Assignment Set 0

This is a kind of "self–test" based on standard Electromagnetic Theory Courses. Go through the questions, mentally checking whether you know the answer ; when in doubt, try working out the answer explicitly. Some questions require knowledge of a certain "standard" formula. You are, of course, expected to look up these formula in the usual textbooks.

If you find majority of these questions nontrivial please let me know.

You need not turn in the solutions to these problems.

Problem 1. If $\mathbf{r}$ is a vector from the origin to the point (x, y, z). Show that $(1)\ \nabla \times \mathbf{r} = 0$ and, except at $r = 0$, $(2)\ \nabla \cdot (\mathbf{r}/r^3) = 0$, $(3)\ \nabla \times (\mathbf{r}/r^3) = 0$, $(4)\ \nabla(1/r) = -\mathbf{r}/r^3$. From (1) we know that $\mathbf{r}$ can be written as $\mathbf{r} = \nabla\phi$. What is ϕ ?

Problem 2. Given 3-dimensional vectors $\mathbf{A}$ and $\mathbf{B}$ we can construct a 'cross product' vector $\mathbf{C} = \mathbf{A} \times \mathbf{B}$. Can this be done in general, i.e. when $\mathbf{A}$ and $\mathbf{B}$ are n-dimensional vectors ?

Problem 3. Make a rough sketch of vector files $\mathbf{A}(\mathbf{x})$, which satisfy (i) $\nabla \cdot \mathbf{A} = 0,\ \nabla \times \mathbf{A} = 0$; (ii)$\nabla \cdot \mathbf{A} = 0,\ \nabla \times \mathbf{A} \neq 0$; (iii)$\nabla \cdot \mathbf{A} \neq 0,\ \nabla \times \mathbf{A} = 0$; (iv) $\nabla \cdot \mathbf{A} \neq 0,\ \nabla \times \mathbf{A} \neq 0$;

Problem 4. An irrigation canal has straight parallel sides 2b metres apart. At the surface of the water in the canal, the velocity of the water is greatest in the mid-stream, decreasing to zero at each edge. Suppose that the flow happens to be such that the surface velocity is approximately given by

$$v = v_0(1 - y^2/b^2)$$

where y is the distance from the centre and $\mathbf{v}$ is everywhere directed downstream. A chip of wood floating on the water half-way between the centre and one side is observed to rotate as it drifts downstream. Explain this. How far downstream will the chip drift while rotating through 360° ?

Problem 5.(a) The divergence of a vector $\mathbf{F}$ is a scalar, as we know. Suppose we try to define a vector, different from curl, in this way :

$$\mathbf{G} = \hat{\mathbf{x}}\frac{\partial F_x}{\partial x} + \hat{\mathbf{y}}\frac{\partial F_y}{\partial y} + \hat{\mathbf{z}}\frac{\partial F_z}{\partial z}$$

where $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ are unit vectors in the respective directions. Can we certify that this creature is a vector, or should we call it instead an 'imposter' ?

(b) The propagation vector $\mathbf{k} = (k_x,\, k_y,\, k_z)$ is defined as the change of phase in radians per unit length along the direction of propagation of the wave. Suppose we try to define a 'wavelength vector' through the following equation :

$$\mathbf{A} = (\nu\lambda_x,\, \nu\lambda_y,\, \nu\lambda_z)$$

where ν is the frequency of the wave and $\lambda_x \equiv (2\pi/k_x)$ etc. Can $\mathbf{A}$ be accepted as a vector ?

Problem 7. (a) If you comb your hair on a dry day, the comb readily picks up small scrapes of paper; why ? Also explain why sometimes the paper comes upto the comb and then flies away ? (b) If a magnet is brought near iron filings, they jump and stick to the magnet. This suggests that the magnet can impart kinetic energy to the iron filings. But can magnetic force do work?

Problem 8. A thin uniform metal disk lies on an infinite conducting plane. A uniform gravitational field is oriented normal to the plane. Initially the disc and plane are uncharged. Charge is now slowly added. What value of charge density is required to cause the disk to "levitate" ?

Problem 9. A charge Q is located h cm above a conducting plane. Asked to predict the amount of work that would have to be done to move this charge out to infinite distance from the plane, one student says that it is the same as the work required to seperate to infinite distance two charges Q and $-Q$ which are initially $2h$ cm apart, hence $W = Q^2/2h$. Another student calculates the force that acts on the charge as it is being moved and integrates Fdx, but gets a different answer. What did the second student get, and who is right?

Problem 10. A standard result in electrostatics is that a point charge cannot be held in stable equilibrium under the action of electrostatic forces. Prove this by showing that the electrostatic potential cannot have a minimum at a point in space where there are no charges.

Problem 11. In section 5.10 of Feynman lectures in physics Vol II, there is a discussion regarding shielding of electric fields by conductors. Feynman says " ... The same arguments can be used to show that no static distribution of charges *inside* a closed conductor can produce any fields *outside*. Shielding works both ways ! ..." . Do you think Feynman is correct? Give reasons for your answer.

Problem 12. Let $\mathbf{E_1}$ and $\mathbf{E_2}$ be the electric fields producced by two point charges q_1 and q_2 kept at a distance b apart. Find the value of the integral

$$I = \int_{all\,space} \mathbf{E_1} \cdot \mathbf{E_2} \; d^3\mathbf{x}.$$

[Hint : All the problems in this assignment are very simple !]

Problem 13. What is the drift speed of the conduction electrons in a copper wire carrying a current of $1.3A$? The diameter of the copper wire can be taken to be $1.8mm$. Can the drift speed you just calculated explain why room lights turn on so quickly when the switch is flipped on? If it cannot, what is the right explanation?

The molar mass and density of copper are $64 \times 10^{-3}\; kg/mol$ and $9.0 \times 10^3\; kg/m^3$ respectively.

Problem 14. An electric eel kills fish on which it preys with current pulses as high as an ampere. It does this by producing a potential difference of several hundred volts along its length which causes current to flow from near its head to its tail through the surrounding water. The eel generates this current with biological cells called electroplaques which are physiological emf devices. The electroplaques in the South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. Each electroplaque has an emf of $0.15V$ and an internal resistance r of 0.25Ω. (a) If water has a resistance $R_w = 800\Omega$, how much current can the eel send through the water from near its head to its tail? (b) How much current i_{row} travels through each row? Can you now explain how the eel manages to produce such a large current without the danger of electrocuting itself?

Problem 15. In a pin-hole camera the distance of the pin-hole from the photographic place is 10 cm. You want to take a picture of the Sun in the visible spectrum ($\lambda \equiv 5000°$A). What diameter of the pin-hole should you use in order to obtain the sharpest resolution?

Problem 16. A microwave detector is located at the shore of a lake 0.5 m above the water level. As a radio-star emitting monochromatic waves rises slowly above the horizon, the detector indicates successive maxima and minima of signal intensity. At what angle θ above horizon is the radio star when the first maximum is received?

Problem 17. Find the index of refraction of Aluminium for X-rays (of wavelength 1.56×10^{-8} cm). Assume that all the electrons in Aluminium have natural frequencies very much less than the frequency of X-rays.

Problem 18. A flywheel of radius R, with charge Q distributed uniformly along the rim rotates with angular velocity ω. What is the rate at which energy is radiated by the system? [Hint: Read the hint in problem 12].

Problem 19. Find the r.m.s. electric field (in volts/cm) above the earth's atmosphere, due to sunlight. The solar constant (defined for a point above the earth's atmosphere) is 1.94 cal/sq.cm/min. (1 cal = 4.18 J).

Problem 20. A total charge $-Q$ is spread uniformly on the surface of a thin *insulating* spherical shell of radius a. Clearly, the electric field inside is zero. A charge $+Q$ is now placed at the center. The electric field outside is now zero. We now move the inside charge by a distance x inside (with $x < a$). This can be done without the expenditure of any energy because the field due to charge on the surface is zero inside (of course, charges are not expected to exert self-force). However, the system now has a dipole moment Qx and the field outside is nonzero. How do you explain the appearance of external field energy, since no work was done in moving the charge?

Problem 21. Why do we see clouds? You might say we see clouds because the water vapour has condensed; but why do we see the water vapour only after condensation? Is there any relation between the colour of the cloud and the size of the water drop in it ?

Problem 22. Consider an unpolarized beam of light falling on a scatterer. Give an argument to show that the light scattered at $90°$ to the incident beam is plane-polarized.
