QUANTUM GRAVITY AND THE INFORMATION LOSS PROBLEM

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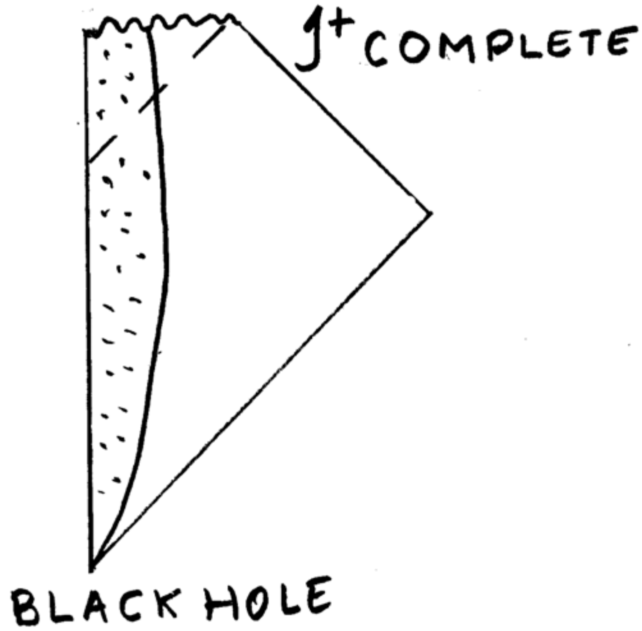
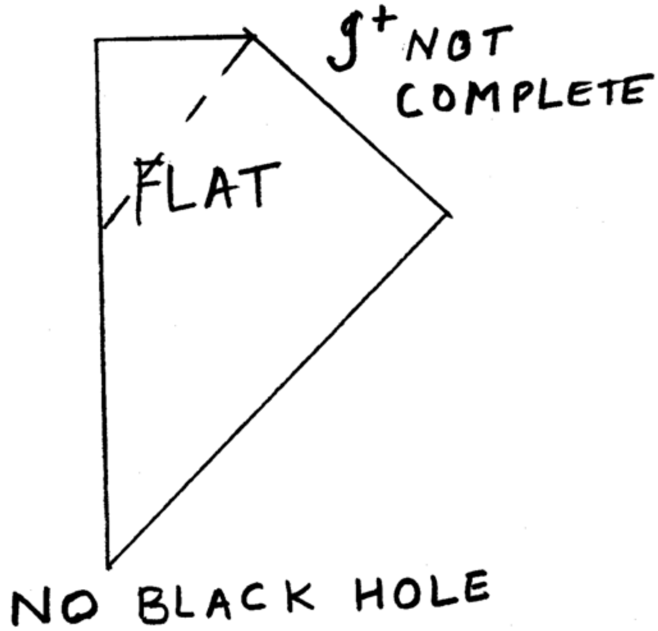


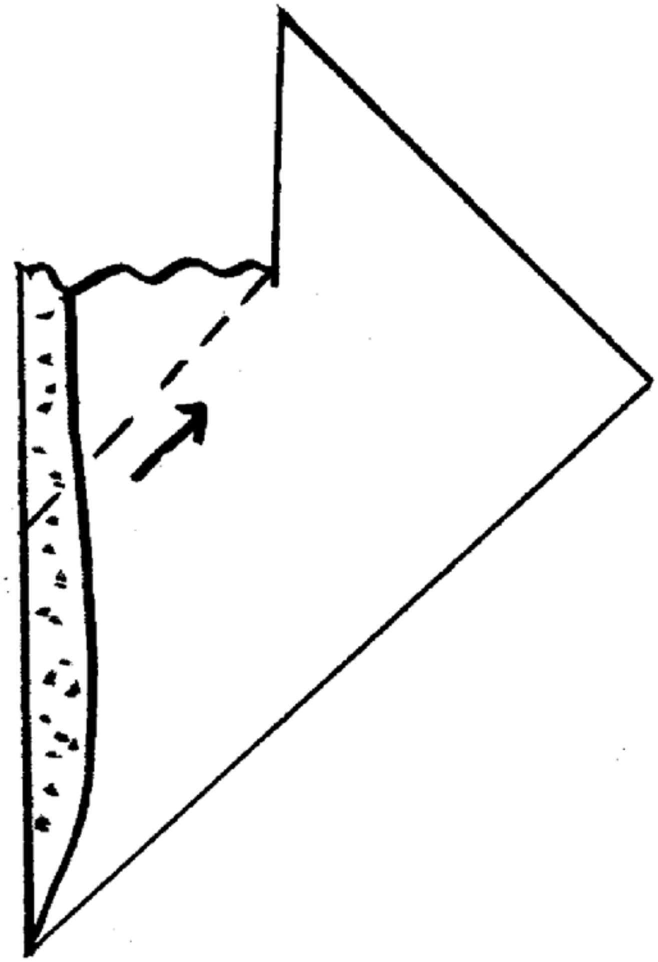
$\mathscr{I}^+$ COMPLETE
BLACK HOLE



$\mathscr{I}^+$ NOT COMPLETE
FLAT
NO BLACK HOLE

Quantum Effects (QFT in CS):

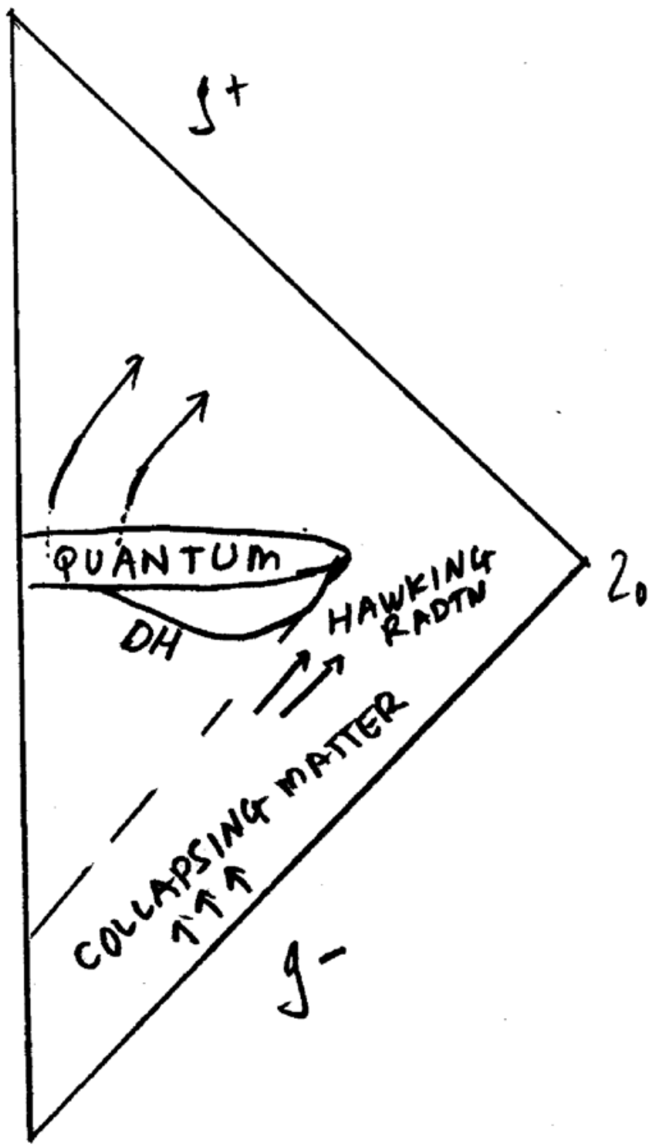
- BH radiates at $kT_H = \frac{m_P}{M} m_P c^2$
- Radtn →Mass Loss → higher temp → more radiatn
- Very slow process:
  $T_{evap} \sim \frac{M}{\dot{M}}, \ \ \dot{M} \sim \sigma \, T_H^4 R_S^2$
  $T_{settling\ down} \sim \frac{R_S}{c}$
  $\frac{T_{settling\ down}}{T_{evap}} \sim \frac{m_P^2}{M^2} << 1$
  ⇒ Quasistatic process



- **Endpt = m_P + Hawking Radiation**
- **Initial matter = pure quantum state**
  ⇒ INFO LOSS.

ALTERNATE VIEW (A-B):

- **Endpt = m_P + radtn + singular boundary**
- **singularity resolved by quantum theory**
  ⇒ Event Horizon not a useful concept (Hajiček)
- **Instead, Dynamical Horizon:**
  - smooth 3d hypersurface foliated by marginally trapped 2-surfaces
  - infalling matter ⇒ splike, area incr.
  - no matter ⇒ null, area const.
  - matter coming out ⇒ timelike, area decr.



- **Info recovered thru correlations of Hawking Radiation with matter on "other side of singularity"**

Brief Digression on Ptcles in QFT:

Ptcle concept is

- Nonlocal: $\hat{\phi}(\mathbf{x},\mathbf{t})$ = sum of plane waves with $\hat{\mathbf{a}}(\mathbf{k}),\hat{\mathbf{a}}^\dagger(\mathbf{k})$ coefficients. $\hat{\mathbf{a}}^\dagger(\mathbf{k})$ creates particle with momentum $\hbar\mathbf{k}$ with wave function $\sim \mathbf{e}^{\mathbf{ikx}-\mathbf{i}\omega\mathbf{t}}$

  $\rightarrow$ very spread out in spacetime.

- Observer Dependent:2 observers $(\mathbf{x},\mathbf{t}),(\mathbf{y},\mathbf{T})$ expand same field oprtr $\hat{\phi}$ in their plane wave bases: $\mathbf{e}^{\mathbf{ikx}-\mathbf{i}\omega\mathbf{t}}$, $\mathbf{e}^{\mathbf{iky}-\mathbf{i}\omega\mathbf{T}}$

  $\Rightarrow$ creation-ann opertrs for the two are different $\hat{\mathbf{a}}(\mathbf{k}) \neq \hat{\mathbf{b}}(\mathbf{k})$

  $\Rightarrow |\mathbf{0_a}\rangle = \Sigma\, |\mathbf{ptcles_b}\rangle$.

CGHS Model:

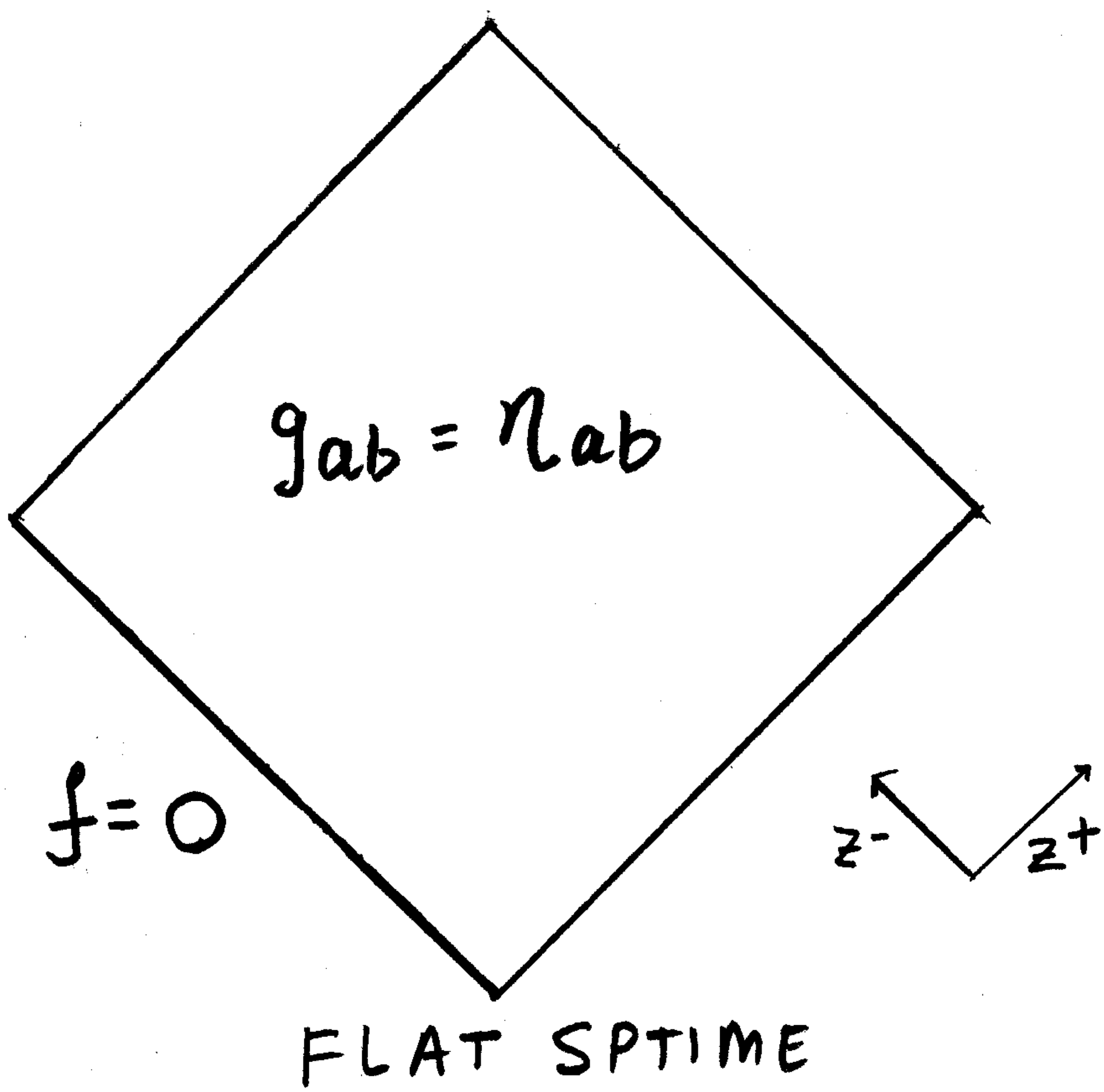
$$\mathbf{S} = \frac{1}{2G}\int d^2x\sqrt{g}e^{-2\phi}[R + 4(\nabla\phi)^2 + 4\kappa^2]$$
$$-\frac{1}{2}\int d^2x\sqrt{g}g^{ab}\nabla_a f\nabla_b f$$

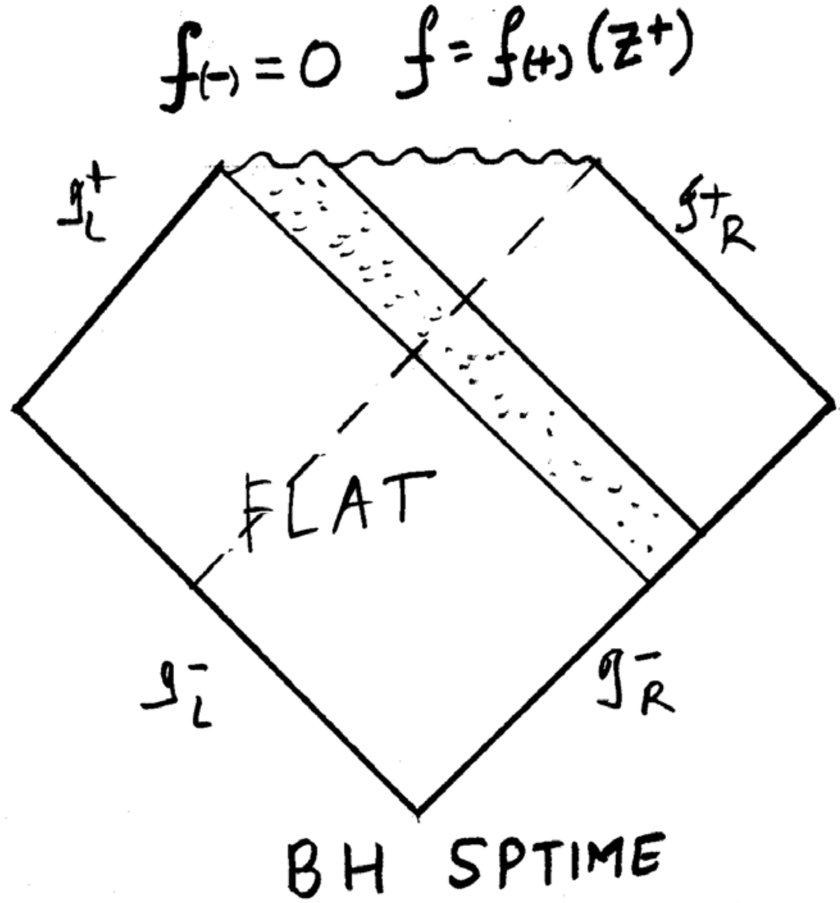
- $[G] = M^{-1}L^{-1}$ $[\kappa] = L^{-1}$
- **2d:** $g^{ab} = \Omega\eta^{ab}$, $\eta \to -(dt)^2 + (dz)^2$, **null coordinates:** $z^{\pm} = t \pm z$
- **Varble Redefn:** $\Phi = e^{-2\phi}$
  $\Theta = \Phi\Omega^{-1}$ $(\Omega = \Phi\Theta^{-1})$
- **Equations of Motion:**
  $\partial_+\partial_- f = 0 \Rightarrow f = f_+(z^+) + f_-(z_-)$
  Evolution eqns:
  $\partial_+\partial_-\Phi + \kappa^2\Theta - \Phi\partial_+\partial_- \ln\Theta = 0$
  $\partial_+\partial_-\Phi + \kappa^2\Theta = 2GT_{+-}$
  $+$ **Boundary Conditions**
- **Can solve for** Φ, Θ **in terms of stress energy of f. Thus, true degrees of freedom** $= f_+(z^+), f_-(z^-)$.

Basic Points:

- **Varbles:dilaton, metric, matter**
- **2d implies cfmal flatness, metric specified by cfml factor**
- **Variable redefns in dilaton-metric sector: Φ, Θ. Cfmal factor = $\Phi\Theta^{-1}$**
- **Matter cnfmally coupled so doesnt see cfmal factor. Hence matter satisfies free wave eqn on flat spacetime.**
- **Remaining equations enable Φ, Θ to be solved in terms of matter stress energy, thus true d.o.f. are parametrised by matter data $f_+(z^+), f_-(z_-)$.**

Solutions:





QFT on BH Sptime (QFT in CS):

- **Calculation a'la Hawking** (Giddings, Nelson) **yields Hawking radiation at $\mathcal{I}^+_{\mathbf{R}}$ with $\mathbf{kT_H} = \kappa\hbar$ indep of mass.**
- **Evaluate $\langle \hat{\mathbf{T}}_{\mathbf{ab}} \rangle$ on BH background.**
  $\langle \hat{\mathbf{T}}_{\mathbf{ab}} \rangle$= classical part $+\hbar$ correction
  (E.g. $\langle \hat{\mathbf{T}}_{+-} \rangle = -(\hbar/48)\mathbf{R}\mathbf{g}_{+-}$)
  $\langle \hat{\mathbf{T}}_{--} \rangle|_{\mathcal{I}^+_{\mathbf{R}}}$ = Hawking flux.

FULL QUANTUM THEORY:

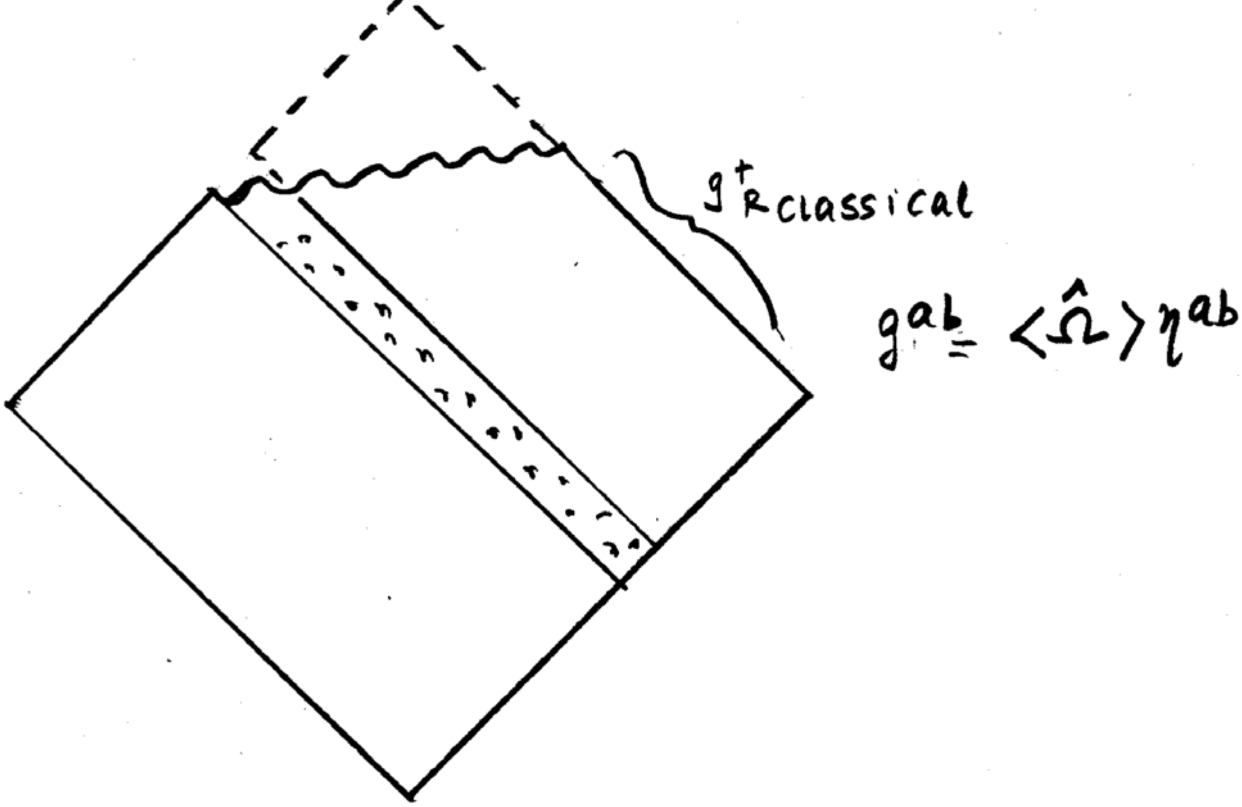
- $\partial_+\partial_-\hat{\mathbf{f}} = \mathbf{0} : \hat{\mathbf{f}} = \hat{\mathbf{f}}_+(\mathbf{z}^+) + \hat{\mathbf{f}}_-(\mathbf{z}_-)$
  $\hat{\mathbf{f}}$ = free scalar field on $\eta_{\mathbf{ab}}$.
  Fock repn:$\mathcal{F}^+ \times \mathcal{F}^-$.
  Arena for Quantum Theory is entire Minkowskian Plane
- Oprtr Eqns for $\hat{\Phi}, \hat{\Theta}$:
  $\partial_+\partial_-\hat{\Phi} + \kappa^{\mathbf{2}}\hat{\Theta} - \hat{\Phi}\partial_+\partial_- \ln \hat{\Theta} = \mathbf{0}$
  $\partial_+\partial_-\hat{\Phi} + \kappa^{\mathbf{2}}\hat{\Theta} = \mathbf{2G}\hat{\mathbf{T}}_{+-}$
  + Oprtr valued Boundary Conditions.
- Open Issue:qft on quantum sp-time, $\hat{\mathbf{T}}_{\mathbf{ab}} = \hat{\mathbf{T}}_{\mathbf{ab}}(\hat{\Phi}\hat{\Theta}^{-\mathbf{1}})$
- Despite this, framework itself allows an analysis of Info Loss Problem.
  NOTE: $\mathcal{F}^+ \times \mathcal{F}^-$ is Hilbert space for gravity-dilaton-matter system, not only for matter.

Info Loss Issue Phrased in Full Quantum Theory Terms:

- **Choose "quantum black hole" state $|\mathbf{f}_+\rangle \times |\mathbf{0}_-\rangle$ - analog of classical data $\mathbf{f} = \mathbf{f}_+(\mathbf{z}^+), \mathbf{f}_- = \mathbf{0}$**
- **Info loss issue takes the form:** ***What happens to $|\mathbf{0}_-\rangle$ part of the state during BH evaporation?***

Trial Solution to Oprtr Eqns:

- Use η_{ab} to define $\hat{T}_{ab}$.Then $\hat{T}_{+-} = 0$, can solve oprtr equations explicitly
- Exp value $\langle \hat{\Omega} \rangle = \langle \hat{\Phi} \hat{\Theta}^{-1} \rangle = \Omega_{classical}$!
- On singularity $\langle \hat{\Phi} \hat{\Theta}^{-1} \rangle = 0$ but $\hat{\Phi}, \hat{\Theta}$ still well defined as operators. Large fluctuations of $\hat{\Omega}$ near classical singularity,
- $\hat{\Phi}, \hat{\Theta}$ well defined on whole Minkowskian plane, even "above" singularity: Quantum Extension of Classical Spacetime.



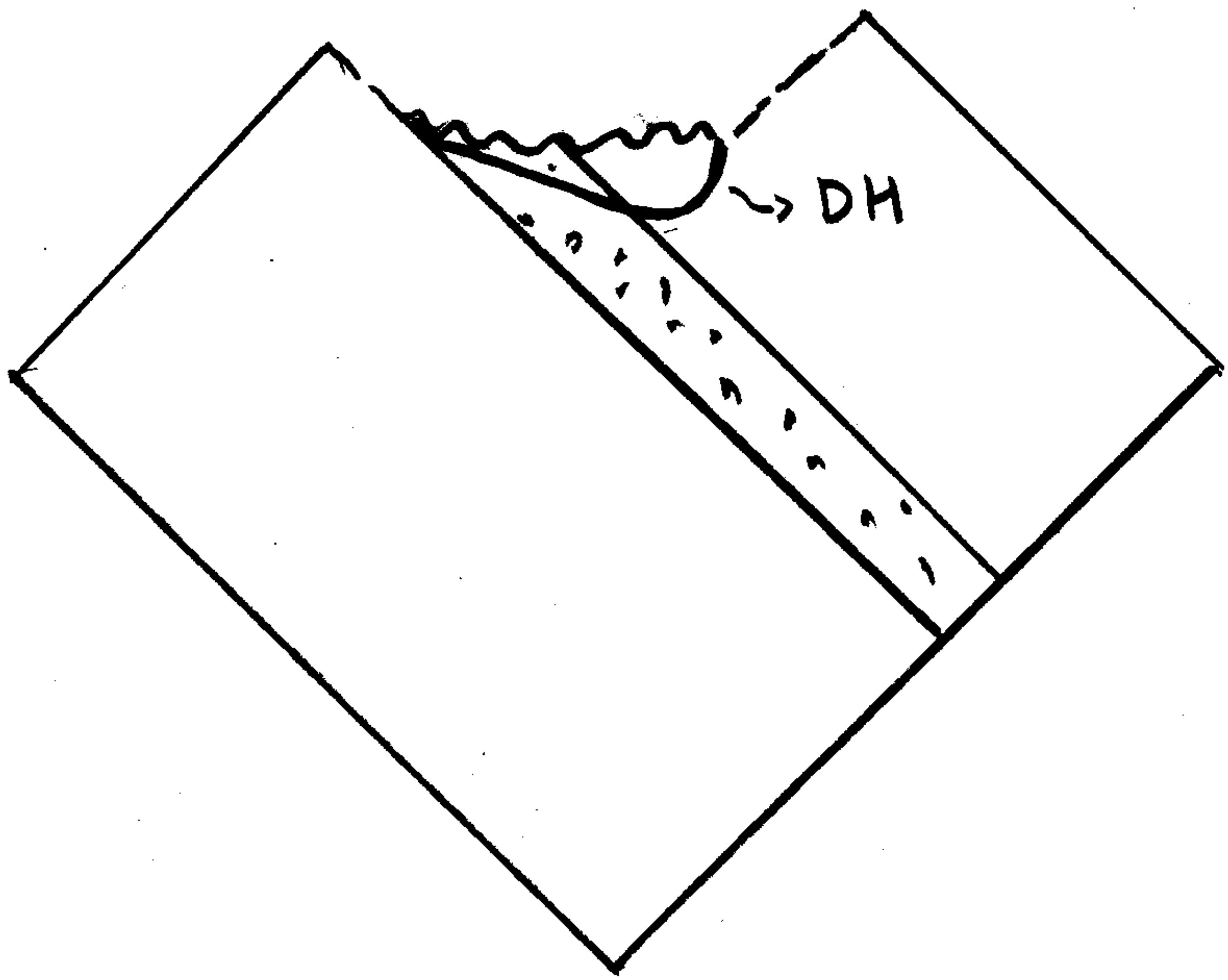
- Hawking Effect: Quantum State of gravity-dilaton-matter system $|f_+\rangle \times |0_-\rangle$. $|0_-\rangle$ interpreted by asymptotic inertial observers in expectation-value- geometry at $\mathcal{I}^+_{\mathrm{Rclassical}}$ as Hawking radiation!

- *But*: No backreaction of this radtn

- Can try to improve on soln to oprtr equations by "bootstrapping" but not useful for info loss issues.

Mean Field Approximation:

- Take exp value of oprtr equations w.r.to $|f_+\rangle \times |0_-\rangle$.
- Neglect fluctuations of gravity-dilaton but not of matter
- Get exact analog of "semiclassical gravity" 4d eqns, "$G_{ab} = 8\pi G\langle\hat{T}_{ab}\rangle$".
- MF eqns for CGHS studied numerically by Piran-Strominger-Lowe, analytically by Susskind-Thorlacius.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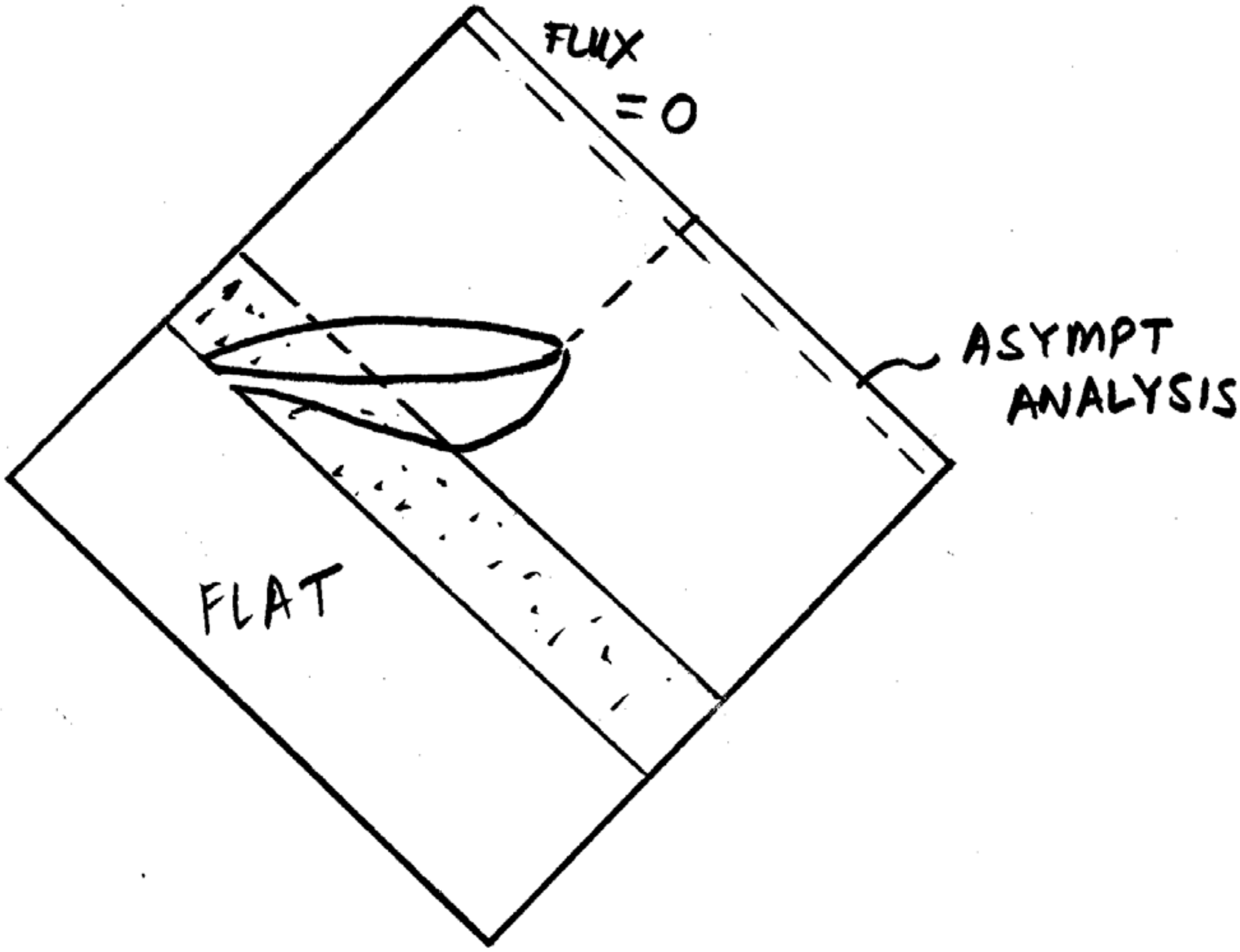
Asymptotic Analysis near $\mathcal{I}^+_R$:

Knowledge of underlying quantum state of CGHS system + MFA eqns near $\mathcal{I}^+_{\mathbf{R}}$ dictate the response of asympt geometry to energy flux at $\mathcal{I}^+_{\mathbf{R}}$. Analysis of eqns implies (almost) uniquely:

- **If Hawking flux smoothly vanishes along $\mathcal{I}^+_{\mathbf{R}}$ then $\mathcal{I}^+_{\mathbf{R}}|_{\mathbf{g_{ab}}}$ *coincides* with $\mathcal{I}^+_{\mathbf{R}}|_{\eta_{\mathbf{ab}}}$**

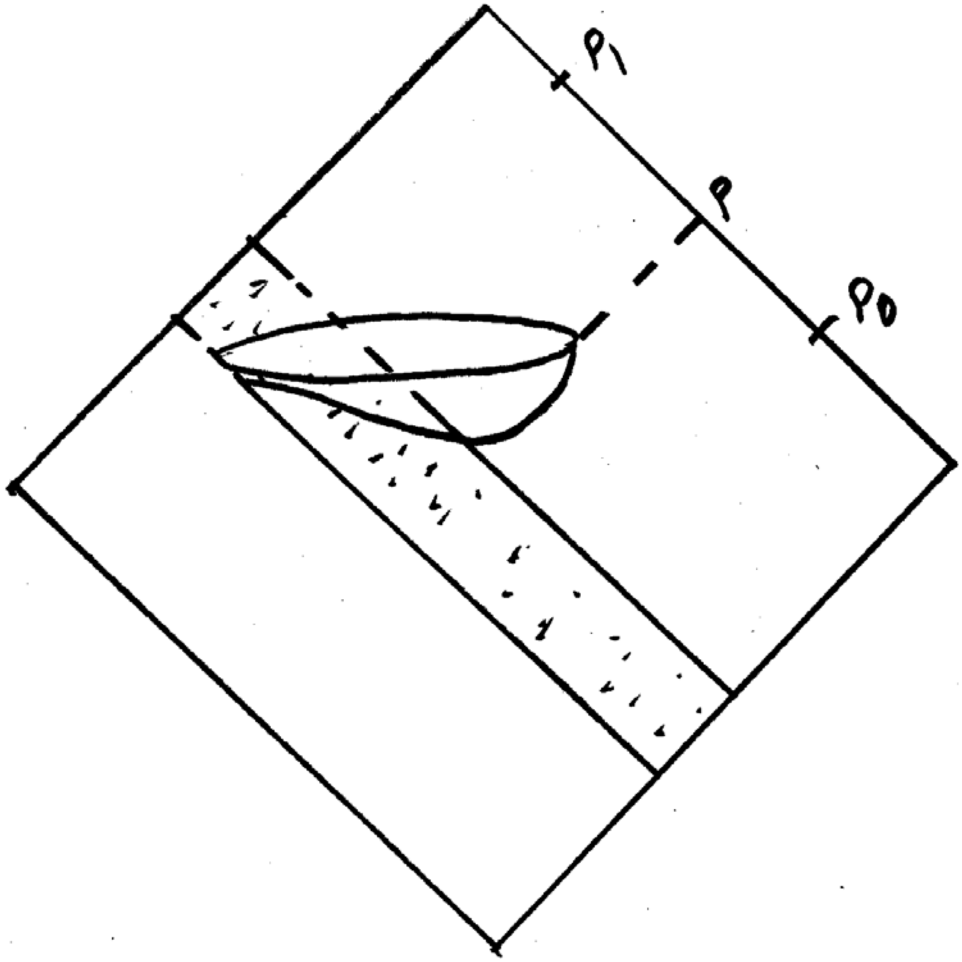
- **Let $\mathbf{g_{ab}}|_{\mathcal{I}^+_{\mathbf{R}}} = \mathbf{dy}^+_{(\mathbf{a}}\mathbf{dy}^-_{\mathbf{b})}$ with $\mathbf{y}^- = \mathbf{y}^-(\mathbf{z}^-)$. Then $|\mathbf{0}_-\rangle$ is a normalized, pure state in Hilbert space of y observers $\Rightarrow$ No Info Loss!.**

FINAL PICTURE:



- **Interior to past of MFA singularity: MFA numerics.**
- **Near $\mathcal{I}^+_{\mathbf{R}}$: Asymptotic Analysis**
- **Conceptual underpinnings provided by oprtr equations suggest:**
  - singularity resolution
  - extension of classical sptime

INFO LOSS PROBLEM:



- $|0_-\rangle$ **is pure state in Hilbert space of y observers**
- **No energy flux beyond P, no remnant with large number of internal states. Nevertheless:** $\langle \hat{f}(P_1)\hat{f}(P_0)\rangle \neq 0$ **- Correlations!**

- **Entropy:**

  - **Defn involves "tracing over ptcle modes to future of P"**
  - **Ptcle modes spread out, have "tails" about P**
  - **With such modes $|0_-\rangle \sim \Sigma_{m=0}^{\infty} |2m \text{ ptcles}\rangle_{\mathcal{I}_R^+}$.**

  Trace over ptcles appearing after P to get $\hat{\rho}$

  - $S = -\mathrm{Tr}\hat{\rho}\ln\hat{\rho}$, $S \to 0$ in remote past, increases to future, then decreases to 0 beyond P. Info in correlations between pairs of ptcles emitted at different times

  NOTE: MFA requires large N, can be taken care of.

SUMMARY:

Non-pert quantization + MFA numerics + asympt analysis point to unitary pic of BH evaporation with key features:

- **Singularity Resolution.**
- **Extension of Classical Sptime.**
- **No such thing as classically empty sptime.**

CGHS wrk in collaboration with Abhay Ashtekar and Victor Taveras.