Beyond the Einstein-Hilbert Action

I. Introduction : Why beyond GR

II. Towards the EH action

III. The case with Weyl curvature terms

IV. Summary

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I. Introduction : Why beyond GR

Einstein theory (General Relativity)

simple and beautiful theory

gravity : geometrical object

confirmed by several experiments and observation

three famous tests in the solar system

the light bending, the perihelion shift, Shapiro delay

binary pulsars

So far NO DOUBT from observation and experiments

Some discussions about "gravity" from a theoretical view point

⇓

We may consider gravitational theories beyond the EH action

It might solve some fundamental problems.

Dark Energy

singularity

(1) Extension of GR

- unification of gravity and e.m. field

  Kaluza-Klein theory ➡ a scalar field in 4D

- motivated by "Mach principle"

  Scalar-tensor theory of gravity (Brans-Dicke theory)

- violation of Lorentz invariance

  Einstein-Aether theory (vector-tensor theory) T. Jacobson

- MOND (dark matter)

  TeVeS J. Bekenstein

➡ universally coupled scalar or vector field as well as tensor ($g_{\mu\nu}$)

(2) Effective action in QFT

Quantization of matter fields

→ divergence → finite theory

renormalization

counter terms

a cosmological constant Λ
The EH action R

+

Non-minimal coupling term $\frac{\xi}{2}\phi^2 R$

Higher curvature terms $R^2, R_{AB}^2, R_{ABCD}^2$

(3) Effective model of unified theory

(i) Supergravity in higher dimensions (e.g. N=1, D=11 SG)

Higher-dimensional KK theories

dilaton, compactification (moduli)

⇒ Scalar-tensor type theory

(ii) Superstring in 10D (or M theory in 11D)

Supergravity in the FT limit ⇒ Scalar-tensor type theory

One-loop corrections ⇒ higher curvature terms

(iii) brane world

a singular hypersurface in a bulk

⇒ non-trivial contribution

As for a model of **"Beyond the EH action"**

We may discuss a gravitational theory with the action

$$S = \int d^D x \sqrt{-g} F(g^{\mu\nu}, R_{\mu\nu}, C^{\mu}{}_{\nu\rho\sigma}, \psi)$$

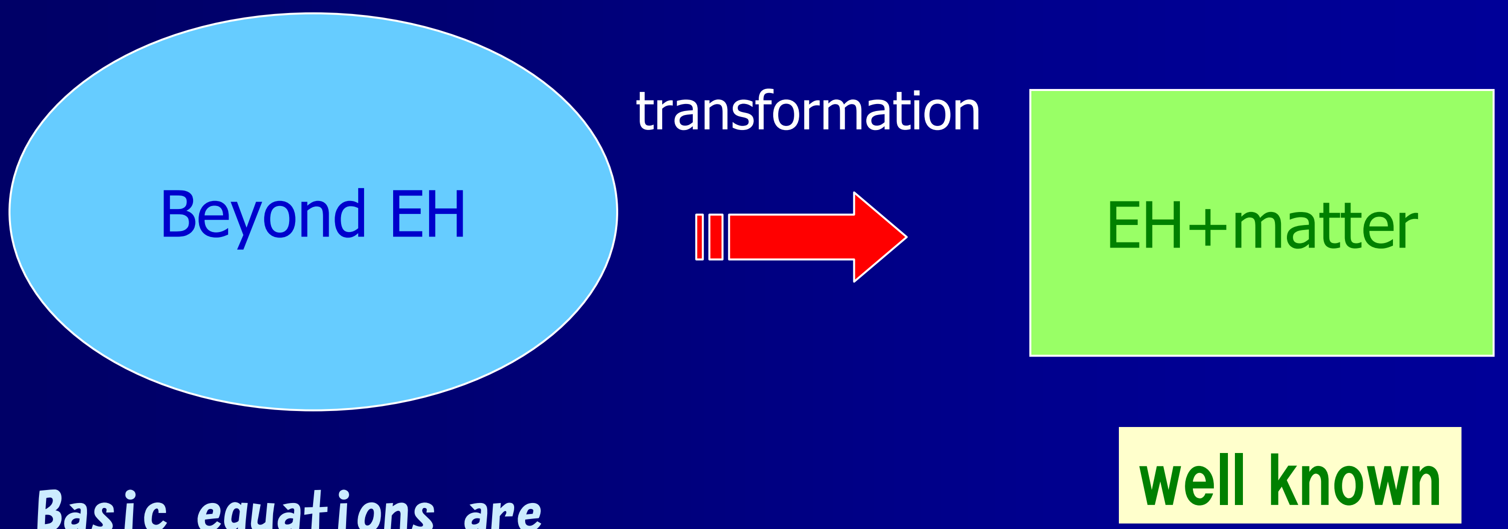
$F(\cdots)$: An arbitrary function

ψ : matter fields including scalar fields

It may show some interesting properties of gravity
But it is too complicated to analyze it

II. Towards the EH action :

If we can find an equivalent gravitational theory only with the EH action by some transformation, it makes our discussion simpler.



Basic equations are very complicated

1. A scalar-tensor type theory

KM PRD 39 (1989) 3159

$$S = \int d^D x \sqrt{-g} \left[f(\phi) R - \frac{\epsilon_\phi}{2} (\nabla \phi)^2 - V(\phi) \right]$$

$$\hat{g}_{\mu\nu} = e^{2\omega} g_{\mu\nu}$$

a conformal transformation

$$\omega = \frac{1}{D-2} \ln(2\kappa^2 |f(\phi)|)$$

$$S = \int d^D x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} R(\hat{g}) - \frac{1}{2} (\nabla \sigma)^2 - U(\sigma) \right]$$

$$\kappa\sigma = \int d\phi \left[\frac{\epsilon_\phi (D-2) f(\phi) + 2(D-1)(f'(\phi))^2}{2(D-2) f^2(\phi)} \right]^{1/2}$$

$$U(\sigma) = \epsilon_f [2\kappa^2 |f(\phi)|]^{-D/(D-2)}$$

2. F(R, ϕ) theory

$$S=\int d^Dx\sqrt{-g}\left[F(R,\phi)-\frac{\epsilon_\phi}{2}(\nabla\phi)^2\right]$$

KM PRD 39 (1989) 3159

higher derivarives

Jakubiec, Kijowski , GRG 19 (1987) 719 ;
Magnano, Ferraris, Francaviglia, GRG 19 (1987) 465 ;
Ferraris, Francaviglia, Magnano, CQG. 5 (1988) L95

$$\hat{g}_{\mu\nu}=e^{2\omega}g_{\mu\nu}$$ a conformal transformation

$$\omega=\frac{1}{D-2}\ln\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right] \qquad \kappa\sigma=\sqrt{\frac{D-1}{D-2}}\ln\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right]$$

"new degree of freedom"

$$S=\int d^Dx\sqrt{-\hat{g}}\Big[\frac{1}{2\kappa^2}R(\hat{g})-\frac{1}{2}(\hat{\nabla}\sigma)^2 -\frac{\epsilon_\phi\epsilon_F}{2}e^{-\sqrt{\frac{D-2}{D-1}}\kappa\sigma}(\nabla\phi)^2-U(\phi,\sigma)\Big]$$

$$U(\phi,\sigma)=\epsilon_F\left[2\kappa^2\left|\frac{\partial F}{\partial R}\right|\right]^{-D/(D-2)}\left(R\frac{\partial F}{\partial R}-F(R)\right)$$

A simple example

KM PRD 37 (1988) 858

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,\left[R+\alpha R^2\right]$$

: Starobinski type inflation

It contains higher derivarives

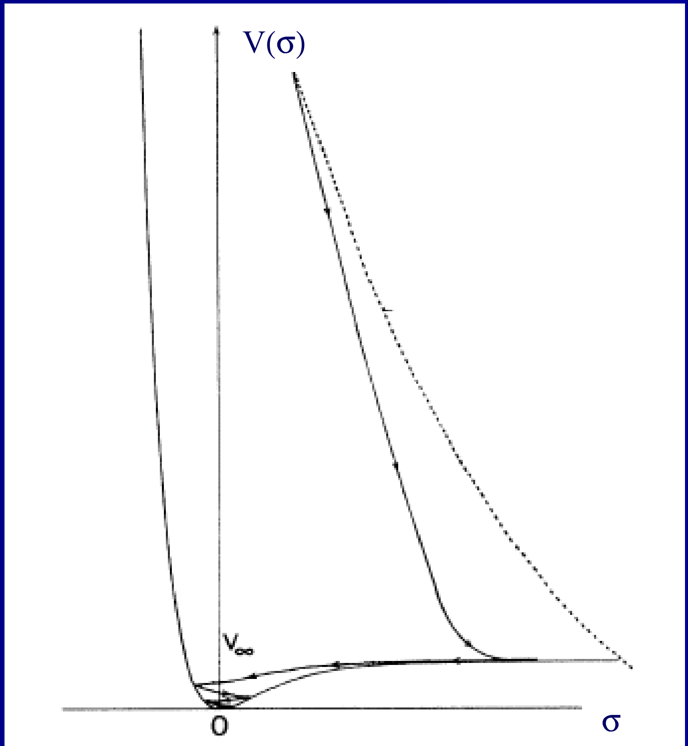
$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}(\nabla\sigma)^2-U(\sigma)\right]$$

GR + a scalar field with a potential U(σ)

$$\kappa\sigma=\sqrt{\frac{3}{2}}\ln(1+2\alpha R)$$

$$U(\sigma)=\frac{1}{8\alpha}\left(1-e^{\sqrt{\frac{3}{2}}\kappa\sigma}\right)^2$$

It is easy to judge
whether inflation occurs or not



3. F($R_{\mu\nu}$) theory

Jakubiec, Kijowski , GRG 19 (1987) 719 ;
Magnano, Ferraris, Francaviglia, GRG 19 (1987) 465 ;
Ferraris, Francaviglia, Magnano, CQG. 5 (1988) L95

$$S = \int d^4x \sqrt{-g} F(g^{\mu\nu}, R_{\mu\nu})$$

$$\sqrt{-q}\, q^{\mu\nu} = 2\kappa^2 \times \sqrt{-g} \frac{\partial F}{\partial R_{\mu\nu}}$$

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-q} \Big[R(q, \partial q, \partial^2 q) + q^{\mu\nu} ({C^\rho}_{\rho\sigma} {C^\sigma}_{\mu\nu} - {C^\rho}_{\mu\sigma} {C^\sigma}_{\rho\nu})$$

$$- q^{\mu\nu} \mathcal{R}_{\mu\nu}(g, q) + \frac{\sqrt{-g}}{\sqrt{-q}} F(\mathcal{R}_{\mu\nu}(g, q), g^{\alpha\beta}) \Big] + S_{\mathrm{matter}}(g^{\alpha\beta}, \psi)$$

$${C^\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\nabla^{(q)}_\mu g_{\nu\sigma} + \nabla^{(q)}_\nu g_{\mu\sigma} - \nabla^{(q)}_\sigma g_{\mu\nu} \right)$$

$$R_{\mu\nu} = \mathcal{R}_{\mu\nu}(g^{\alpha\beta}, q^{\gamma\delta})$$

The EH gravitational action + spin 2 field ($g^{\mu\nu}$) + other matter fields

4. Palatini formalism

GR

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}g^{\mu\nu}R_{\mu\nu}(\Gamma) + S_{\rm matter}$$

metric $g^{\alpha\beta}$, connection ${\Gamma^\alpha}_{\beta\gamma}$: independent

Those variations give the Einstein equations

higher curvature gravitational theories

Allemandi, Borowiec, Francaviglia, PRD 70 (2004) 103503

Palatini formalism is not equivalent to that by the conventional metric variation

$$\nabla_\alpha^{(\Gamma)}(\sqrt{-g}f'(R)g^{\mu\nu}) = 0 \qquad f(R) \equiv 2\kappa^2 F(R)$$

${\Gamma^\alpha}_{\beta\gamma}$ is the Levi-Civita connection of $h_{\mu\nu}$

$$\sqrt{-h}h^{\mu\nu} = \sqrt{-g}f'(R)g^{\mu\nu}$$

F(R) theory in vacuum

metric variation: GR + a scalar field with a potential U(σ)

Palatini formalism: GR + a cosmological constant

5. Black hole entropy

GR

Bekenstein-Hawking entropy formula

$$S_{BH} = \frac{A}{4G}$$

Gravitational theory with higher curvature terms

$$S = \int d^4x \sqrt{-g}\, F(g^{\mu\nu}, R^{\mu}{}_{\nu\rho\sigma})$$

Wald's entropy formula $S_{BH}^{[\mathrm{W}]}$ is not given by BH formula

Wald, PRD **48** (1993) 3427
Iyer, Wald, PRD **50** (1994) 846

Jacobson, Kang, Myers,
PRD **49** (1994) 6587;
PRD **52** (1995) 3518

Koga, KM, PRD, 58 (1998) 064020

$$S = \int d^4x \sqrt{-g}\, F(g^{\mu\nu}, R_{\mu\nu})$$

$$\sqrt{-q}q^{\mu\nu} = 2\kappa^2 \times \sqrt{-g}\frac{\partial F}{\partial R_{\mu\nu}}$$

q -frame

(Einstein frame)

$$S_{BH}^{[\mathrm{W}]} = \frac{A_q}{4G}$$

A_q : area of horizon in *q* -frame

Cai. Chen, KM, Ohta, Pang, in preparation

$$S = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\Big[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{\alpha\phi}F^2$$

$$+e^{\alpha\phi}\left(aR^2 - bR_{AB}^2 + cR_{ABCD}^2\right)\Big]$$

near horizon $AdS_2 \times S^{D-2}$

$$ds^2 = v_1\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + v_2 d\Omega_{D-2}^2$$

$$R = -\frac{2}{v_1} + \frac{(D-2)(D-3)}{v_2} \qquad R_{AB}^2 = \frac{2}{v_1^2} + \frac{(D-2)(D-3)^2}{v_2^2}$$

$$R_{ABCD}^2 = \frac{4}{v_1^2} + \frac{2(D-2)(D-3)}{v_2^2}$$

$$S_{BH}^{[\mathrm{W}]} = \frac{D(D-3)b - 2(D^2-5D+8)c}{2[(D-3)b-2c]} \times \frac{A}{4G}$$

$$R^2_{ABCD} \approx \frac{2}{D^2(D-3)}\Big[-(D-4)^2R^2 + D(D^2-5D+8)R^2_{AB} -2(D-4)R\sqrt{2(D-2)(DR^2_{AB}-R^2)}\Big]$$

$$S \simeq \frac{1}{16\pi G}\int d^Dx\sqrt{-g}\Big[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{\alpha\phi}F^2 + e^{\alpha\phi}\left(\tilde{a}R^2 - \tilde{b}R^2_{AB} - \tilde{d}R\sqrt{2(D-2)(DR^2_{AB}-R^2)}\right)\Big]$$

$$\tilde{a} = a - \frac{2(D-4)^2}{D^2(D-3)}c\,, \qquad \tilde{b} = b - \frac{2(D^2-5D+8)}{D(D-3)}c\,, \qquad \tilde{d} = \frac{4(D-4)}{D^2(D-3)}c$$

$$\sqrt{-q}q^{AB} = \sqrt{-g}\left[\left(1 - \tilde{d}e^{\alpha\phi}\sqrt{2D(D-2)R^2_{CD}}\right)g^{AB} - 2\tilde{b}e^{\alpha\phi}R^{AB}\right]$$

field redefinition

$$\Rightarrow \quad A_q = \frac{D(D-3)b - 2(D^2-5D+8)c}{2[(D-3)b-2c]}A_g$$

$$S_{BH}^{[\mathrm{W}]} = \frac{A_q}{4G}$$

III. The case with Weyl curvature terms

String theory with higher-order correction terms

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2\kappa^2} R + c_1 \alpha' e^{-2\Phi} L_2 \right.$$
$$\left. + c_2 \alpha'^2 e^{-4\Phi} L_3 + c_3 \alpha'^3 e^{-6\Phi} L_4 + \cdots \right]$$

$$L_2 = R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$: Gauss-Bonnet term

$$L_m = (\mathrm{Lovelock})_m + (\text{higher derivative terms}) \ (m = 3, 4, \cdots)$$

Bento, Bertolami, Phys. Lett. B 368 (1996) 198

theories	c_1	c_2	c_3
bosonic string	$\frac{1}{4}$	$\frac{1}{48}$	$\frac{1}{8}$
heterotic string	$\frac{1}{8}$	0	$\frac{1}{8}$
type II string	0	0	$\frac{1}{8}$

Type II string (or M theory)

The effective action

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g}\, (R + \alpha_4 E_8 + \gamma J_0)$$

$$E_8 = -\frac{1}{2^4 \times (D-8)!} \epsilon^{\alpha_1 \alpha_2 \cdots \alpha_{D-8} \rho_1 \sigma_1 \cdots \rho_4 \sigma_4} \epsilon_{\alpha_1 \alpha_2 \cdots \alpha_{D-8} \mu_1 \nu_1 \cdots \mu_4 \nu_4}$$

$$\times R^{\mu_2 \nu_2}{}_{\rho_2 \sigma_2} R^{\mu_3 \nu_3}{}_{\rho_3 \sigma_3} R^{\mu_4 \nu_4}{}_{\rho_4 \sigma_4}$$

$$J_0 = C^{\lambda\mu\nu\kappa} C_{\alpha\mu\nu\beta} C_\lambda{}^{\alpha\rho\sigma} C^\beta{}_{\rho\sigma\kappa} + \frac{1}{2} C^{\lambda\kappa\mu\nu} C_{\alpha\beta\mu\nu} C_\lambda{}^{\rho\sigma\alpha} C^\beta{}_{\rho\sigma\kappa}$$

$$\alpha_4 = \frac{\kappa_{11}^2\; T_2}{3^2 \times 2^9 \times (2\pi)^4} \qquad \gamma = \frac{\kappa_{11}^2\; T_2}{3 \times 2^4 \times (2\pi)^4}$$

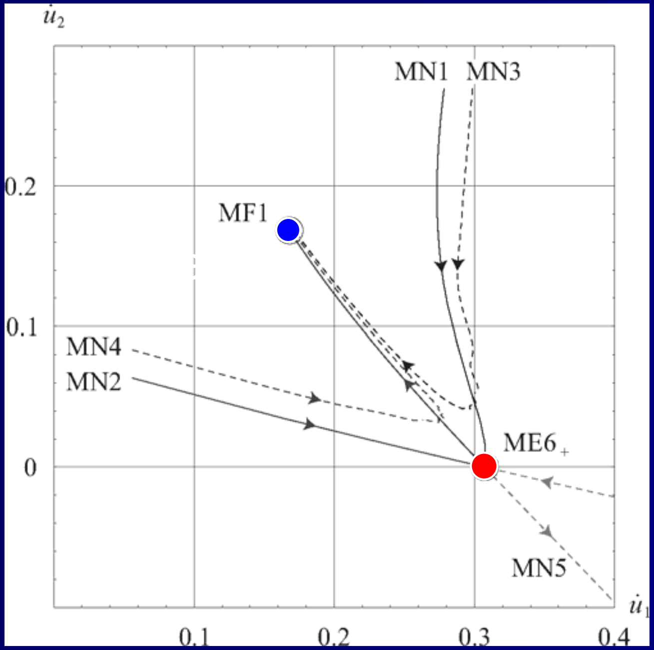
$$T_2 = (2\pi^2/\kappa_{11}^2)^{1/3}$$ membrane tension

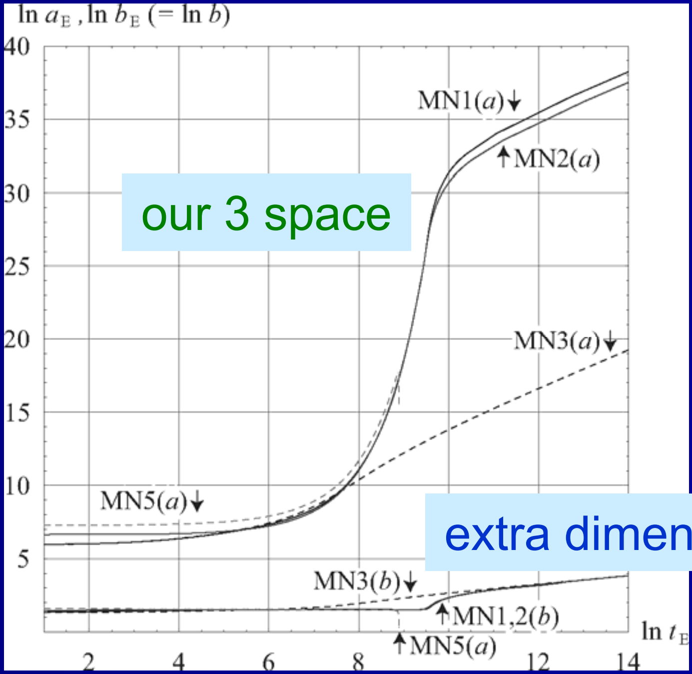
Cosmology with higher curvature

KM, N. Ohta, PLB (04), PRD (05)
K. Akune, KM, N. Ohta, PRD (06)

$$ds_{11}^2 = -dt^2 + a^2 \sum_{i=1}^{3} (dx^i)^2 + b^2 \sum_{a=5}^{11} (dy^a)^2$$

de Sitter : trangent attractor





our 3 space

extra dimensions

inflationary phase

- reheating ?
- graceful exit ?

Black Hole ?

preliminary   with N. Ohta, Y. Sasagawa

$$ds_D^2 = -f(r)e^{2\delta(r)}dt^2 + \frac{1}{f(r)}dr^2 + r^2 h_{ij}dx^i dx^j$$

maximally symmetric

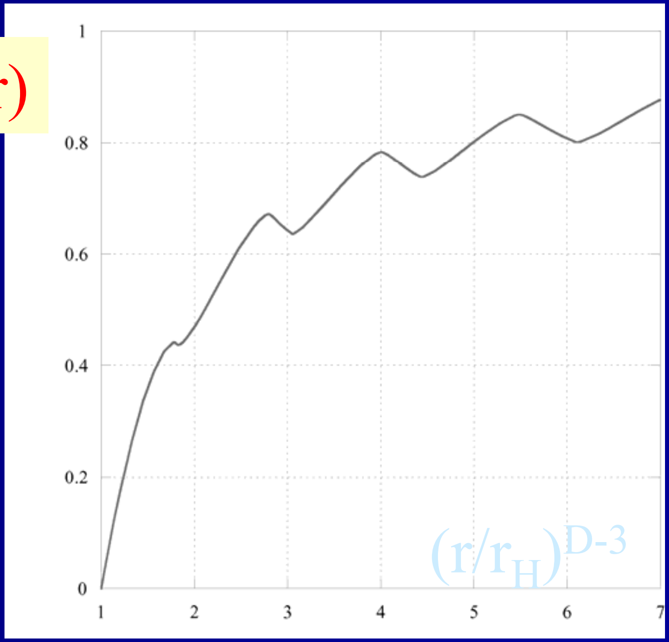
numerical solution

- regular horizon
- ~~asymptotically flat (or AdS)~~

oscillation

decays slower than mass term M/r^{D-3}

f(r)



A. Sen: hetrotic string with HD correction, JHEP (05)
B. V. Hubeny, A. Maloney, M. Rangamani: 4D with HD, JHEP(05)
A. Castro, J.L. Davis, P. Kraus, F. Larsen: 5D SG (07)

Sen: remove it by field redefinition ?

IV. Summary

We discuss gravitational theories beyond the EH action

(1) Why we consider "beyond the EH action"

Such theories might solve some fundamental problems.

(2) How to discuss such theories

Towards the EH action

(3) The case with Weyl curvature terms

Better approach ?