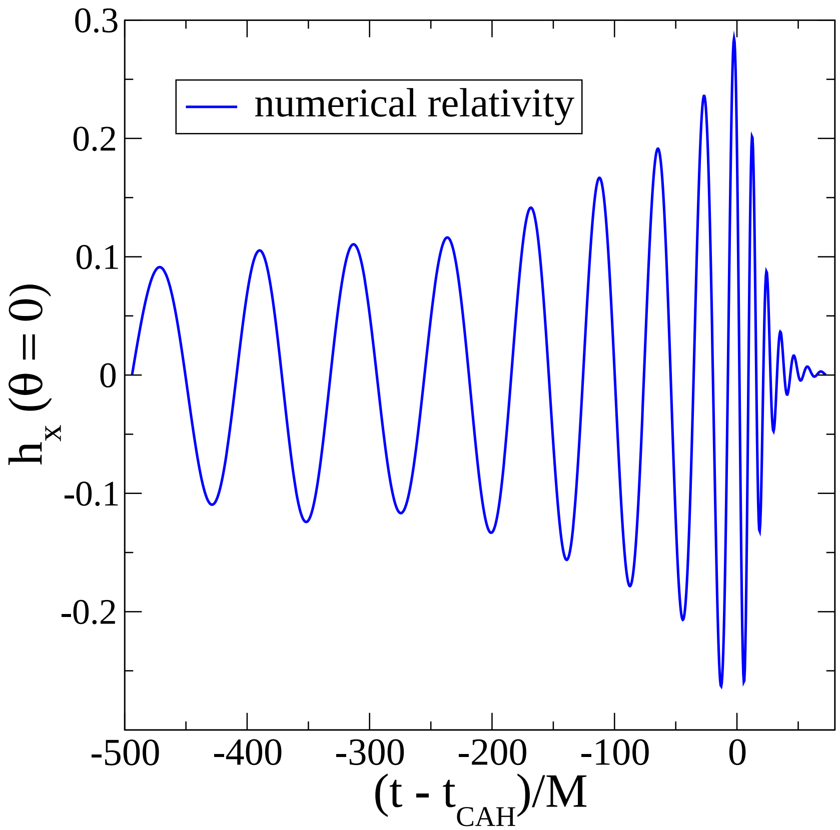
- **The plunge (~ 1.5 GW cycles) is a smooth continuation of the inspiral phase**
- **The transition merger to ringdown was assumed** *very short*
- One single **QNM matched using** $M_{\rm BH} = E_{\rm LR} = 0.976\, M$, $a_{\rm BH} = \boldsymbol{J}_{\rm LR}/E_{\rm LR}^2 = 0.77$



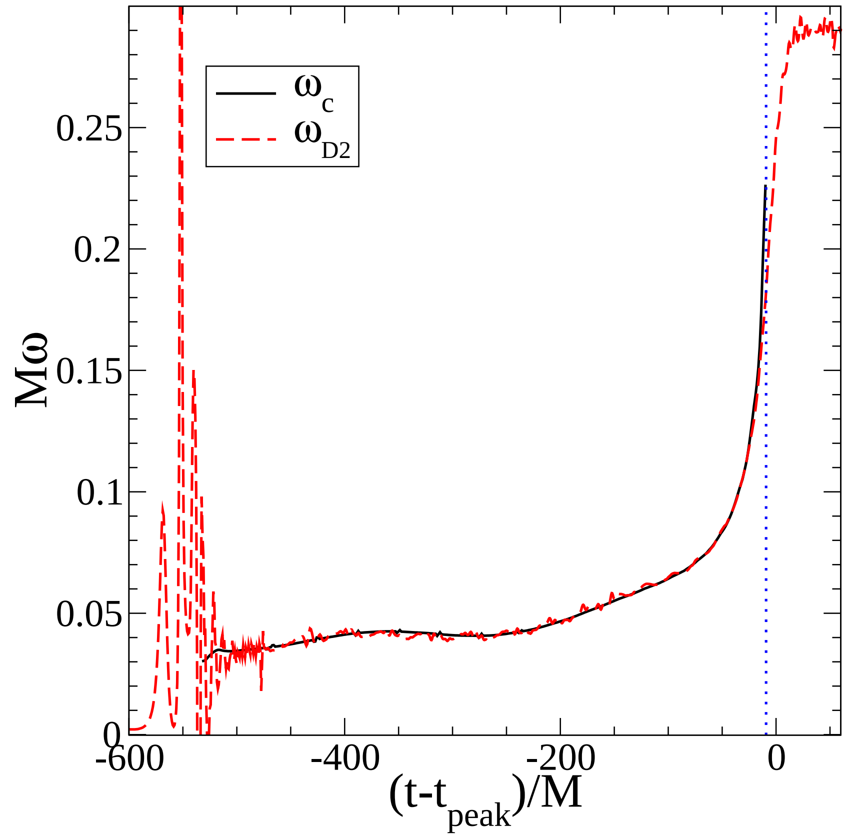


[AB & Damour 99, 00; Damour, Jaranowski & Schafer 00; Damour 01; AB, Chen & Damour 06]

Numerical simulations of equal-mass binary: *one* dominant frequency



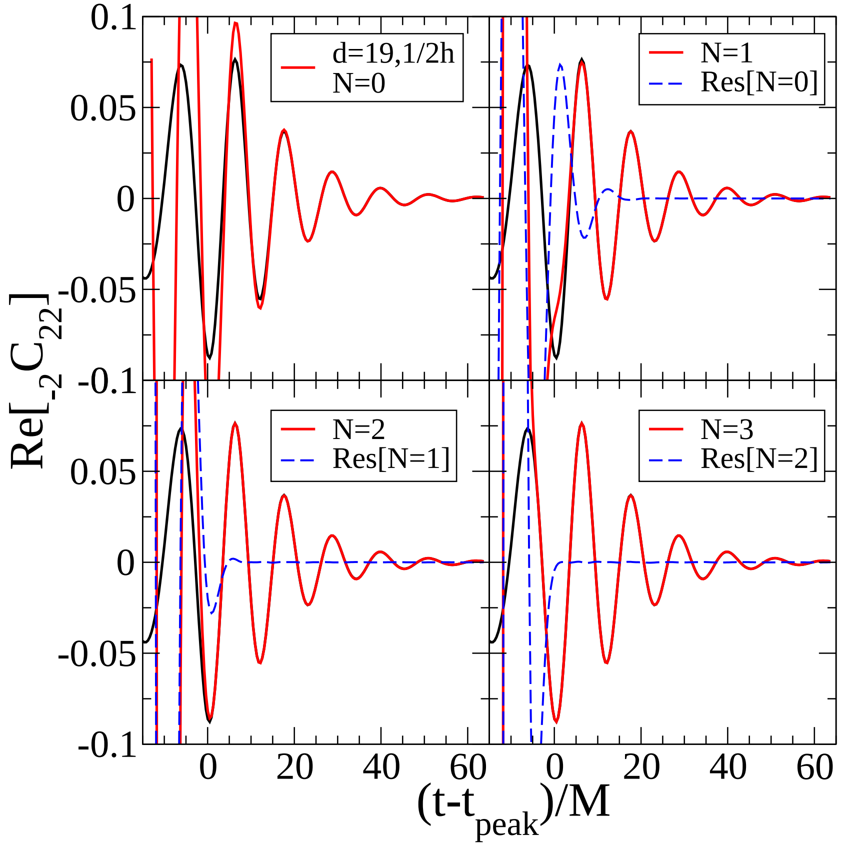
[AB, Cook & Pretorius 06]

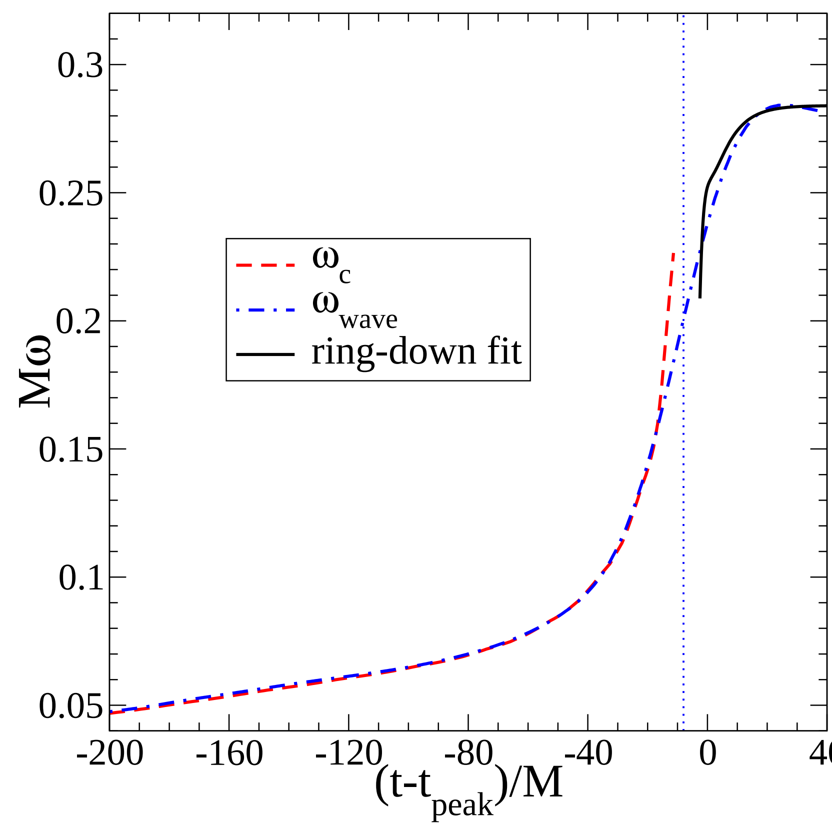


- $\omega_c \Leftarrow$ **from the coordinate separation**
- $\omega_{\rm D2} = -\frac{1}{2}{\rm Im}\left[\frac{\dot{C}_{22}}{C_{22}}\right] \Leftarrow$ **from the wave**

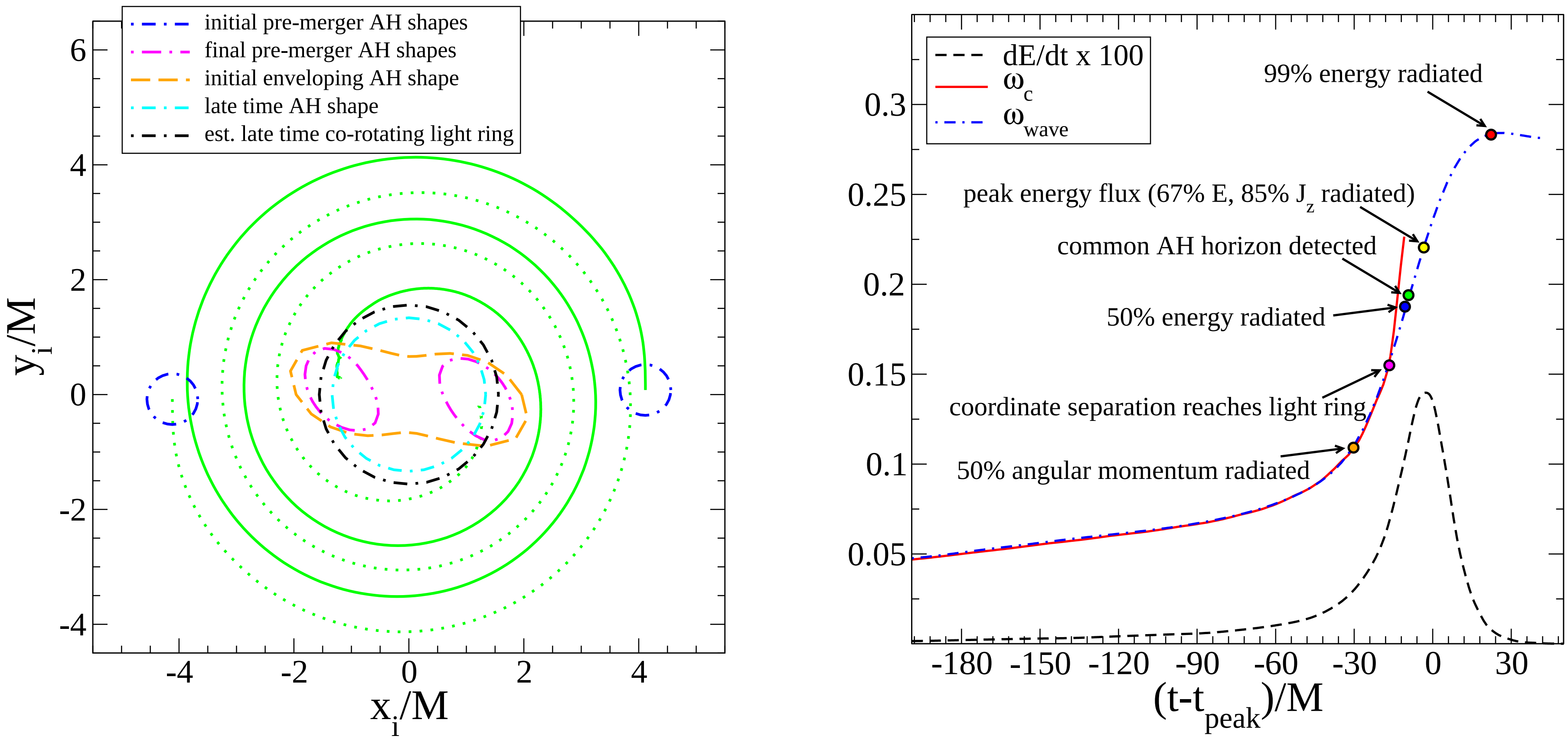
When the ringdown phase starts. Higher overtones.

[AB, Cook & Pretorius 06; see also Berti et al. 07]





The (plunge and) merger

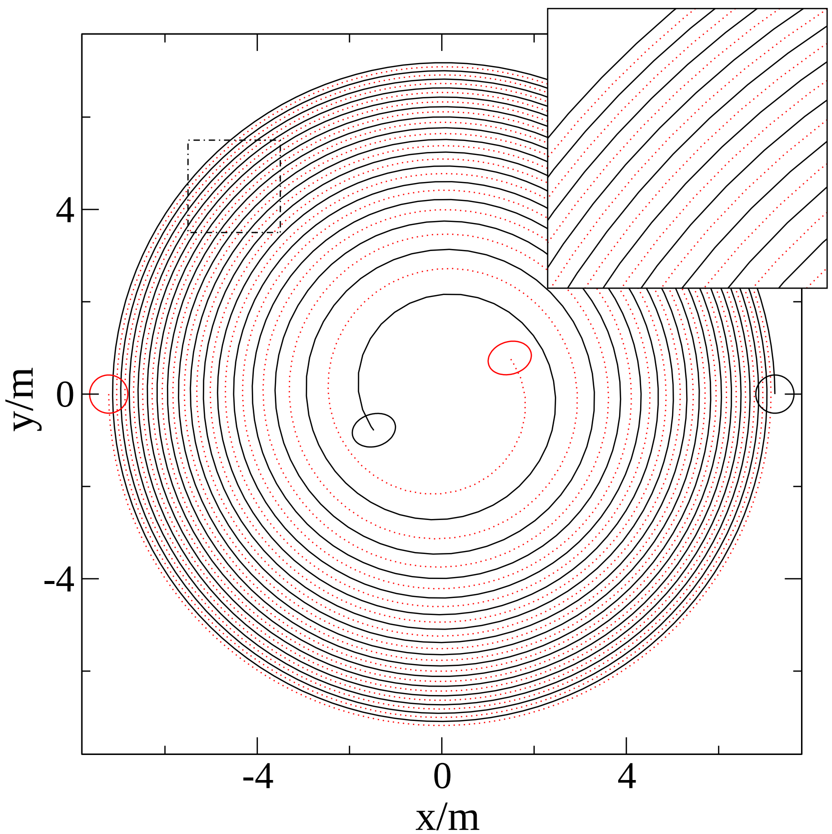


[AB, Cook & Pretorius 06]

- *Short* **transition merger–ringdown**
- **Energy and angular-momentum quickly released during merger**

Extremely accurate NR simulation using spectral methods

- **Equal-mass non-spinning black-hole binary** `Caltech-Cornell collaboration`



- **During the first 15 GW cycles all PN models agree with NR within 0.05 rad**

- **Different PN models differ by the way of solving**:

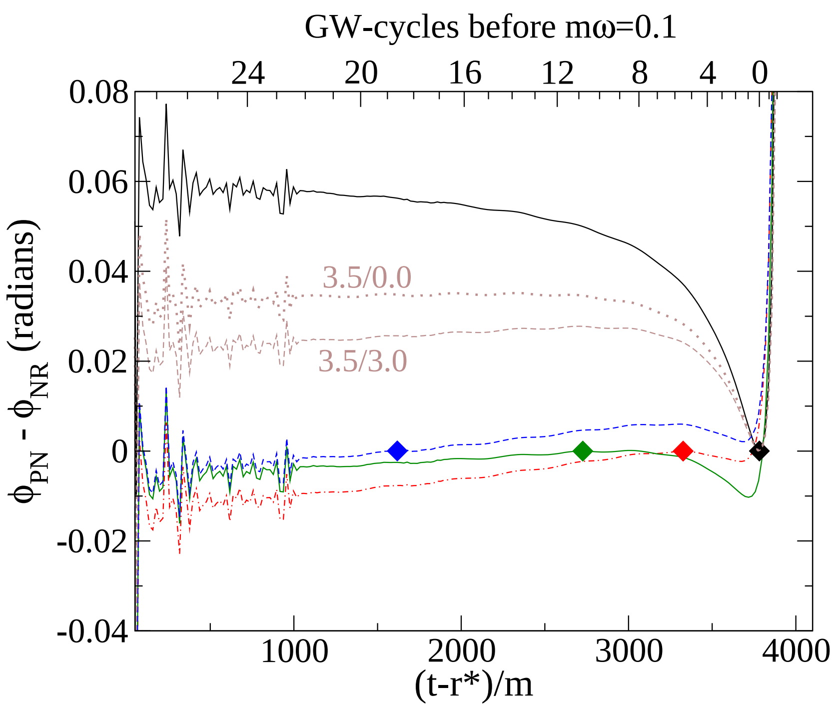
$$\dot{\omega} = -\frac{F(\omega)}{[dE(\omega)/d\omega]}$$

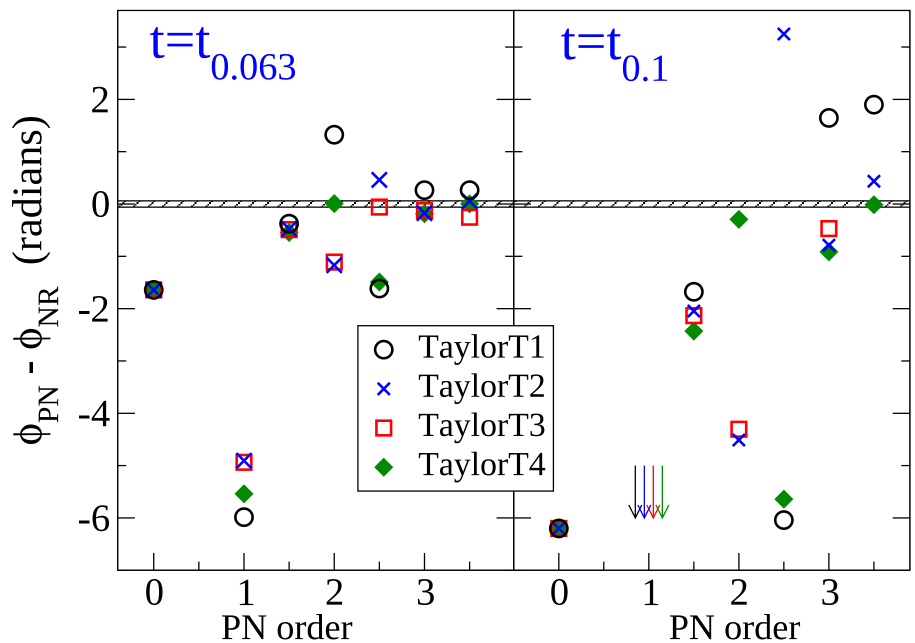
Comparison PN-adiabatic models and extremely accurate numerical simulations

- **Equal-mass non-spinning black-hole binary** `Caltech-Cornell collaboration`

- **Later on the PN models accumulate a dephasing of few rads, except for one model**

`[see also Nasa-Goddard 07; Jena 07]`

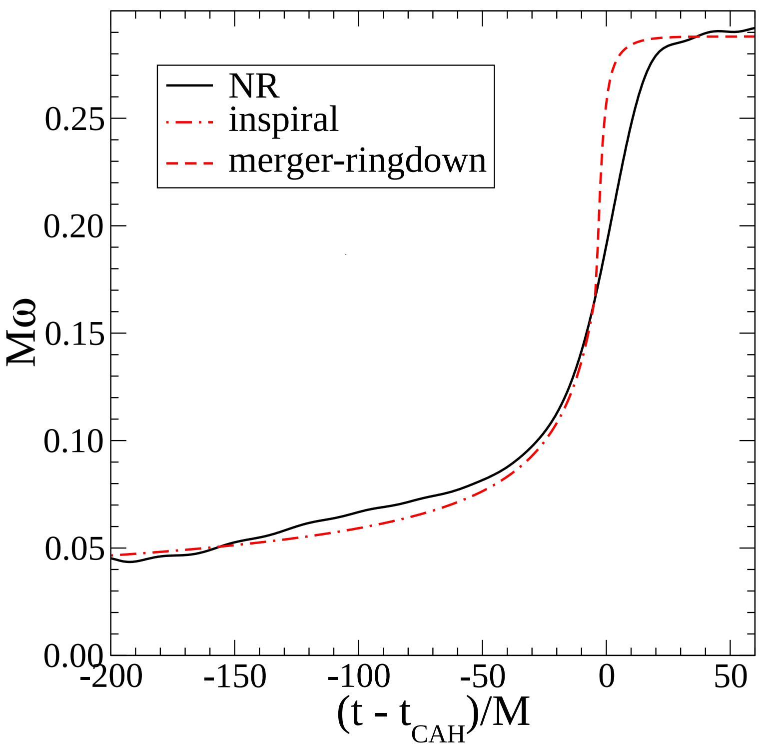


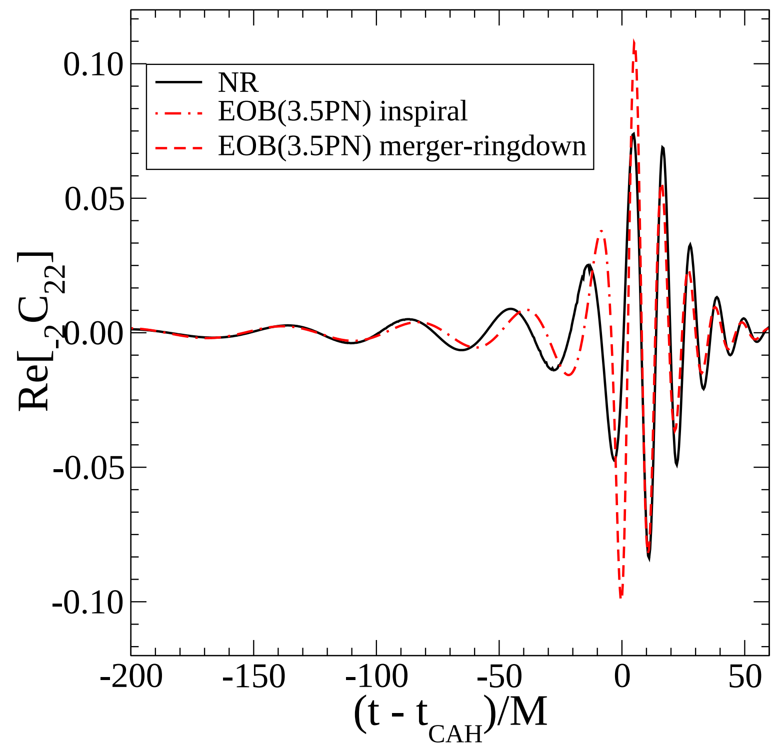


Comparing NR and EOB waveforms: *effectualness*

[AB, Cook & Pretorius 06; see also Pan, AB & NASA-Goddard 07]

- **Fundamental QNM mode and two overtones included**

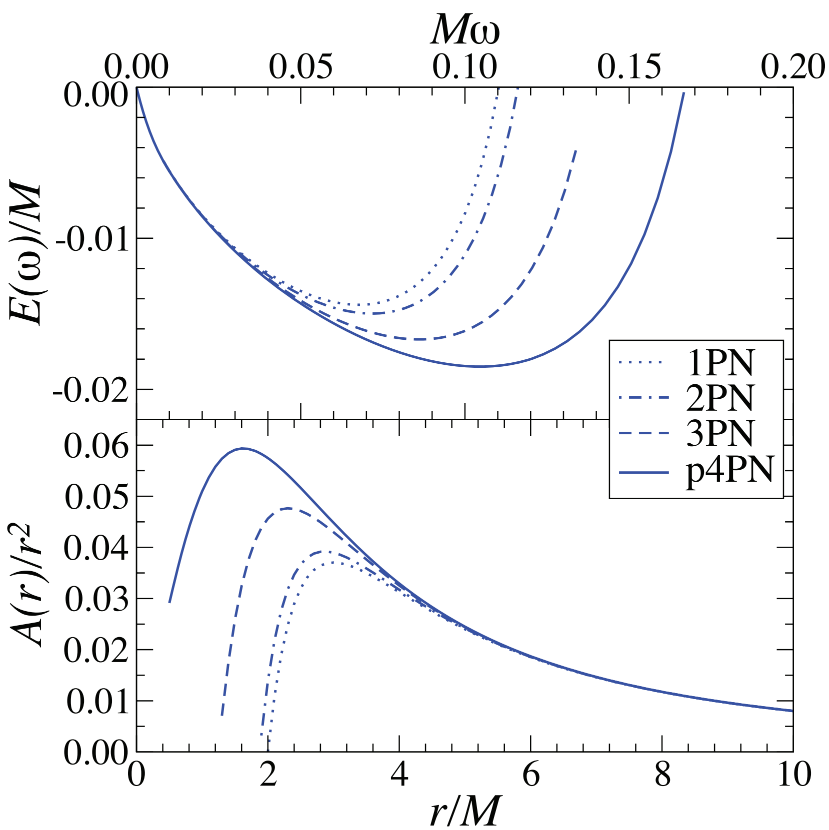




- **overlap $\gtrsim$ 0.97 maximizing on binary parameters, time-of-arrival, initial phase**

Improving EOB model using NR as *guide*

[AB, Pan & NASA-Goddard 07]



[Damour, Iyer, Jaranowski & Sathyaprakash 03]

- $A^{\rm p4PN}(r) = A^{\rm 3PN}(r) + \frac{\lambda\,\eta}{r^5},\ \lambda = 60$
- **Apply Padé resummation to ensure presence of LSO and light ring**
- **Analytic inspiral/ringdown matching point**

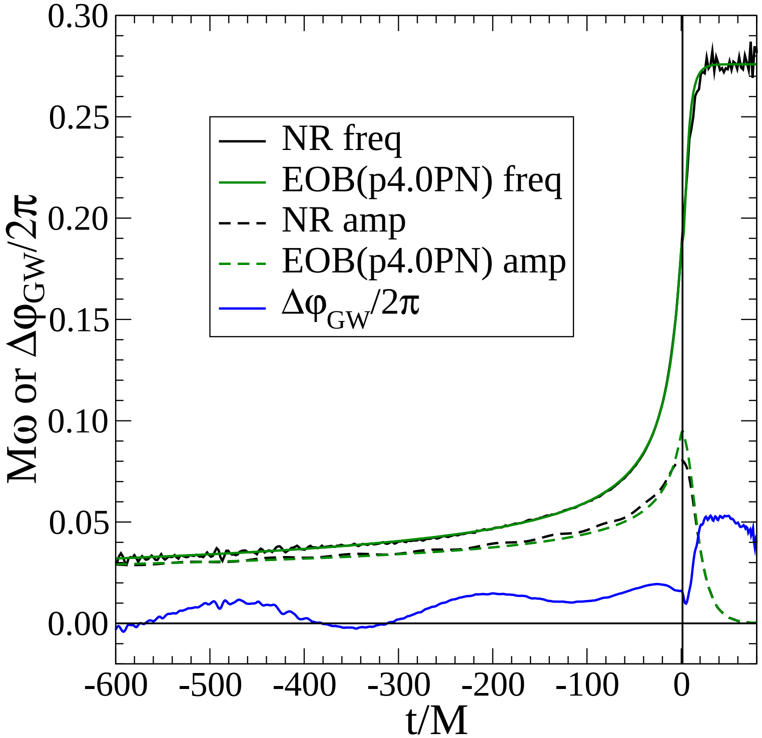
  $$M\,\omega_{\rm match} = 0.133 + 0.183\,\eta + 0.161\eta^2$$
- **QNM frequency and decay time depend *only* on $M_{\rm BH}/M$ and $a_f/M_{\rm BH}$**

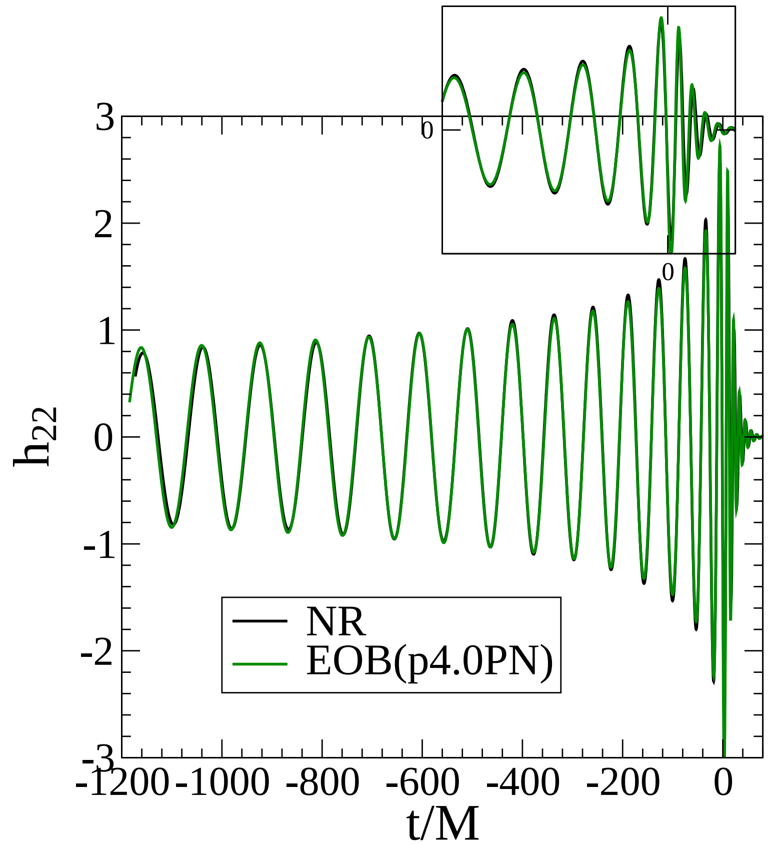
  $$\frac{M_{\rm BH}}{M} = 1 + (\sqrt{8/9} - 1)\,\eta - 0.498\,\eta^2$$

  $$\frac{a_f}{M_{\rm BH}} = \sqrt{12}\,\eta - 2.90\,\eta^2$$

NR and EOB waveforms for equal-mass binary: *faithfulness*

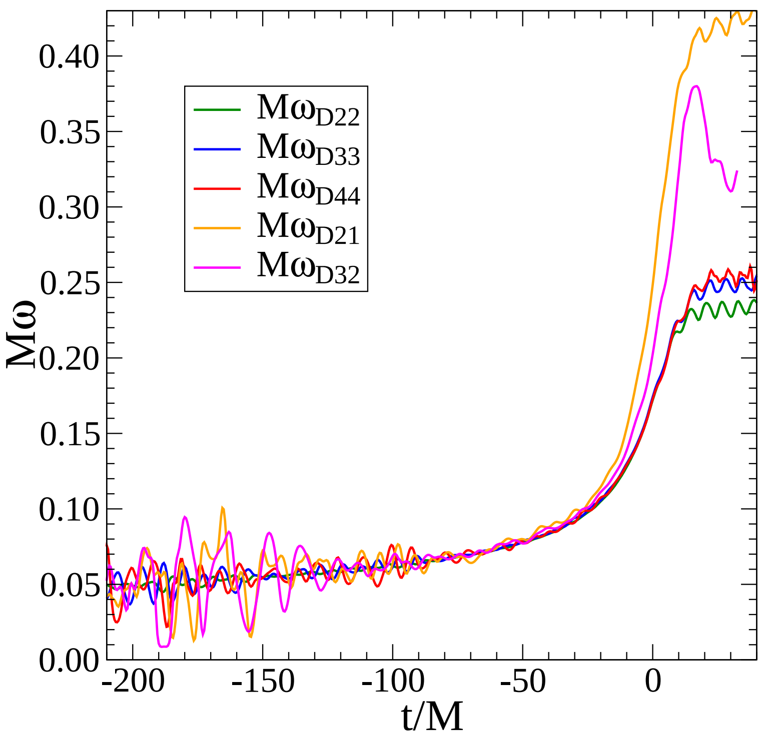
- **Phase difference in GW cycles of $\sim 5\%$** [AB, Pan & NASA-Goddard 07]
- **overlap $\gtrsim 0.98$ maximizing *only* on time-of-arrival and initial phase**

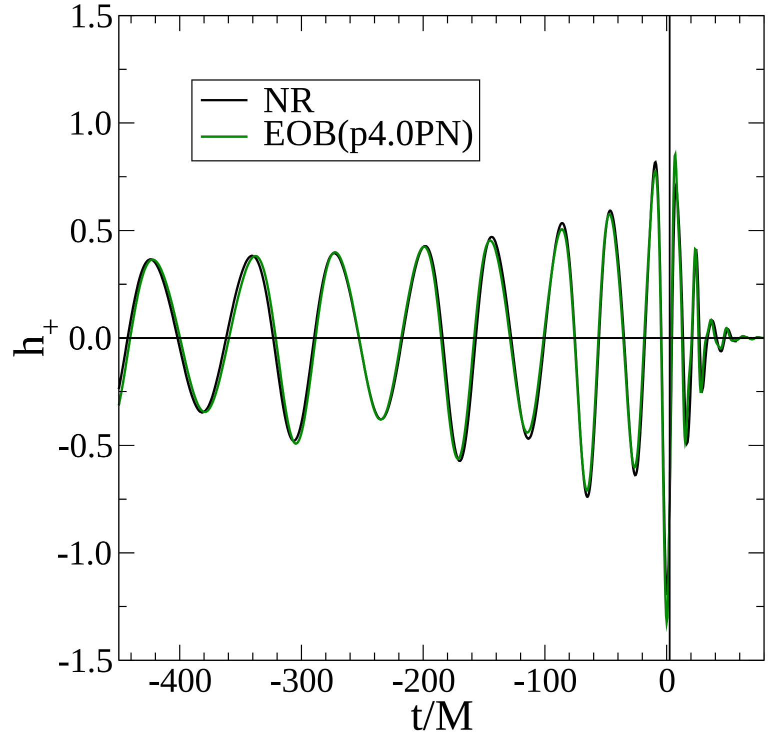




NR and EOB waveforms for unequal-mass binary: *faithfulness*

- **Phase difference in GW cycles of** $\sim 8\%$ [AB, Pan & NASA-Goddard 07]
- **overlap** $\gtrsim 0.98$ **maximizing** *only* **on time-of-arrival and initial phase**

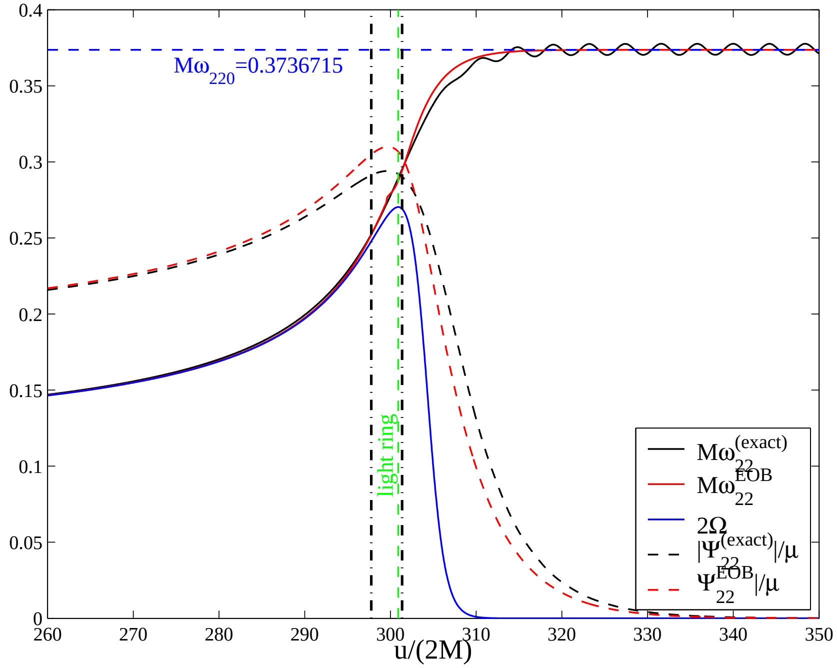


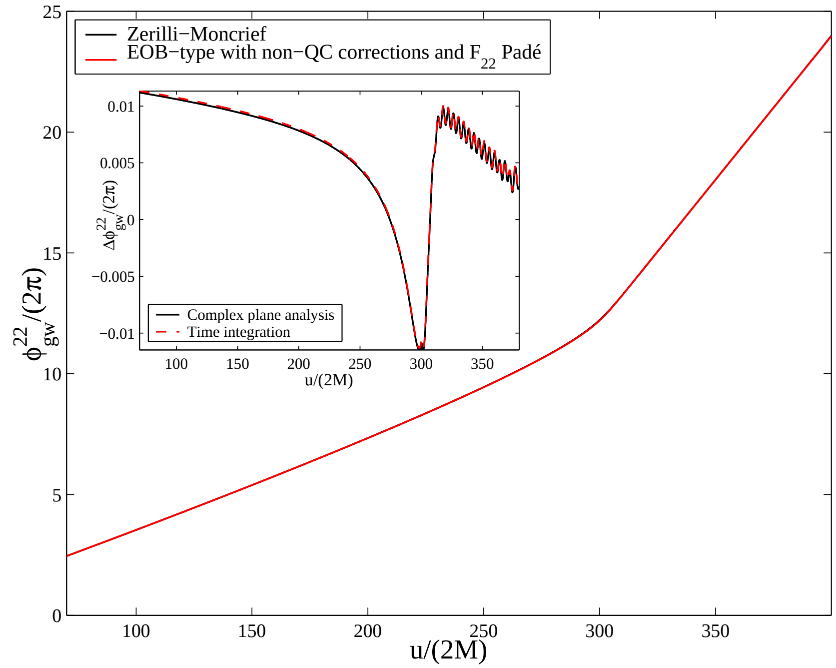


[talk by Ajith on frequency-domain template family for inspiral-merger-ringdown]

Comparison Regge-Wheeler-Zerilli and EOB in the test-mass limit

[Damour, Nagar & Tartaglia 06; Damour & Nagar 07]



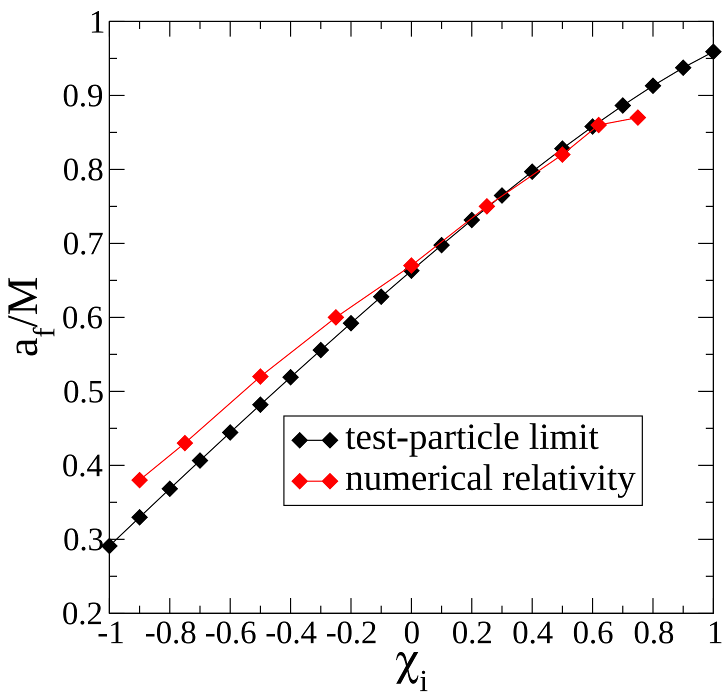


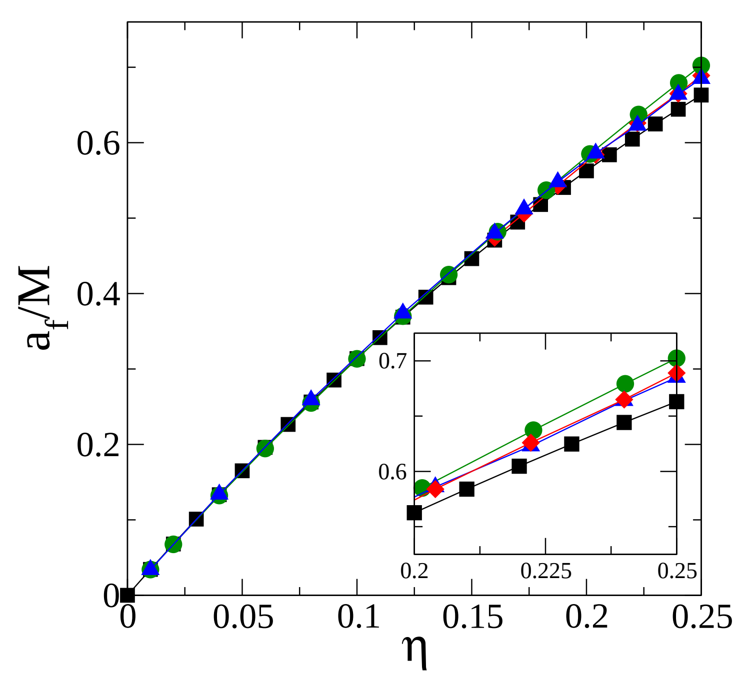
- **Several improvements: resummed higher-order amplitude corrections; deviations from quasi-circular motion; matching inspiral to ringdown on a *comb* instead of a point**

What is the final black hole spin and mass?

[Berti et al. 07; Damour & Nagar 07; AB et al. 07; Pollney et al. 07; Boyle et al. 07; Sperhake et al. 07]

- $\frac{a_f}{M} = \frac{L_{\rm orb}^{\rm ISCO}(a_f,\eta)}{M^2} + \frac{S_1}{M^2} + \frac{S_2}{M^2}$ $\Leftarrow$ **using Kerr spacetime!** [AB, Kidder & Lehner 07]
- **Good estimations also for precessing, spinning binaries**

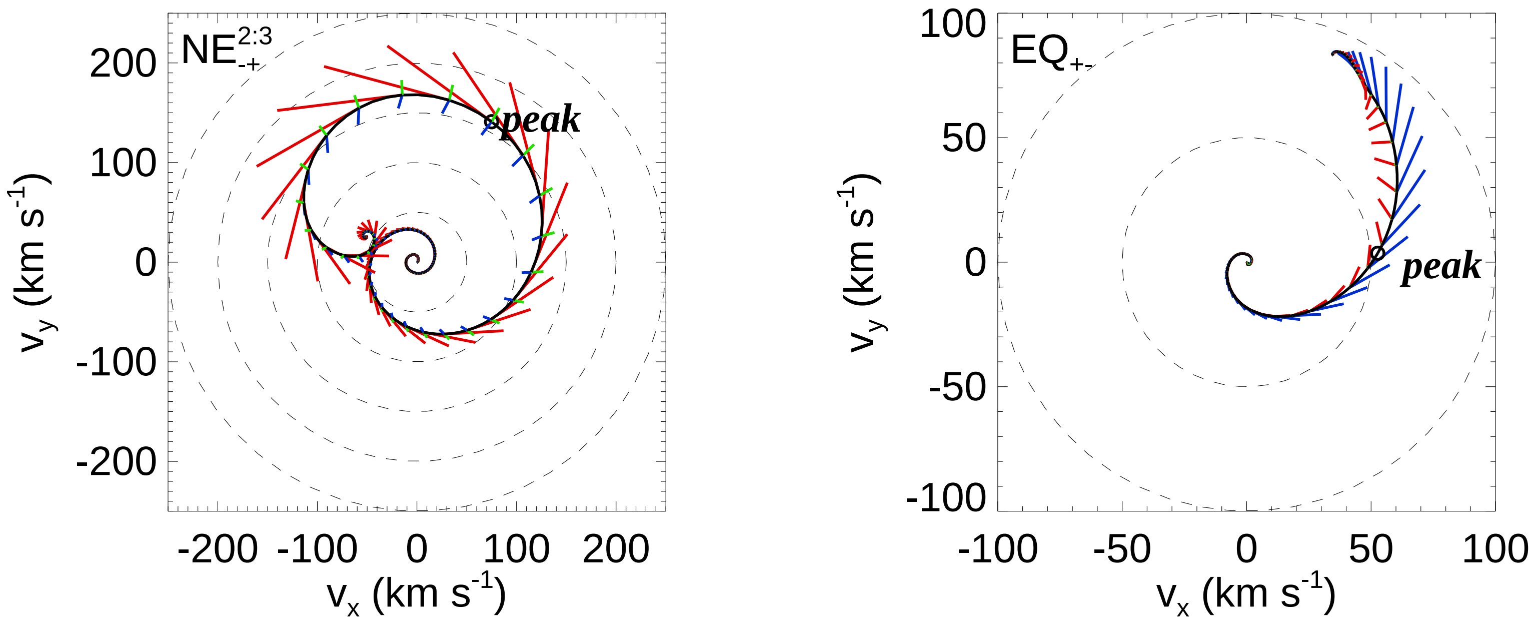




Anatomy of the kick (inspiral–merger–ringdown)

- **Analytic predictions for the kick** [Bekenstein; Fitchett & Detweiler; Kidder; Blanchet et al.] [Damour & Gopakumar 06; Schnittman, AB & NASA-Goddard 07]

$$\mathbf{V}_{\rm kick} \simeq \int \left[\hat{\mathbf{V}} \cdot \frac{d\mathbf{P}}{dt}(I^{22}\, S^{21}) + \hat{\mathbf{V}} \cdot \frac{d\mathbf{P}}{dt}(I^{22}\, I^{33}) + \hat{\mathbf{V}} \cdot \frac{d\mathbf{P}}{dt}(I^{33}\, I^{44}) \right] dt$$



Anatomy of the kick and anti-kick

[Schnittman, AB & NASA-Goddard 07]

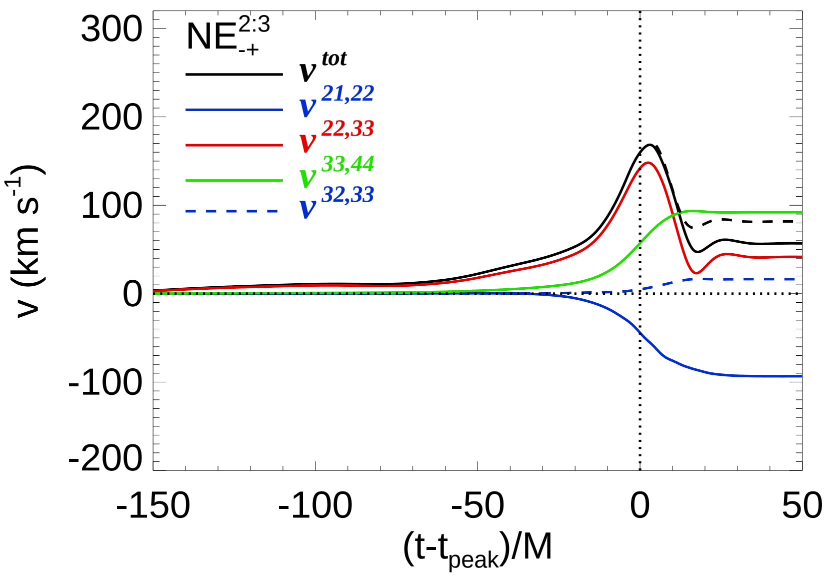
- **Magnitude of anti-kick depends on QNM-frequencies associated to dominant modes**

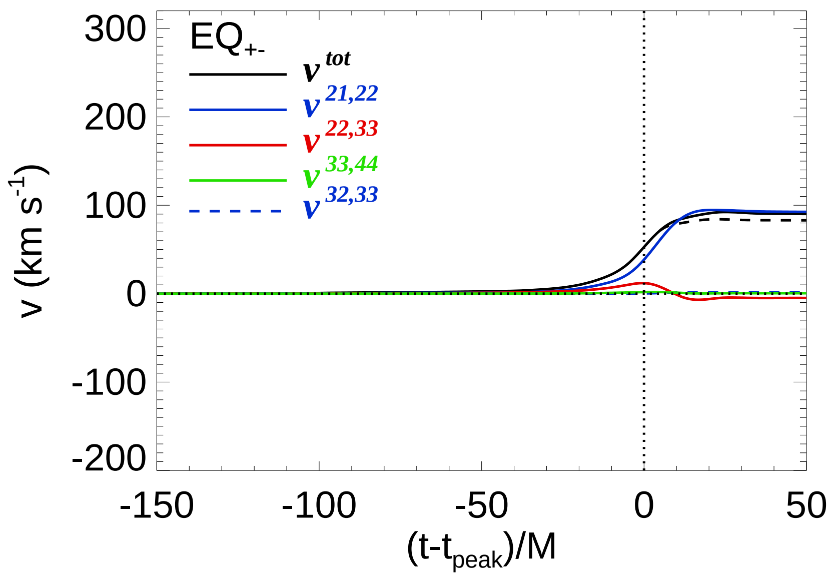
$I^{22}\, I^{33\,*}$: $(\omega_{33}^{\rm QNM} - \omega_{22}^{\rm QNM})$ **is large**

$\Rightarrow$ **spiral back inward**

$I^{22\,*}\, S^{21}$: $(\omega_{21}^{\rm QNM} - \omega_{22}^{\rm QNM})$ **is small**

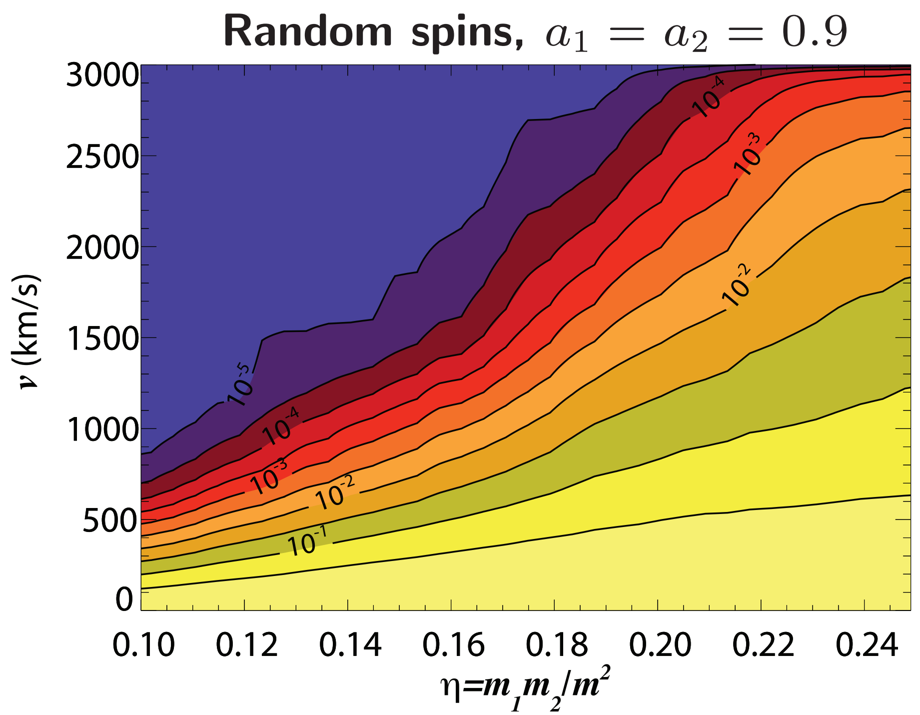
$\Rightarrow$ **drifts off**

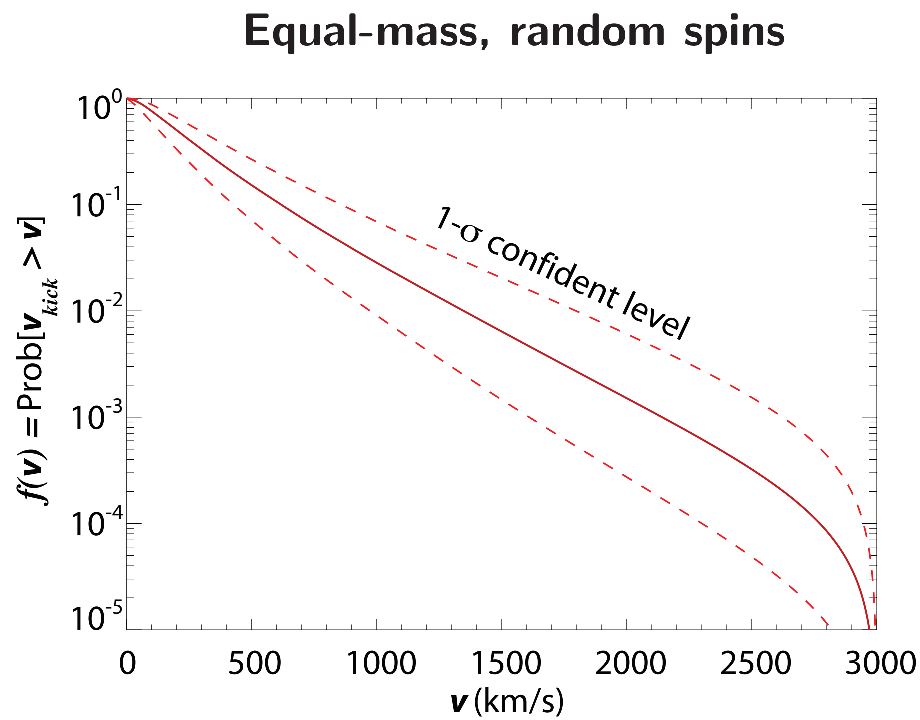




Cumulative probability distribution for recoil velocities using EOB approach

[Schnittman & AB 07]





$$f_{v_{\rm kick}>500} = 0.12^{+0.06}_{-0.05}$$

$$f_{v_{\rm kick}>1000} = 0.027^{+0.021}_{-0.014}$$

Conclusions

- **Intriguing (anticipated)** *simplicity* **of (non-spinning) binary coalescence: details of merger hidden behind the curvature potential barrier.**

- **Consistency between PN calculations through 3PN order and numerical simulations**

- **Several progresses in estimating the final black-hole mass and spin**

- **Guided by NR simulations and by PN theory (at earlier times), notably the EOB model, we have a first example of analytical model for inspiral, merger, and ringdown** *to be further improved* **and extended to longer and accurate simulations.**

- **Gravitational recoil determined mainly by merger-ringdown phases.**

- **Improvement of analytic modeling to reduce uncertainties in Monte Carlo simulations of recoil velocity distribution.**